Lect. 14: Second-Order Passive Filters (S&S 12.5)

2nd-order filter $$T(s) = \frac{a_2 S^2 + a_1 S^1 + a_0}{S^2 + b_1 S^1 + b_0} = \frac{a_2 (s - z_1)(s - z_2)}{(s - p_1)(s - p_2)}$$

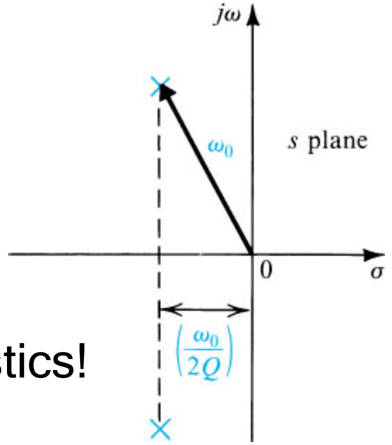
If p_1, p_2 are real,

➔ 2nd-order filters can be easily made by cascading two 1st-order filters

Consider T(s) with complex poles $$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + (\omega_0 / Q)s + \omega_0^2}$$

$$p_1, p_2 = -\frac{\omega_0}{2Q} \pm j\omega_0 \sqrt{1 - (1/4Q^2)}$$

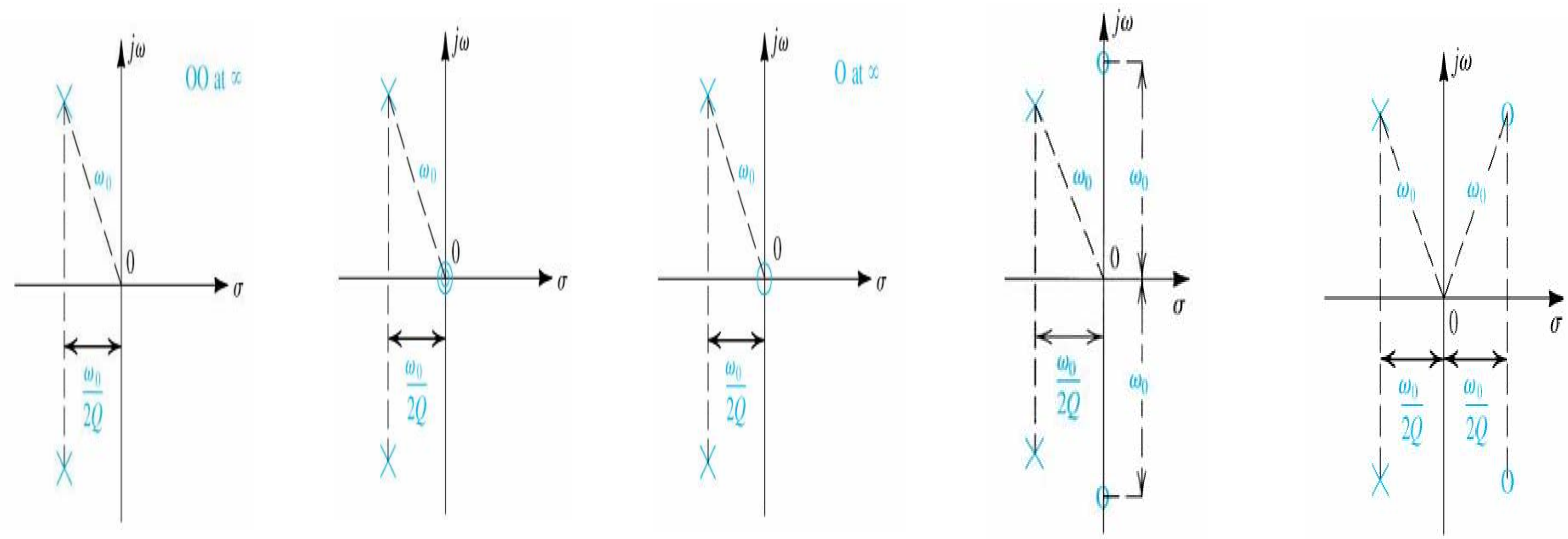
(Q > ½)



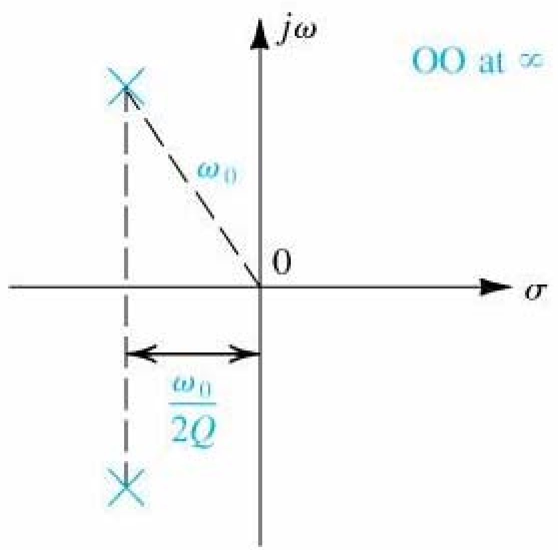
Depending on zero locations, various filter characteristics!

