What happens step voltages are applied to \( G \) with \( S, B \) grounded?

It takes time to charge up voltages!

\( \Rightarrow \) capacitive effects between

\( G \) and \( S \) (\( C_{GS} \)), \( G \) and \( D \) (\( C_{GD} \)), and many other places
In saturation,

\[ C_{GS} = \frac{2}{3} WLC_{ox} + WL_{ov}C_{ov} \quad \text{(about 20fF for L=1.0\mu m)} \]

\[ C_{GD} = WL_{ov}C_{ov} \quad \text{(about 2fF for L=1.0\mu m)} \]

\[ C_{GS} > C_{GD} \]
Lect. 13: High-Frequency Model of MOSFET

How does a capacitor affect frequency response of a circuit?

Consider an RC circuit

\[
\frac{v_i(t) - v_o(t)}{R} = i(t) = C \frac{dv_o(t)}{dt}
\]

\[
\frac{dv_o(t)}{dt} + \frac{v_o(t)}{RC} = \frac{v_i(t)}{RC}
\]

If \(v_i(t) = V_i \exp(j\omega t)\), \(v_o(t) = V_o \exp(j\omega t)\) (why?)

\(V_o = ?\)

\(j\omega \cdot V_o \exp(j\omega t) + \frac{V_o \exp(j\omega t)}{RC} = \frac{V_i \exp(j\omega t)}{RC}\)

\(V_o(j\omega + \frac{1}{RC}) = V_i \frac{1}{RC}\)

\(V_o = V_i \frac{1}{j\omega C}\)

\(\frac{1}{R + \frac{1}{j\omega C}}\)

Voltage divider with capacitor impedance of \(\frac{1}{j\omega C}\)!
For any circuit having capacitors and inductors, use $Z = \frac{1}{j\omega C}$ (or $\frac{1}{sC}$) for capacitor impedance and $Z = j\omega L$ (or $sL$) for inductor impedance.

\[ \frac{V_o(\omega)}{V_i(\omega)} \quad \text{(or)} \quad \frac{V_o(s)}{V_i(s)} = ?; \quad \text{Frequency response, System function, Transfer function} \]

\[ \frac{V_o(\omega)}{V_i(\omega)} = \frac{1}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\frac{\omega}{\omega_0}} \quad (\omega_0 = \frac{1}{RC}) \]

\[ \left| \frac{V_o(\omega)}{V_i(\omega)} \right| = \frac{1}{\sqrt{1 + \frac{\omega^2}{\omega_0^2}}} , \quad \angle \frac{V_o(\omega)}{V_i(\omega)} = -\tan^{-1} \frac{\omega}{\omega_0} \]
Lect. 13: High-Frequency Model of MOSFET

Bode Plots

\[
\frac{V_o(\omega)}{V_i(\omega)} = \frac{1}{\frac{j\omega C}{R} + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\frac{\omega}{\omega_0}} \quad (\omega_0 = \frac{1}{RC})
\]

\[
\left| \frac{V_o(\omega)}{V_i(\omega)} \right| = \frac{1}{\sqrt{1 + \omega^2 / \omega_0^2}}, \quad \angle \frac{V_o(\omega)}{V_i(\omega)} = -\tan^{-1} \frac{\omega}{\omega_0}
\]

\[
f_{3\text{-dB}} = \frac{\omega_0}{2\pi}
\]
Lect. 13: High-Frequency Model of MOSFET

How fast can a MOSFET transistor operate?

Unit-Gain Frequency ($f_t$):
Frequency at which magnitude of the short-circuit current gain of CS configuration becomes 1

Or $\text{Mag}(I_o(\omega)/I_i(\omega)) = 1$

Why $V_{gs}$, not $v_{gs}$? Frequency-domain analysis
How fast can a MOSFET transistor operate?

\[ I_0 = g_m V_{gs} - \frac{V_{gs}}{1/j\omega C_{gd}} = g_m V_{gs} - j\omega C_{gd} V_{gs} \approx g_m V_{gs} \ (\because g_m >> \omega C_{gd}) \]

\[ V_{gs} = I_i \cdot \left( \frac{1}{j\omega C_{gd}} || \frac{1}{j\omega C_{gs}} \right) = I_i \cdot \frac{1}{j\omega (C_{gd} + C_{gs})} \]

\[ \frac{I_0}{I_i} = \frac{g_m}{j\omega (C_{gd} + C_{gs})} \]

For \[ \left| \frac{I_0}{I_i} \right| = 1, \quad \omega = \frac{g_m}{C_{gd} + C_{gs}} \]

Or \[ f_T = \frac{g_m}{2\pi(C_{gd} + C_{gs})} \] (Unit-gain Frequency)

How to make MOSFET faster?
Which is faster NMOS or PMOS?
Current state-of-the-art NMOS has \( f_T \) approaching 100 GHz.
Lect. 13: High-Frequency Model of MOSFET

How fast can a MOSFET transistor operate?

\[ I_0 = g_m V_{gs} - \frac{V_{gs}}{1/sC_{gd}} = g_m V_{gs} - j\omega C_{gd} V_{gs} \approx g_m V_{gs} \quad (\because g_m >> \omega C_{gd}) \]

\[ V_{gs} = I_i \cdot \left( \frac{1}{j\omega C_{gd}} \parallel \frac{1}{j\omega C_{gs}} \right) = I_i \cdot \frac{1}{j\omega (C_{gd} + C_{gs})} \]

\[ \therefore \frac{I_0}{I_i} = \frac{g_m}{j\omega (C_{gd} + C_{gs})} \]

For \[ \left| \frac{I_0}{I_i} \right| = 1, \quad \omega = \frac{g_m}{C_{gd} + C_{gs}} \]

Or \[ f_T = \frac{g_m}{2\pi(C_{gd} + C_{gs})} \] (Unit-gain Frequency)

How to make MOSFET faster?
Which is faster NMOS or PMOS?
Current state-of-the-art NMOS has \( f_T \) approaching 100 GHz.
Homework: (Due before Tutorial on 10/17)
For the circuit given below,
1. Determine the transfer function $V_i(\omega)/V_s(\omega)$.
2. Plot magnitude and phase of $V_i(w)/V_s(w)$ in dB scale (Bode plot).
3. What is the 3-dB frequency?