So far we have considered only sinusoidal responses. (Cheng 9–5.2, 5.3)

Now let’s consider transient responses: step responses.

i) At $t = 0^+$

Initially, the voltage wave “sees” only $R_0$. => Voltage divider
Lect. 17: Transients in Transmission Lines

ii) At \( t = \frac{L}{u} \)

\[
V_1^- = \Gamma_L V_1^+, \quad \Gamma_L = \frac{R_L - R_0}{R_L + R_0}
\]

iii) At \( t = \frac{2L}{u} \)

\[
V_2^+ = \Gamma_G V_1^-, \quad \Gamma_G = \frac{R_G - R_0}{R_G + R_0}
\]

iv) At \( t = \frac{3L}{u} \)

\[
V_2^- = \Gamma_L V_2^+, \quad \ldots
\]

\[
V_{\text{total}(t=\infty)} = V_1^+ + V_1^- + V_2^+ + V_2^- + \ldots
\]

\[
= V_1^+ \left(1 + \Gamma_L + \Gamma_L \Gamma_G + \Gamma_L^2 \Gamma_G + \Gamma_L^2 \Gamma_G^2 + \Gamma_L^3 \Gamma_G^2 + \ldots\right)
\]

\[
= V_1^+ \left[(1 + \Gamma_L \Gamma_G + \Gamma_L^2 \Gamma_G^2 + \ldots) + \Gamma_L \left(1 + \Gamma_L \Gamma_G + \Gamma_L^2 \Gamma_G^2 + \ldots\right)\right]
\]

\[
= V_1^+ \left(1 + \frac{\Gamma_L}{1 - \Gamma_L \Gamma_G}\right) \left(\frac{R_0}{R_0 + R_G}\right) V_0 \left(1 + \frac{\Gamma_L}{1 - \Gamma_L \Gamma_G}\right)
\]

\[
= \left(\frac{R_L}{R_L + R_G}\right) V_0
\]

No TL effect:
All the wave characteristics have died out!
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Example) \( R_L = 3R_0, \ R_G = 2R_0 \)

\[
\Gamma_L = \frac{R_L - R_0}{R_L + R_0} = \frac{2R_0}{4R_0} = \frac{1}{2}, \ \Gamma_G = \frac{R_G - R_0}{R_G + R_0} = \frac{R_0}{3R_0} = \frac{1}{3}
\]

\( V_{tot(t=\infty)} = \frac{3}{5} V_0 \)
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\[ R_0 + \]

\[ t=0 \]

\[ 3R_0 \]

\[ v_0 \]

\[ 2R_0 \]

\[ R_0 \]

\[ L \]

\[ \text{voltage} \]

\[ \text{voltage} \]

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Lect. 17: Transients in Transmission Lines

\[ R_L = 3R_0, \quad R_G = 2R_0, \Gamma_L = \frac{1}{2}, \quad \Gamma_G = \frac{1}{3} \]

How about currents?

\[ I_1^+ = \frac{V_1^+}{R_0} = \frac{V_0}{3R_0} \]

\[ I_1^- = -\frac{V_1^-}{R_0} = -\frac{V_0}{6R_0} \]

\[ I_2^+ = \frac{V_2^+}{R_0} = \frac{V_0}{18R_0} \]
Lect. 17: Transients in Transmission Lines

Example) \( R_L = 3R_0, \ R_G = 2R_0 \)

\[
\Gamma_L = \frac{R_L - R_0}{R_L + R_0} = \frac{2R_0}{4R_0} = \frac{1}{2}, \quad \Gamma_G = \frac{R_G - R_0}{R_G + R_0} = \frac{R_0}{3R_0} = \frac{1}{3}
\]

\[
I_{total(t=\infty)} = I_1^+ + I_1^- + I_2^+ + I_2^- + \ldots
\]

\[
= I_1^+ \left( 1 - \Gamma_L \Gamma_G - \Gamma_L^2 \Gamma_G^2 - \Gamma_L^3 \Gamma_G^2 + \ldots \right)
\]

\[
= I_1^+ \left[ (1 + \Gamma_L \Gamma_G + \Gamma_L^2 \Gamma_G^2 + \ldots) - \Gamma_L (1 + \Gamma_L \Gamma_G + \Gamma_L^2 \Gamma_G^2 + \ldots) \right]
\]

\[
= I_1^+ \left( \frac{1 - \Gamma_L}{1 - \Gamma_L \Gamma_G} \right)
\]

\[
= \frac{1}{3} \frac{V_0}{R_0} \left( \frac{1 - \frac{1}{2}}{1 - \left( \frac{1}{2} \right) \left( \frac{1}{3} \right)} \right) = \frac{1}{5} \frac{V_0}{R_0}
\]
Lect. 17: Transients in Transmission Lines

![Diagram of a transmission line circuit with a voltage source, resistor, and inductor.](image)

**voltage**

**current**

$\begin{align*}
\text{voltage} & \quad \text{current} \\
\end{align*}$
Lect. 17: Transients in Transmission Lines

Homework: Due Nov. 3

A transmission line circuit is connected to a constant current source with $I_0$ at $t=0$ as shown below. The length of the line is $L$ and the velocity of wave propagation on the line is $v$.

(a) Sketch the voltage on the line, $V(z)$, for $t = 1.5 \frac{v}{L}$.
(b) Sketch the current on the line, $I(z)$, for $t = 3.5 \frac{v}{L}$.
(c) Sketch the voltage on the line, $V(z)$, for $t = \infty$.

Make sure you express the magnitude of waves in terms of $I_0$. 

![Circuit Diagram]