Lect. 14: Second-Order Passive Filters

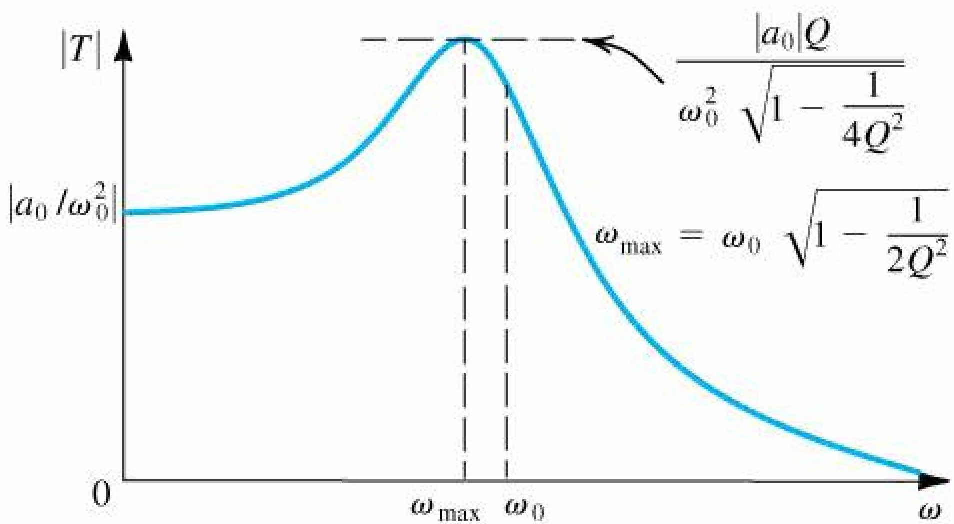
Second-order filter pole-zero diagrams



Lect. 14: Second-Order Passive Filters



$$T(s) = \frac{a_0}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$



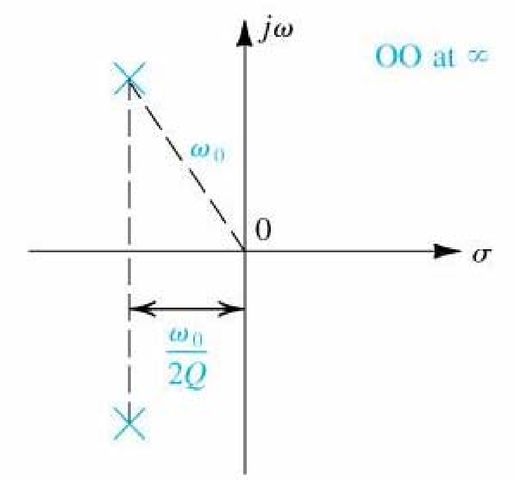
LP Filter

At $\omega=\omega_0$, $|T| = |a_0/\omega_0^2|\, Q$

Larger Q ➔ More peaking

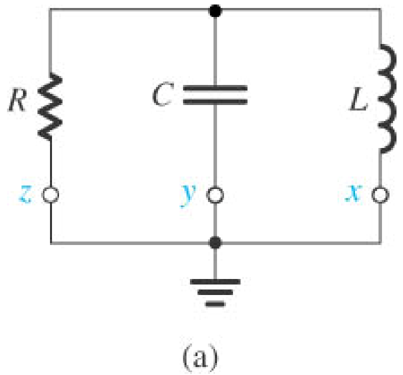
Lect. 14: Second-Order Passive Filters

How to realize passive second-order LP filter

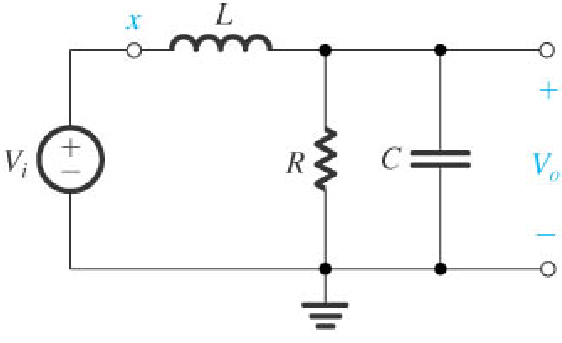


$$T(s) = \frac{a_0}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

Use LCR



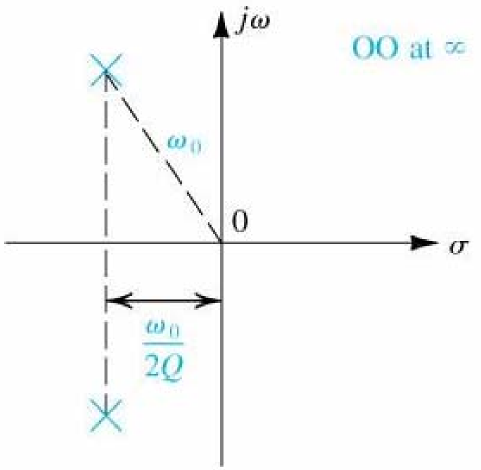
(a)

