Analyzing Nuclear Magnetic Resonance and their application to Magnetic Resonance Imaging (MRI) Systems

Abstract
In this document, we focus on analyzing and representing the operating theory of nuclear magnetic resonance. Magnetic resonance imaging (MRI) is used by clinicians to produce tomographic images of the inside of the human body. MRI is also used by scientists to study materials as it is a non-destructive imaging technique. It does no harm to human body since this system does not use ionized radiation like X-rays. Magnetic resonance imaging is an analytical technique based on a property of matter called spin that is based on the absorption and emission of energy in the radio frequency range of the electromagnetic spectrum.

Introduction
Before the development of Magnetic Resonance Imaging, the major way for imaging the internal human body was X-ray based Computer tomography, which we call CT. When CT was first brought out, it was a revolution on imaging body parts which X-ray method could not express. But CT was only capable for transverse photographing. For enabling more accurate examination for the medical doctors, 3-dimensional image was needed, which was impossible with the Computer tomography based on using X-rays. On account of this reason, the method for providing a 3-dimensional image of the internal human body was highly requested, and that leaded the development of Magnetic Resonance Imaging system as an application to Nuclear Magnetic Resonance phenomenon which was used for chemical and physical molecular analysis. Magnetic resonance imaging (MRI) is an imaging technique used primarily in medical settings to produce high quality images of the inside of the human body. MRI is based on the principles of nuclear magnetic resonance (NMR), a spectroscopic technique used by scientists and medical doctors to obtain microscopic chemical and physical information about molecules. The technique was called magnetic resonance imaging rather than nuclear magnetic resonance imaging (NMRI) because of the negative connotations associated with the word nuclear in the late 1970's. MRI started out as a tomographic imaging technique, that is it produced an image of the NMR signal in a thin slice through the human body. MRI has advanced beyond a tomographic imaging technique to a volume imaging technique.

i. Spin Characteristics
Spin phenomenon can be the critical factor in understanding the MRI analysis. Spin is a fundamental property of nature that is trying to maintain the energy at the minimum level. (Fig. A) One of simple example for this phenomenon can be the rotation of the earth during the revolution around the sun. Spin comes in multiples of 1/2 and can be + or -. Protons, electrons, and neutrons possess spin. Individual unpaired electrons, protons, and neutrons each possess a spin of 1/2. For MRI systems, deuterium atom is used to observe the spin phenomenon since 80% of human body is consisted of water. In the deuterium atom (²H), with one unpaired electron, one unpaired proton, and one unpaired neutron, the total electronic spin = 1/2 and the total nuclear spin = 1. Two or more particles with spins having opposite signs can pair up to eliminate the observable manifestations of spin. An example is helium. In nuclear magnetic resonance, it is unpaired nuclear spins that are of importance. When atom is placed in a magnetic field of strength B, a particle with a net spin can absorb a photon, of frequency ω. The frequency ω depends on the gyromagnetic ratio, γ of the particle.

$$\omega = \gamma B$$
For hydrogen, $\gamma = 42.58 \text{ MHz} / \text{T}$. The shell model for the nucleus tells us that nucleons, just like electrons, fill orbital. When the number of protons or neutrons equals 2, 8, 20, 28, 50, 82, and 126, orbital are filled. Because nucleons have spin, just like electrons do, their spin can pair up when the orbital are being filled and cancel out. Almost every element in the periodic table has an isotope with a non zero nuclear spin. NMR can only be performed on isotopes whose natural abundance is high enough to be detected. Considering a proton, this proton has the property called spin. Think of the spin of this proton as a magnetic moment vector, causing the proton to behave like a tiny magnet with forming a north and a south pole. When the proton is placed in an external magnetic field, the spin vector of the particle aligns itself with the external field, just like a magnet would. This particle can undergo a transition between the two energy states by
the absorption of a photon. A particle in the lower energy state absorbs a photon and ends up in the upper energy state. The energy of this photon must exactly match the energy difference between the two states. The energy, \( E \), of a photon is related to its frequency, \( \omega \), by Plank’s constant (\( h = 6.626 \times 10^{-34} \) J s). (Fig.B)

\[
E = h \omega
\]

In NMR and also in MRI, the quantity \( \omega \) is called the resonance frequency and the Larmor frequency. The energy of the two spin states can be represented by an energy level diagram. We have seen that \( \omega = \gamma B \) and \( E = h \omega \), therefore the energy of the photon needed to cause a transition between the two spin states can be expressed as shown below.

\[
E = h \gamma B \quad (\text{Fig.1})
\]

