Lect. 6: Diode Small-Signal Model and Frequency Response

Piece-wise linear model

\[ i_D = I_S \left[ \exp \left( \frac{V_D}{V_T} \right) - 1 \right] \]

What is the correct value of \( r_D \)?

\( \Rightarrow \) It depends on \( i_D \)

Expression for \( r_D \) as function of \( i_D \)?
Lect. 6: Diode Small-Signal Model and Frequency Response

\[ i_D = I_S \left[ \exp\left(\frac{V_D}{V_T}\right) - 1 \right] \]

Expression for \( r_D \) as function of \( i_D \)?

\[
\begin{align*}
  r_d & = \frac{1}{\frac{\partial i_D}{\partial V_D}} \bigg|_{i_D=I_D} \\
  \left[ \frac{\partial i_D}{\partial V_D} \right]_{i_D=I_D} & = \frac{1}{V_T} \cdot I_S \exp\left(\frac{V_D}{V_T}\right) \approx \frac{I_D}{V_T} \\
  \therefore r_d & \approx \frac{V_T}{I_D}
\end{align*}
\]
Lect. 6: Diode Small-Signal Model and Frequency Response

\[ v_S = V_{DC} + v_{ac}\sin(\omega t), \quad V_{DC} \gg 0, \quad v_{ac} \]

Diode ON

\[ i_D = ? \]

\[ r_d \text{ depends on } i_D \]
\[ \text{which depends on } r_d \]

Divide the problem into two:

- DC or Large-Signal analysis
- AC or Small-Signal analysis
- Add above two results
$v_S = V_{DC} + v_{ac}\sin(\omega t)$, $V_{DC} >> 0$, $v_{ac}$

DC or Large-Signal analysis

$I_D = (V_{DC} - 0.7) / R$
Lect. 6: Diode Small-Signal Model and Frequency Response

\[ v_S = V_{DC} + v_{ac}\sin(\omega t), \quad v_{DC} \gg 0, \quad v_{ac} \]

AC or Small-Signal analysis

\[ i_d = \frac{v_{ac}\sin(\omega t)}{R + r_d} \]

\[ r_d = \frac{V_T}{I_D} \]
Lect. 6: Diode Small-Signal Model and Frequency Response

\[ v_S = V_{DC} + v_{ac}\sin(\omega t), \quad v_{DC} \gg 0, \quad v_{ac} \]

\[ i_D = \frac{v_{ac}\sin(\omega t)}{R + \frac{V_T}{I_D}} \]

\[ i_D = \frac{(V_{DC} - 0.7)}{R} \]

\[ i_d = \frac{v_{ac}\sin(\omega t)}{R + \frac{V_T}{I_D}} \]

\[ i_D (\text{total}) = i_D (\text{large signal}) + i_d (\text{small signal}) \]
Lect. 6: Diode Small-Signal Model and Frequency Response
v_s = V_{DC} + v_{ac}\sin(\omega t), \quad v_{DC} \gg 0, \quad v_i

Determine i(t) \quad \Rightarrow \quad \text{Small-signal analysis}

i = ?

Let \( R' = R + r_d \)
\[
\frac{V_i(\omega)}{I(\omega)} = R' \parallel \frac{1}{j\omega C} \\
= \frac{R'}{j\omega C} = \frac{R'}{1 + j\omega R'C} = \frac{R'}{1 + j\frac{\omega}{\omega_0}} (\omega_0 = \frac{1}{R'C}) \\
\left| \frac{V_i(\omega)}{I(\omega)} \right| = \frac{R'}{\sqrt{1 + \frac{\omega^2}{\omega_0^2}}} \\
\angle \frac{V_i(\omega)}{I(\omega)} = -\tan^{-1} \frac{\omega}{\omega_0}
\]
Lect. 6: Diode Small-Signal Model and Frequency Response

Bode Plots

\[
\frac{V_i(\omega)}{I(\omega)} = \frac{R'}{\sqrt{1 + \omega^2/\omega_0^2}}
\]

\[f_{3\text{-dB}} = \frac{\omega_0}{2\pi}\]
\[ \angle \frac{V_I(\omega)}{I(\omega)} = -\tan^{-1} \frac{\omega}{\omega_0} \]