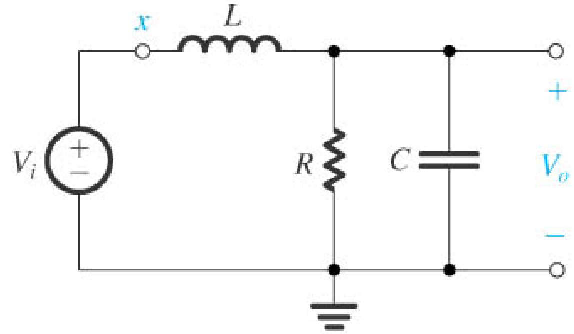


(b) LP

➔ Two zeros at s infinity

How to realize passive second-order LP filter





(b) LP

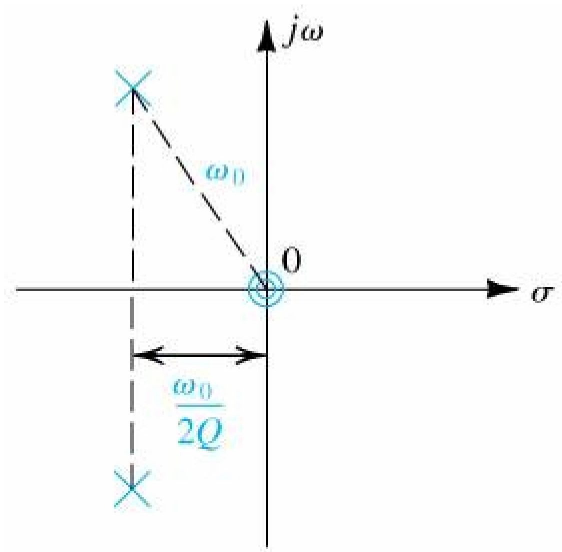
$$T(s) = \frac{a_0}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$T(s) \equiv \frac{\mathbf{V_o}}{\mathbf{V_i}} = \frac{Z_2}{Z_1 + Z_2} = \frac{Y_1}{Y_1 + Y_2} = \frac{1/sL}{(1/sL) + sC + (1/R)}$$

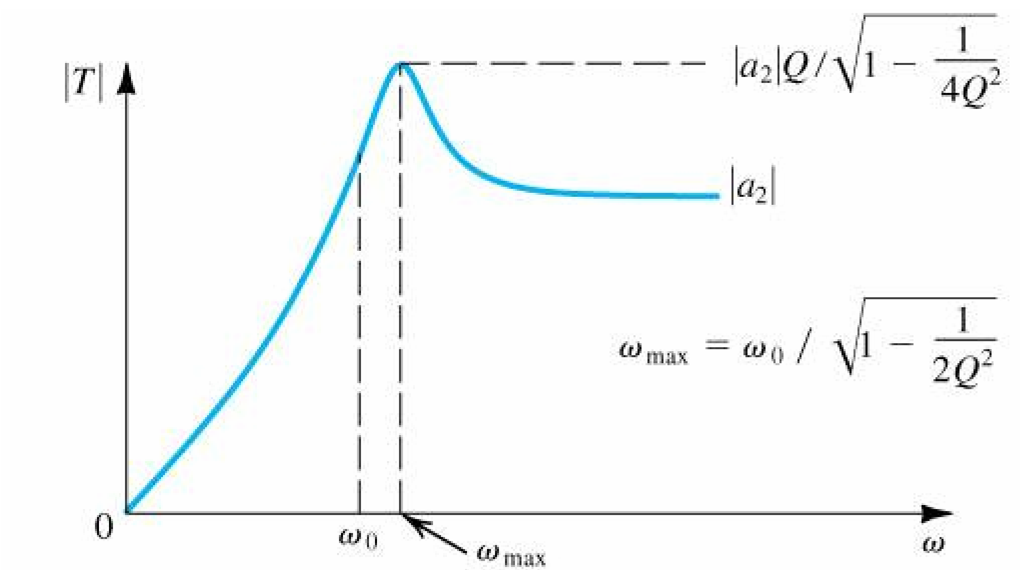
$$= \frac{1/LC}{s^2 + s(1/CR) + (1/LC)} = \frac{\omega_0^{\ 2}}{s^2 + \frac{\omega_0}{Q}s + \omega_0^{\ 2}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad \frac{\omega_0}{Q} = \frac{1}{CR} \qquad Q = \omega_0 CR = \sqrt{\frac{C}{L}}R$$

Lect. 14: Second-Order Passive Filters

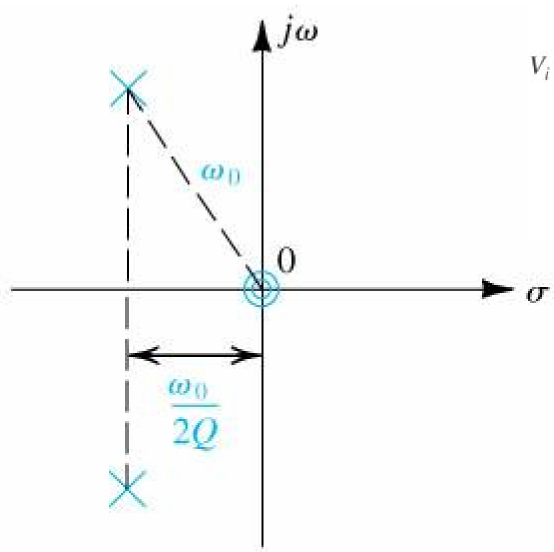


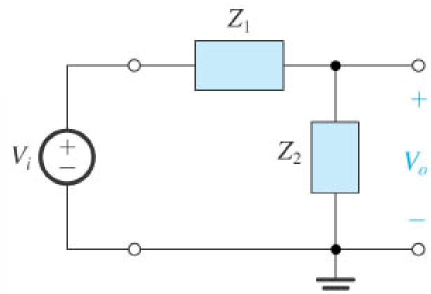
$$T(s) = \frac{a_2 s^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$



