Lect. 23: PLL Dynamics

PLL Block Diagram

\[ V_{in} = \sin(\omega_{in}t) \]

Phase Detector \[ V_{PD} \]

Low Pass Filter \[ V_C \]

Voltage Controlled Oscillator \[ V_{out} = \sin(\omega_{out}t + \theta) \]

Linear Model

\[ \phi_{in} \]

\[ \sum \]

\[ K_{PD} \]

\[ V_{PD} \]

\[ \frac{\omega_p}{s + \omega_p} \]

\[ V_C \]

\[ K_{VCO} \]

\[ \frac{\omega_{out}}{s} \]

\[ \phi_{out} \]

\[ K_{PD}K_{VCO} \frac{\omega_p}{s(s + \omega_p)} \]

\[ \phi_{in} \]

\[ \sum \]

\[ \phi_{out} \]
Lect. 23: PLL Dynamics

Open loop gain:

$$G(s) = K_{PD}K_{VCO} \frac{\omega_p}{s(s + \omega_p)}$$

Closed loop gain

$$H(s) = \frac{\phi_{out}}{\phi_{in}} = \frac{G(s)}{1 + G(s)} = \frac{K_{PD}K_{VCO} \frac{\omega_p}{s(s + \omega_p)}}{1 + K_{PD}K_{VCO} \frac{\omega_p}{s(s + \omega_p)}} = \frac{K_{PD}K_{VCO} \omega_p}{s^2 + \omega_p s + K_{PD}K_{VCO} \omega_p}$$

⇒ 2nd order LPF!
Lect. 23: PLL Dynamics

\[ H(s) = \frac{\phi_{out}}{\phi_{out}} = \frac{K_{PD}K_{VCO}\omega_p}{s^2 + \omega_p s + K_{PD}K_{VCO}\omega_p} \]

\[ |H(s)| = \left| \frac{\phi_{out}}{\phi_{in}} \right| \]

Note that input and output are ‘phase’.

What does \( \omega \) mean in x-axis?
In LPF,

\[ |H(s)| = \left| \frac{V_{out}}{V_{in}} \right| \]
In PLL,

\[ |H(s)| = \left| \frac{\phi_{out}}{\phi_{in}} \right| \]
Lect. 23: PLL Dynamics

\[ H(s) = \frac{\phi_{out}}{\phi_{in}} = \frac{K_{PD}K_{VCO}\omega_p}{s^2 + \omega_p s + K_{PD}K_{VCO}\omega_p} \]

2\textsuperscript{nd} order system \hspace{1cm} \[ H(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + (\omega_0 / Q)s + \omega_0^2} \]

\[ \omega_0 = \sqrt{\omega_p K_{PD} K_{VCO}} \] Use \( \omega_n \), natural frequency \((\omega_n = \omega_0)\)

\[ Q = \sqrt{\frac{K_{PD}K_{VCO}}{\omega_p}} \] Use damping factor \( \zeta = \frac{1}{2Q} = \sqrt{\frac{\omega_p}{K_{PD}K_{VCO}}} \)

\[ H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \]

\[ \frac{\omega_{out}}{\omega_{in}} = \frac{s\phi_{out}}{s\phi_{in}} = \frac{\phi_{out}}{\phi_{in}} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \]
Damping factor dependence

\[ H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \]

\[ \omega_n = 2\pi \]

\( \xi = 0.1 \)
\( \xi = 0.3 \)
\( \xi = 0.7 \)
\( \xi = 1 \)
\( \xi = 1.5 \)
Lect. 23: PLL Dynamics

Natural frequency dependence

\[ H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \]

\[ \xi = 1 \]

\[ \omega_n = 2\pi \times 1 \]
\[ \omega_n = 2\pi \times 2 \]
\[ \omega_n = 2\pi \times 3 \]
\[ \omega_n = 2\pi \times 4 \]
\[ \omega_n = 2\pi \times 5 \]
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Step response

\[ \omega_{in}(t) = \Delta \omega \cdot u(t) \]

\[ \omega_{out}(t) = \left\{ 1 - e^{-\zeta \omega_n t} \left[ \cos(\omega_n \sqrt{1-\zeta^2} \cdot t) + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin(\omega_n \sqrt{1-\zeta^2} \cdot t) \right] \right\} \Delta \omega \cdot u(t) \]
Lect. 23: PLL Dynamics

Damping factor dependence: step response

\[ H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \]

\( \omega_n = 2\pi \)

- \( \xi = 0.1 \)
- \( \xi = 0.3 \)
- \( \xi = 0.7 \)
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Lect. 23: PLL Dynamics

Natural frequency dependence: Step Response

\[ H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \]

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