When the energy of the photon matches the energy difference between the two spin states absorption of energy occurs. In the NMR experiment, the frequency of the photon is in the radio frequency (RF) range. In NMR spectroscopy, \( \omega \) is between 60 and 800 MHz for hydrogen nuclei. In clinical MRI, \( \omega \) is typically between 15 and 80 MHz for hydrogen imaging. When a group of spins is placed in a magnetic field, each spin aligns in one of the two possible orientations. (Fig.2) The number of spins in the lower energy level, \( M' \), slightly outnumbers the number in the upper level, \( M \). Boltzmann statistics can be written as shown below.

\[
M/M' = e^{E/kT}
\]

\( E \) is the energy difference between the spin states; \( k \) is Boltzmann’s constant, \( 1.3805 \times 10^{-23} \) J/Kelvin; and \( T \) is the temperature in Kelvin. As the temperature increases, the ratio approaches to 1. The signal in NMR spectroscopy results from the difference between the energy absorbed by the spins which make a transition from the lower energy state to the higher energy state, and the energy emitted by the spins which simultaneously make a transition from the higher energy state to the lower energy state. The signal is thus proportional to the population difference between the states. NMR is a rather sensitive spectroscopy since it is capable of detecting these very small population differences. It is the resonance, or exchange of energy at a specific frequency between the spins and the spectrometer, which gives NMR its sensitivity. Though there are 2 factors, the natural abundance of the isotope and biological abundance that influence the result of MRI signal, the influences of these factors are small enough to ignore. The natural abundance of an isotope is the fraction of nuclei having a given number of protons and neutrons, or atomic weight. For example, there are three isotopes of hydrogen which we use at the MRI system, \(^1\)H, \(^2\)H, and \(^3\)H. At this time, the natural abundance of \(^1\)H is 99.985%. The biological abundance is the fraction of one type of atom in the human body. Since a number of spins acts identically in the exposed magnetic field, we can observe the MRI in a macroscopic way. A spin packet is a group of spins experiencing the same magnetic field strength. At any instant in time, the magnetic field due to the spins in each spin packet can be represented by a magnetization vector, and the size of each vector is proportional to \( (N' - N) \). The vector sum of the magnetization vectors from all of the spin packets is the net magnetization. (Fig.3) Adapting the conventional NMR coordinate system, the external magnetic field and the net magnetization vector at equilibrium are both along the Z axis. (Fig.4)

ii. \( T_1 \) and \( T_2 \) Processes

The main concept of MRI is analyzing the energy wave emitted from nucleus. To understand to method, we need to validate some very important process which is called \( T_1 \) and \( T_2 \) Process. Vertically placed net magnetization in the XY plane rotates about the Z axis at a frequency equal to the frequency of the photon which would cause a transition between the two energy levels of the spin. This is called the Precession and this frequency is called as the Larmor frequency. Electron rotates about the nucleus of the atom, but the direction of the rotation axis changes the spin. This is called the Precession and this frequency is called as the Larmor frequency.

\[
E = E_o + V, \quad V = -\mu B \cos
\]

When magnetic wave with frequency \( f_o \) that satisfies \( hf_o = E_o E_i \), the hydrogen atom absorbs the wave and the potential level of the energy becomes to \( E_2 \) from \( E_1 \) which is unstable, which is called Resonance absorption. As a matter of fact, the nearer the shock of this magnetic field is to Larmor frequency, the smoother the transfer of energy is. At the equilibrium condition, the net magnetization vector lies along the direction of the applied magnetic field \( B_o \) and is called the equilibrium magnetization \( M_e \). It is possible to change the net magnetization by exposing the nuclear spin system to energy of a frequency equal to the energy difference
between the spin states. If enough energy is put into the system, it is possible to saturate the spin system and make longitudinal magnetization $M_2 = 0$. (Fig.5) The time constant which describes how $M_1$ returns to its equilibrium value is called the spin lattice relaxation time ($T_1$). The equation governing this behavior as a function of the time $t$ after its displacement is shown as below.

$$M_1 = M_e (1 - e^{-t/T_1}) \text{ (Fig.6)}$$

$T_1$ is therefore defined as the time required for changing the vertical component of magnetization by a factor of e. If the net magnetization is placed along the -Z axis, it will gradually return to its equilibrium position along the +Z axis at a rate governed by $T_1$. (Fig.7) through this process, the final equation governing this behavior as a function of the time $t$ after its displacement is shown as below.