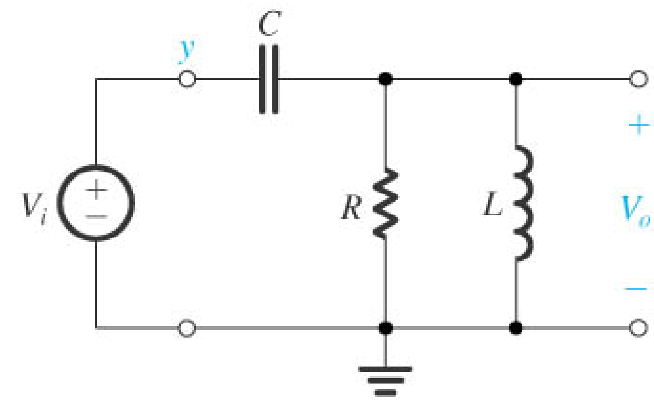
HP Filter

Lect. 14: Second-Order Passive Filters





Two zeros at s=0



(c) HP

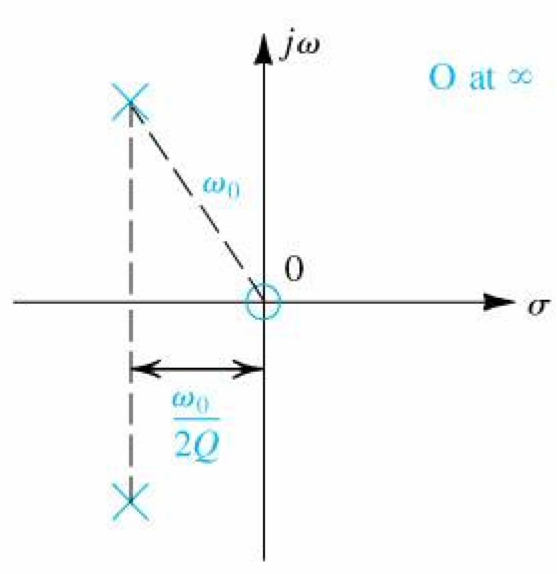
$$T(s) \equiv \frac{\mathbf{V_o}}{\mathbf{V_i}} = \frac{Z_2}{Z_1 + Z_2} = \frac{Y_1}{Y_1 + Y_2} = \frac{sC}{sC + (1/R) + (1/sL)}$$

$$= \frac{s^2}{s^2 + s(1/CR) + (1/LC)}$$

$$T(s) = \frac{a_2 s^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

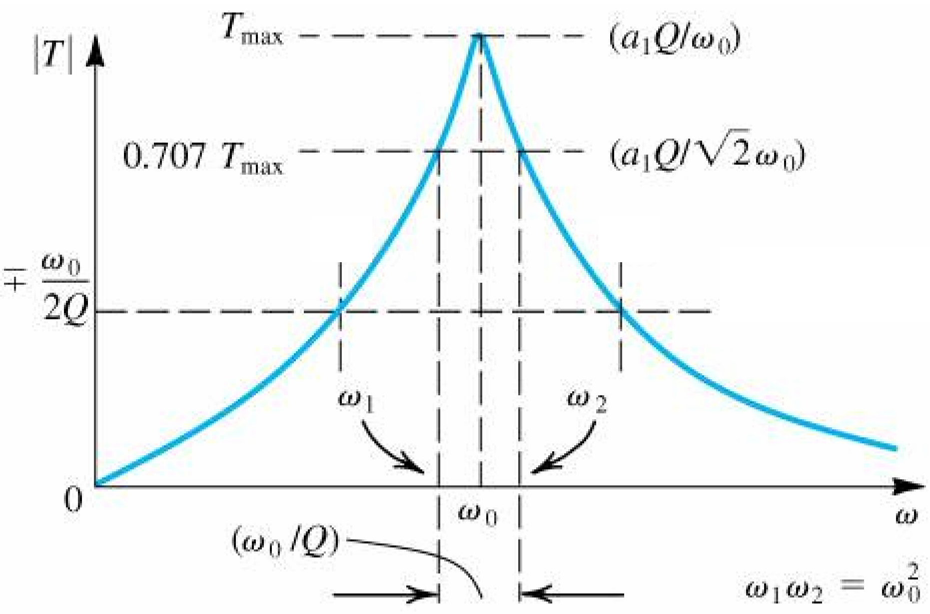
Same pole characteristics as LP ➔ Same natural modes

Lect. 14: Second-Order Passive Filters



$$T(s) = \frac{a_1 s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$\omega_1, \omega_2 = \omega_0\sqrt{1 + \frac{1}{4Q^2}} \mp \frac{\omega_0}{2Q}$$

