Lect. 17: First-Order Filters

(14.1, 14.2 in Razavi)

Filter: A device that alters the frequency spectrum of signals passing through it

Input: $V_i(s)$

Filter

$T(s) = V_o(s) / V_i(s)$

Output: $V_o(s)$

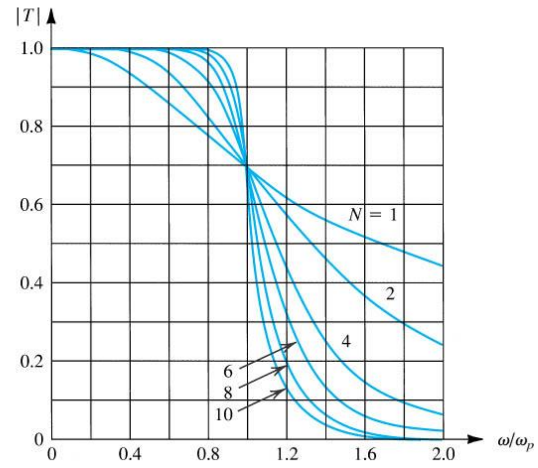
$V_o(s) = V_i(s) \times T(s)$

$$T(s) = \frac{a_M S^M + a_{M-1} S^{M-1} + \cdots + a_0}{S^N + b_{N-1} S^{N-1} + \cdots + b_0} = \frac{a_M (s - z_1)(s - z_2) \cdots (s - z_M)}{(s - p_1)(s - p_2) \cdots (s - p_N)}$$

poles, zeros

N: filter order

In general, higher N gives sharper filter response



Lect. 17: First-Order Filters

First-order filter $$T(s) = \frac{a_1 s + a_0}{s + \omega_0}$$

$$T(s) = \frac{a_0}{s + \omega_0}$$

$$T(s) = \frac{a_1 s}{s + \omega_0}$$

$$T(s) = \frac{a_1 s + a_0}{s + \omega_0}$$

$$T(s) = -a_1 \frac{s - \omega_0}{s + \omega_0}$$

$$a_1 > 0$$

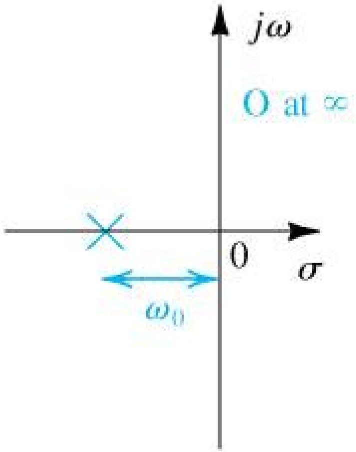
Pole-zero diagrams

Pole, zero locations determine how magnitude and phase of T(ω) change

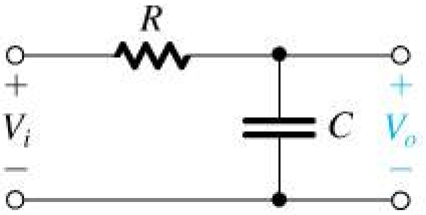
➔ Bode Plot

W.-Y. Choi

Lect. 17: First-Order Filters



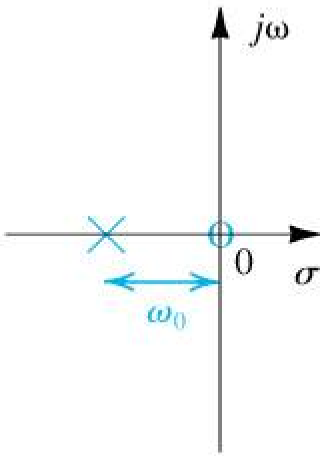
Circuit Implementation



$$T(s) = \frac{a_0}{s + \omega_0}$$

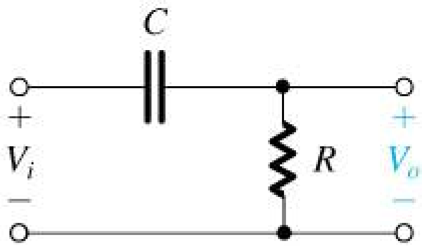
$$\frac{V_o(s)}{V_i(s)} = \frac{1}{sRC + 1}$$

