Lect. 2 Maxwell's Eq.s and Wave Eq.

- Derivation of wave equation

✓ Assumptions

1) source free → $\rho = 0,\ \overline{J} = 0$

2) uniform medium → $\varepsilon, \mu \neq f(x, y, z)$, constant.

✓ From Maxwell's Equations, $\nabla \times \overline{E} = -\dfrac{\partial \overline{B}}{\partial t}$

✓ 양변에 curl을 취하면

$$\nabla \times \left(\nabla \times \overline{E}\right) = -\nabla \times \left(\frac{\partial \overline{B}}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla \times \overline{B}) = -\mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2}$$

Lect. 2 Maxwell's Eq.s and Wave Eq.

✓ Source free라는 가정과 vector identity에 의해 좌변은

$$\nabla \times \left(\nabla \times \overline{E}\right) = \nabla\left(\nabla \cdot \overline{E}\right) - \nabla^2 \overline{E} = -\nabla^2 \overline{E} = -\mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2}$$

$$\Rightarrow \quad \nabla^2 \overline{E} = \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2}$$ Wave Equation!

- Solution of wave equation(E-field)

✓ Assumptions

1) E field has no x, y dependence → $\overline{E} = \overline{E}(z,t)$

2) E field has x-component only. → $\overline{E} = \overline{x}E(z,t)$

Lect. 2 Maxwell's Eq.s and Wave Eq.

✓ 가정에 의해, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ ⇨ $\nabla^2 = \frac{\partial^2}{\partial z^2}$

✓ 예측한 해를 wave eq.에 대입하여 정리하면,

$$\frac{\partial^2}{\partial z^2} E(z,t) = \mu\varepsilon \frac{\partial^2}{\partial t^2} E(z,t)$$

✓ 위의 방정식은 다음과 같은 형태의 해를 가진다.

$E(z,t) = f(\alpha),\ \alpha = t - \sqrt{\mu\varepsilon} \cdot z,\ f(\alpha)$는 임의의 함수

✓ 구해진 해를 식에 대입하여 보면

$$\frac{\partial^2}{\partial z^2} E(z,t) = \frac{\partial}{\partial z}\left(\frac{\partial \alpha}{\partial z} f'(\alpha)\right) = \frac{\partial}{\partial z}\left(-\sqrt{\mu\varepsilon}\, f'(\alpha)\right) = \mu\varepsilon \cdot f''(\alpha)$$ → left side

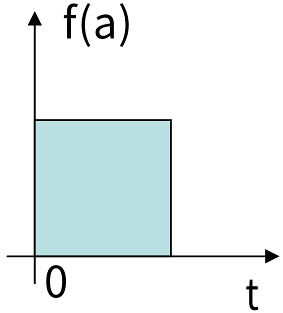
Lect. 2 Maxwell's Eq.s and Wave Eq.

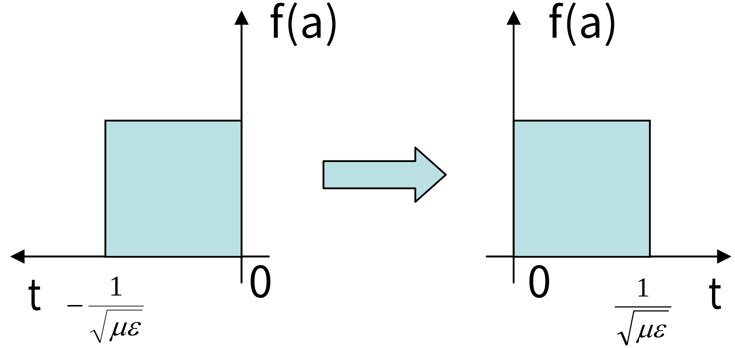
$$\mu\varepsilon\frac{\partial^2}{\partial t^2}E(z,t)=\mu\varepsilon\frac{\partial}{\partial t}\left(\frac{\partial\alpha}{\partial t}f^{'}(\alpha)\right)=\mu\varepsilon\cdot f^{''}(\alpha)$$ → Right side

✓ 좌변과 우변이 같게 되어 방정식을 만족하는 것을 확인할 수 있다.

✓ $f(\alpha),\ \alpha=t-\sqrt{\mu\varepsilon}\cdot z$ 을 다음과 같이 가정하고 시간의 흐름에 따라 관찰해보면

$$\begin{cases} f(\alpha)=1, & 0<\alpha<1 \\ f(\alpha)=0, & \text{otherwise} \end{cases}$$





① t=0일때 ② t=1일때

※ f(a) 형태의 wave가 z축 방향으로 이동

Lect. 2 Maxwell's Eq.s and Wave Eq.

✓ Wave의 전파속도?

→ 속도= 시간/거리

$$v = \frac{1}{\sqrt{\mu\varepsilon}} \Big/ 1 = \frac{1}{\sqrt{\mu\varepsilon}} = C, \text{ speed of light}$$

✓ 그렇다면 wave는 항상 z축 방향으로만 전파되는가?

$$f(\alpha), \begin{cases} \alpha = t - \sqrt{\mu\varepsilon}\cdot z, \ propagates\ along\ +z\ axis \\ \alpha = t + \sqrt{\mu\varepsilon}\cdot z, \ propagates\ along\ -z\ axis \end{cases}$$

→ 두가지 모두 $\frac{\partial^2}{\partial z^2}E(z,t) = \mu\varepsilon \frac{\partial^2}{\partial t^2}E(z,t)$ 를 만족하는 해이다.

HW1) Maxwell방정식으로부터 $\nabla^2 \overline{H} = \mu\varepsilon \frac{\partial^2 \overline{H}}{\partial t^2}$ 를 유도하시오.