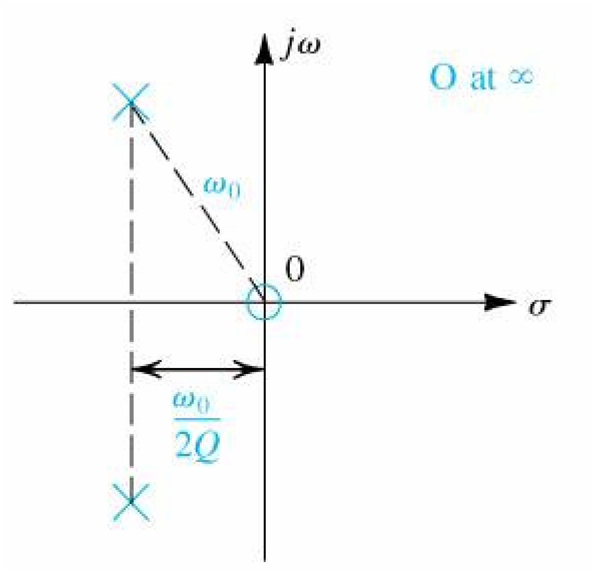


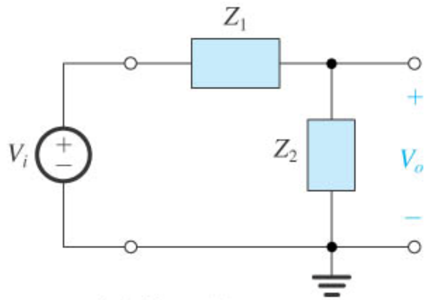
3dB bandwidth

BP Filter

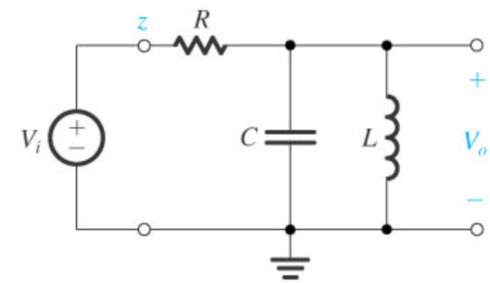
Large Q ➔ Sharper response

Lect. 14: Second-Order Passive Filters





(a) General structure



(d) BP

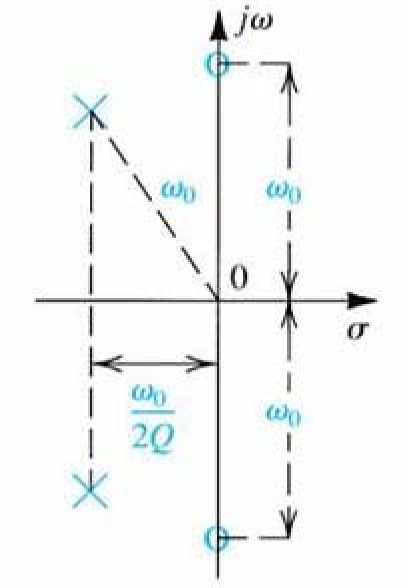
$$T(s) = \frac{a_1 s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

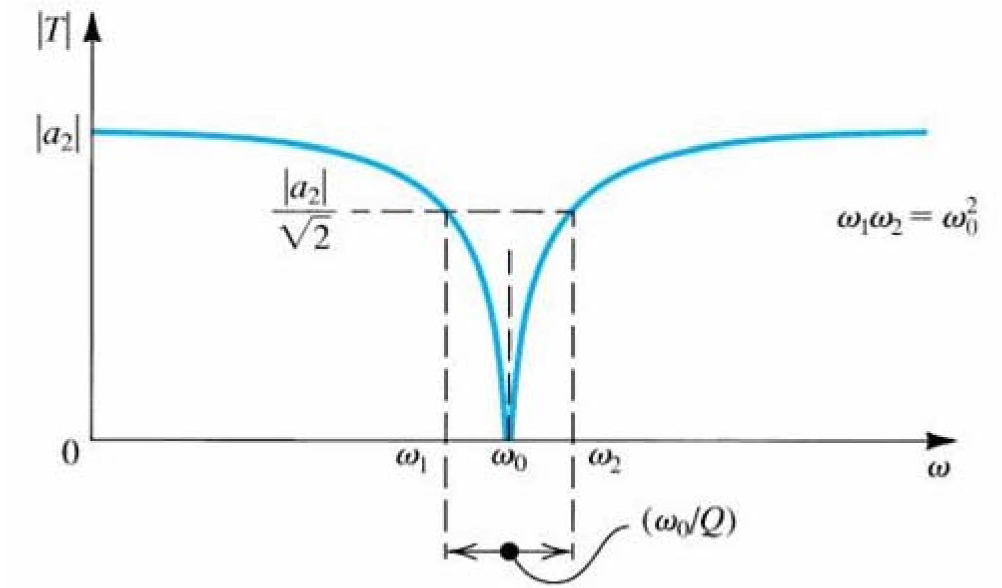
$$T(s) = \frac{\mathbf{V_o}}{\mathbf{V_i}} = \frac{Y_1}{Y_1 + Y_2} = \frac{1/R}{(1/R) + sC + (1/sL)}$$