Lect. 17: First-Order Filters



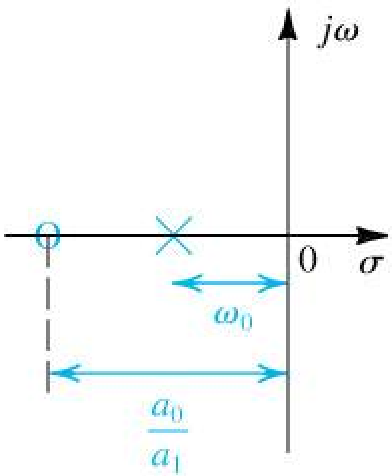
$$T(s) = \frac{a_1 s}{s + \omega_0}$$

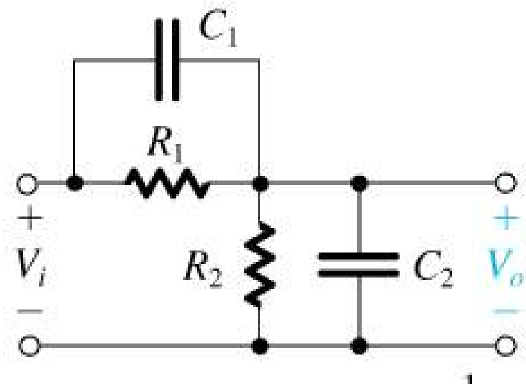
Circuit Implementation



$$\frac{V_o(s)}{V_i(s)} = \frac{sRC}{sRC + 1}$$

Lect. 17: First-Order Filters





$$\frac{V_o(s)}{V_i(s)} = \frac{sR_1R_2C_1 + R_2}{sR_1R_2\left(C_1 + C_2\right) + R_1 + R_2}$$

$$T(s) = \frac{a_1 s + a_0}{s + \omega_0}$$

$a_0/a_1 > \omega_0$

$$\omega_0 =$$

$$\frac{a_0}{a_1} =$$

DC gain:

AC gain:

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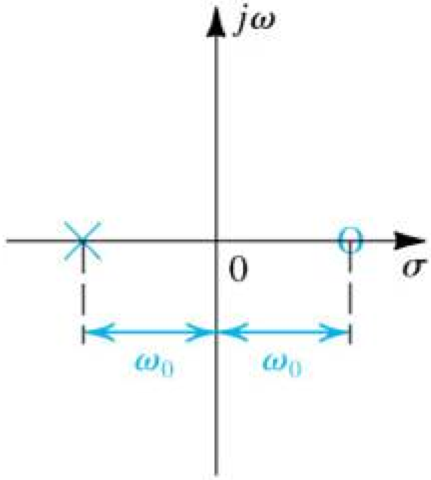


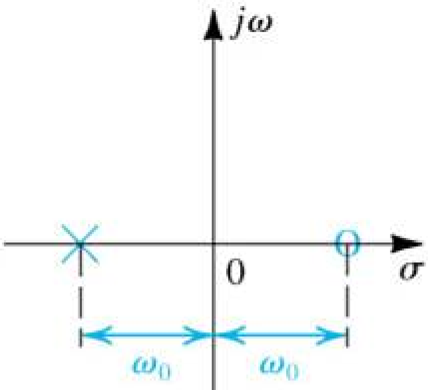
$$T(s) = -a_1 \frac{s - \omega_0}{s + \omega_0}$$

$$a_1 > 0$$

All Pass (AP) Filter: Phase change only

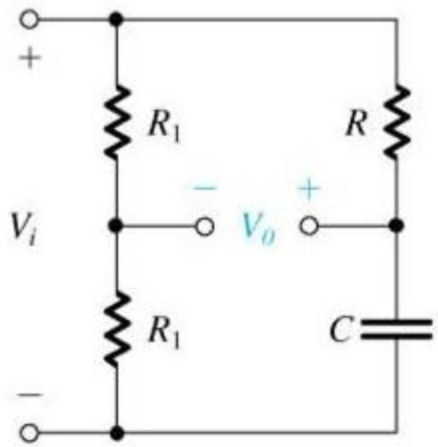
➔ Delay in time domain: Delay line

Lect. 17: First-Order Filters



$$T(s) = -a_1 \frac{s - \omega_0}{s + \omega_0}$$

$$a_1 > 0$$



$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sC}}{\frac{1}{sC} + R} - \frac{1}{2}$$