$$M_1 = M_e (1 - 2e^{-t/T_1}) \text{ (Fig.7)}$$

$T_1$ is therefore the time to reduce the difference between the longitudinal magnetization ($M_2$) and its equilibrium value by a factor of e. Besides the above factors, $T_1$ is decided by various kind element but existence of paramagnetic ion or molecule are most important among them. Besides the rotation, the net magnetization starts to dephase because each of the spin packets made up is experiencing a slightly different magnetic field and rotates at its own Larmor frequency. The longer the elapsed time flows, there is greater phase difference. As $T_2$ become longer, the phase scattering effect becomes slower, and it means the capability of releasing the continuous signal is becoming greater. The time constant which describes the return to equilibrium status of the transverse magnetization, $M_{XY}$, is called the spin-spin relaxation time, which is $T_2$. And $T_2$ is always less than or equal to $T_1$.

$$M_{XY} = M_{XY0} e^{-t/T_2} \text{ (Fig.8)}$$

The net magnetization in the XY plane goes to zero and then the longitudinal magnetization grows in until $M_0$ is parallel to Z axis. (Fig.9) The transverse component rotates about the direction of applied magnetization and dephases.

The core concept of $T_1$, $T_2$ is that, $T_1$ governs the rate of recovery of the longitudinal magnetization. Therefore, the spin-spin relaxation time, $T_2$, is the time to reduce the transverse magnetization by a factor of e, and this is defined as the time of signal decreased to 37% compared to the initial signal. In the previous sequence, $T_2$ and $T_1$ processes are shown separately for clarity. That is, the magnetization vectors are shown filling the XY plane completely before growing back up along the Z axis. Actually, both processes occur simultaneously with the only restriction being that $T_2$ is less than or equal to $T_1$. A coil of wire placed around the X axis provides magnetic field along the X axis when a direct current is passed through the coil. An alternating current produces a magnetic field which alternates in direction. In magnetic resonance, the magnetic field created by the coil passing an alternating current at the Larmor frequency is called the $B_1$ magnetic field. When the alternating current through the coil is turned on and turned off, it creates a pulsed $B_1$ magnetic field along the X axis. The spins respond to this pulse in such a way as to cause the net magnetization vector to rotate about the direction of the applied $B_1$ field. The rotation angle depends on the length of time the field is on $\tau$, and its magnitude $B_1$.

$$\Theta = 2\pi \gamma \tau B_1$$

A 90° pulse is one which rotates the magnetization vector clockwise by 90 degrees about the X' axis. A 90° pulse rotates the equilibrium magnetization down to the Y' axis. A 180° pulse will rotate the magnetization vector by 180 degrees. A 180° pulse rotates the equilibrium magnetization down to the -Z axis. By utilizing and controlling the $T_1$ and $T_2$, medical doctors are able to examine the specific area in the body in a specific form. These methods are called NMR spectroscopy, and there are The Time Domain NMR Signal, The +/- Frequency Convention, The 90-FID Sequence, The Spin-Echo Sequence, The Inversion Recovery Sequence. We will abridge the further research about the individual methods since it does not have to do with understanding the fundamentals of MRI.

### iii. Image Gaining

Gathering the information through $T_1$ and $T_2$ processes and NMR spectroscopy, does not provide the proper images for medical doctors to examine the patient's body. In order to produce the tomographic images of the inside of the human body, a number of mathematical formula and hardware is required. Through Fourier Transform of original signal and hardware such as Magnet, Gradient coils, RF coils, and Quadrature detector, MRI phantom, we can get a clear tomographic image. As a brief description of the hardware, the gradient coils produce the gradients in the $B_0$ magnetic field, and RF coils create the $B_1$ field which rotates the net magnetization in a pulse sequence. They also detect the transverse magnetization as it acts
precession in the XY plane. The quadrature detector is a device which separates out the $M_x$ and $M_y'$ signals from the signal from the RF coil. An MRI phantom is an anthropogenic object that can be imaged to test the performance of the magnetic resonance imaging system. (Fig.10)

iv. Conclusion
MRI is the device that provides the images of human body in a 3-dimensional view based on the spin phenomenon that nucleus radiates potential energy when specific magnetic wave is exposed to the electrons. By the different amount of the radiated potential energy, we are able to image the specific part of the body we want to examine. Since, MRI is harmless, and capable in imaging every part of the human body in various forms, MRI system can be regarded as a revolutionary device in the medical field, which utilized the fundamental Electromagnetic knowledge. As we can assume that the need of medical imaging equipment will increase, since imaging is a critical issue in the medical field. But due to the high price problem, optimizing the increase of the market size is obscure. But considering human nature of interest in health, it is inevitable that continuous R&D in this area will be done in the future.

Reference
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