$$= \frac{s(1/RC)}{s^2 + s(1/RC) + (1/LC)}$$

$$Q = \omega_0 CR = \sqrt{\frac{C}{L}}R$$

Lect. 14: Second-Order Passive Filters

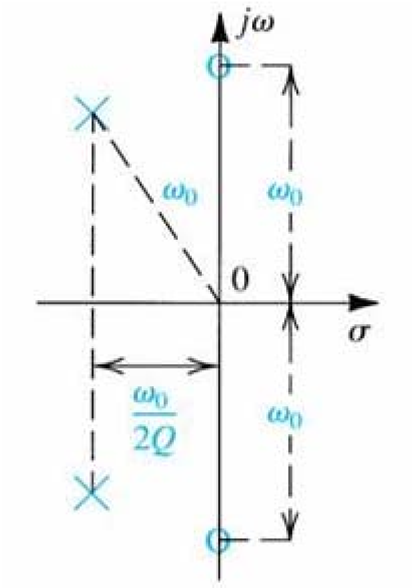


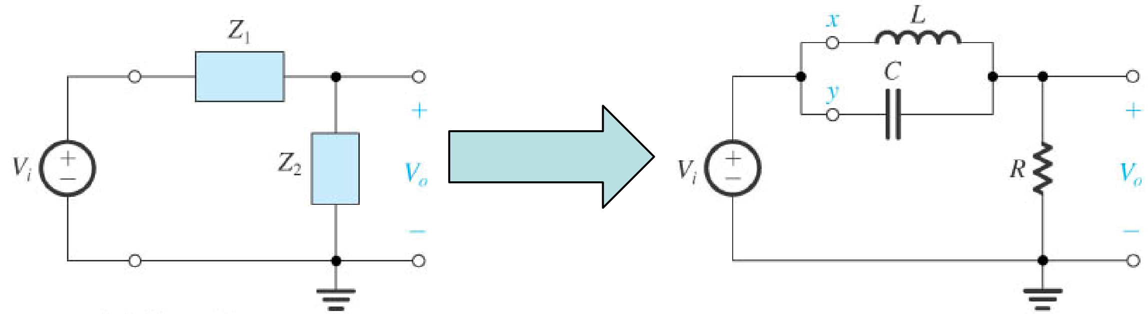


$$T(s) = a_2 \frac{s^2 + \omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

Band-Rejection or Notch Filter

Lect. 14: Second-Order Passive Filters





(a) General structure

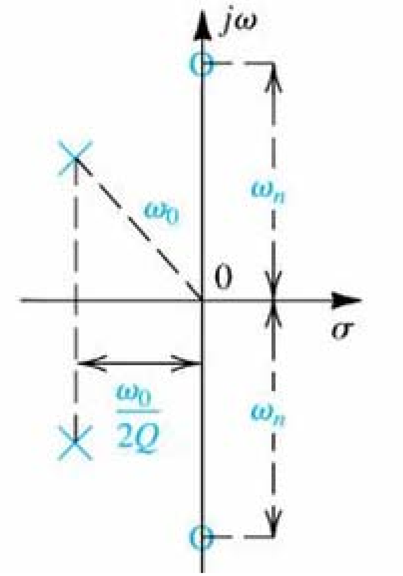
(e) Notch at ω_0

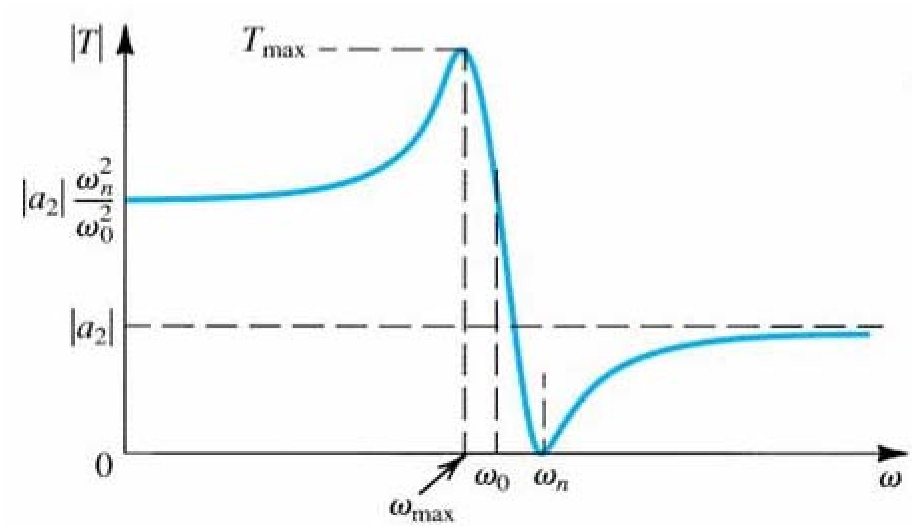
$$T(s) = a_2 \frac{s^2 + \omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$Z_1 = \frac{sL \cdot \frac{1}{sC}}{sL + \frac{1}{sC}} = \frac{sL}{1 + s^2 LC}$$

$$T(s) = \frac{V_o}{V_i} = \frac{Z_2}{Z_1 + Z_2} = \frac{R}{\frac{sL}{1 + s^2 LC} + R} = \frac{s^2 + (1/LC)}{s^2 + s(1/RC) + (1/LC)}$$

Lect. 14: Second-Order Passive Filters





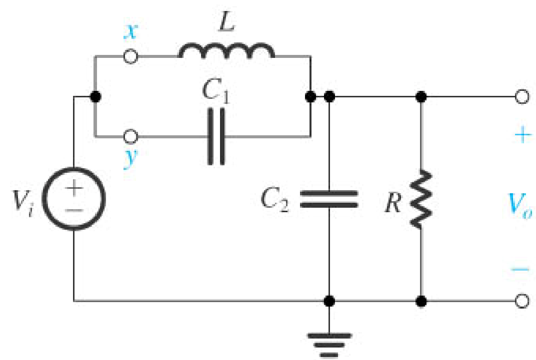
Low-Pass Notch Filter

$$T(s) = a_2 \frac{s^2 + \omega_n^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$\omega_n \ge \omega_0$$

$$\text{DC gain} = a_2 \frac{\omega_n^2}{\omega_0^2}$$

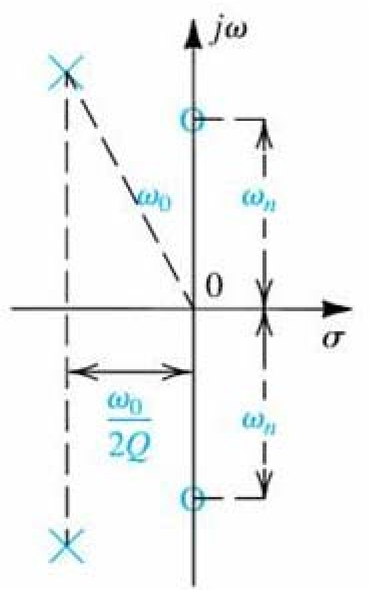
$$\text{High-frequency gain} = a_2$$

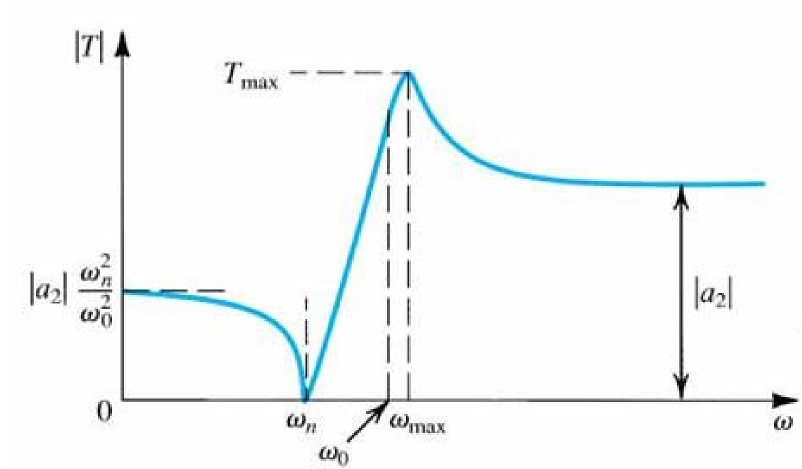


(g) LPN $(\omega_n > \omega_0)$

$$T(s) = \frac{s^2 + (1/LC_1)}{s^2 + s\frac{1}{(C_1 + C_2)R} + \frac{1}{L(C_1 + C_2)}}$$

Lect. 14: Second-Order Passive Filters





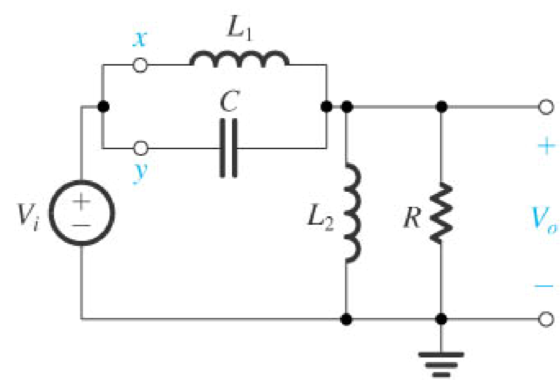
High-Pass Notch Filter

$$T(s) = a_2 \frac{s^2 + \omega_n^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$$

$$\omega_n \le \omega_0$$

$$\text{DC gain} = a_2 \frac{\omega_n^2}{\omega_0^2}$$

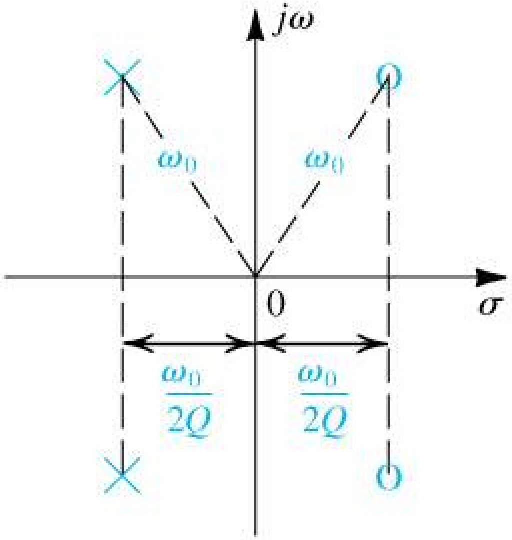
$$\text{High-frequency gain} = a_2$$



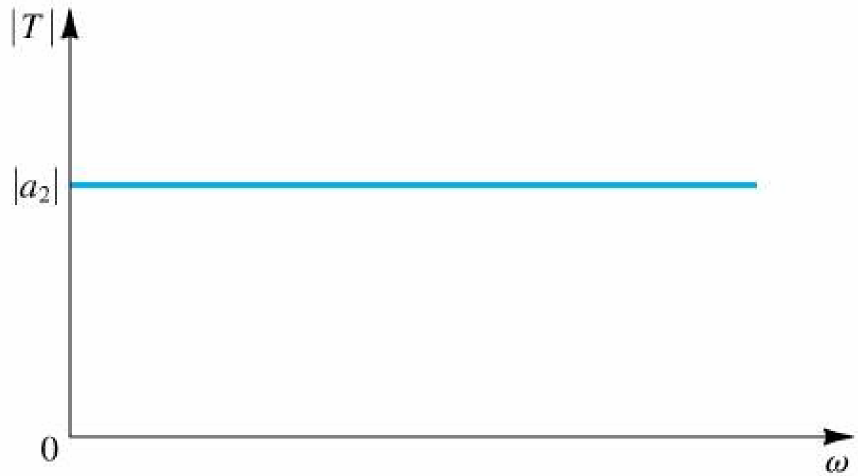
(i) HPN $(\omega_n < \omega_0)$

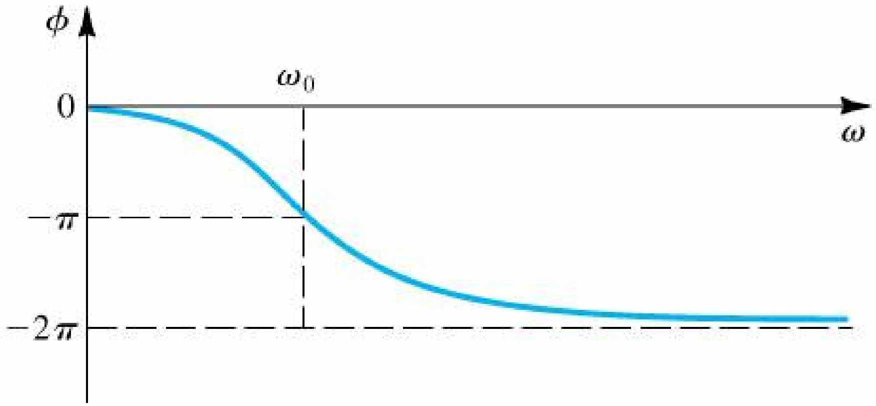
$$T(s) = \frac{s^2 + (1/L_1C)}{s^2 + s(1/CR) + [1/((L_1 \parallel L_2)C)]}$$

Lect. 14: Second-Order Passive Filters

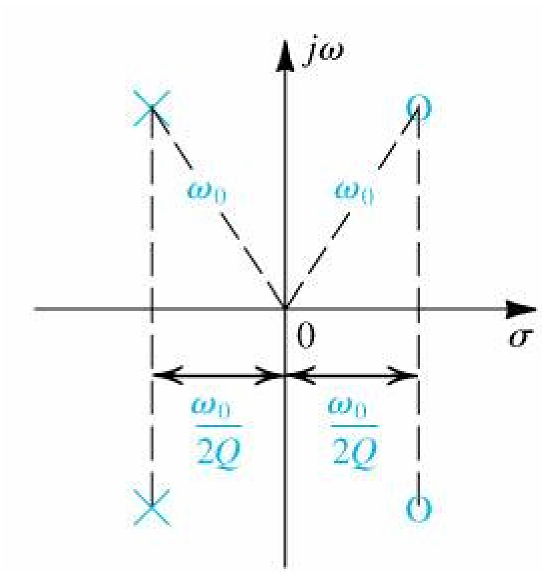


$$T(s) = a_2 \frac{s^2 - s\frac{\omega_0}{Q} + \omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$



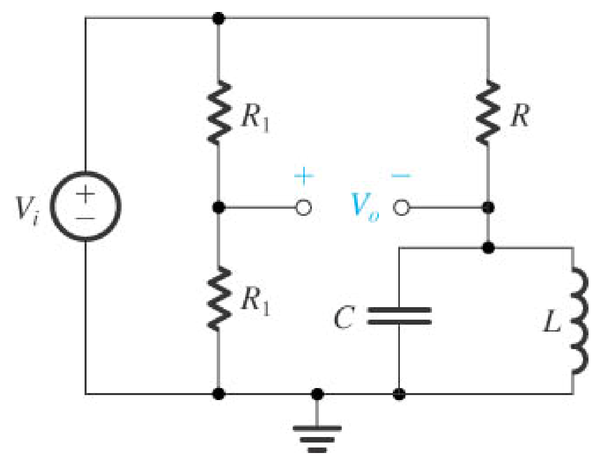


Lect. 14: Second-Order Passive Filters



$$T(s) = \frac{s^2 - s(\omega_0/Q) + \omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2} = 1 - \frac{s2(\omega_0/Q)}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

$$= 2\left(\frac{1}{2} - \frac{s(\omega_0/Q)}{s^2 + s(\omega_0/Q) + \omega_0^2}\right)$$



$$T(s) = a_2 \frac{s^2 - s\frac{\omega_0}{Q} + \omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$