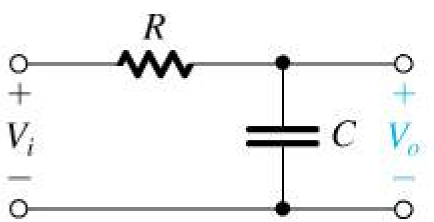
$$= \frac{1}{1 + sRC} - \frac{1}{2} = \frac{1 - sRC}{2 + 2sRC}$$

$$= -\frac{1}{2} \frac{s - \frac{1}{RC}}{s + \frac{1}{RC}}$$

Lect. 17: First-Order Filters

Limitation of passive filters (RC filters): Fixed gain, Loading effect

LP Filter

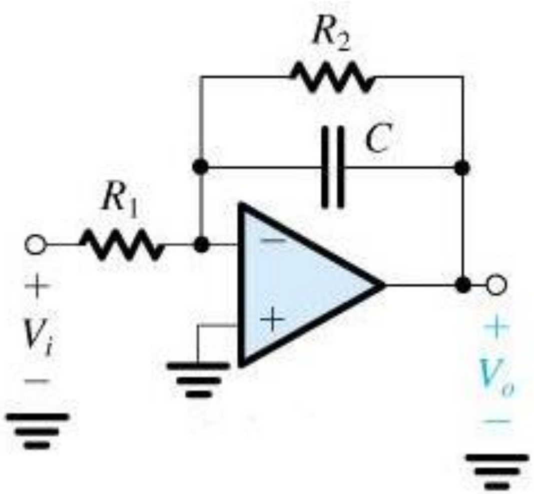


$CR = \frac{1}{\omega_0}$

DC gain = 1

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{sRC+1}$$

➔ Active filters



$$V_o(s) = -\frac{V_i(s)}{R_1} \cdot R_2 \,//\, \frac{1}{sC}$$

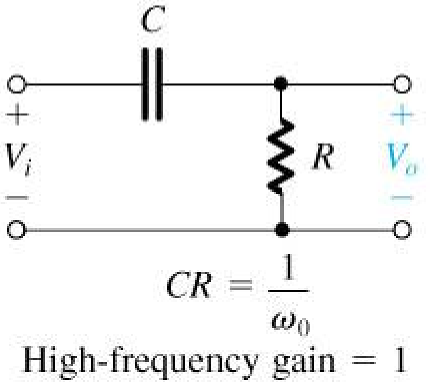
$$\frac{V_o(s)}{V_i(s)} = -\frac{R_2}{R_1}\frac{1}{sR_2C+1}$$

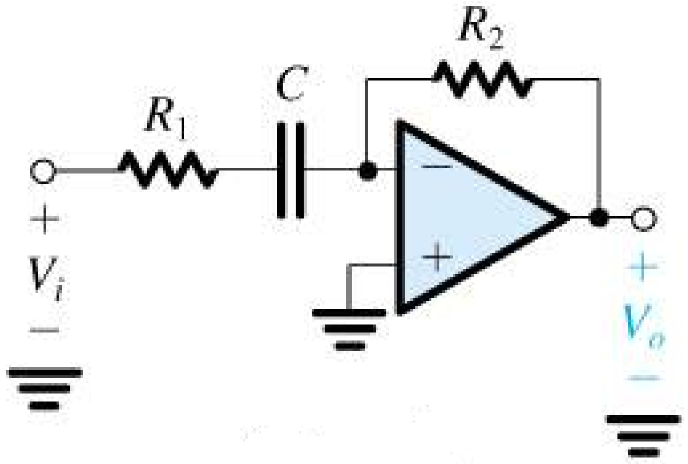
DC gain control possible

No loading effect!

Lect. 17: First-Order Filters

High Pass (HP) Filter

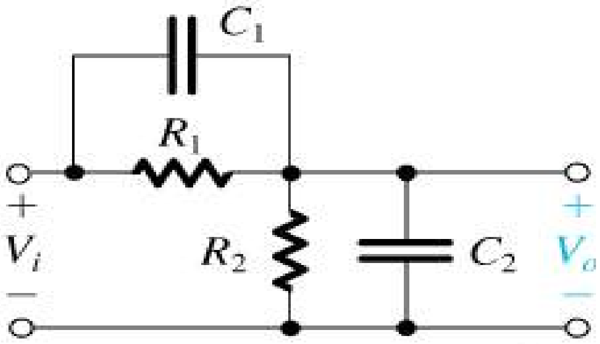


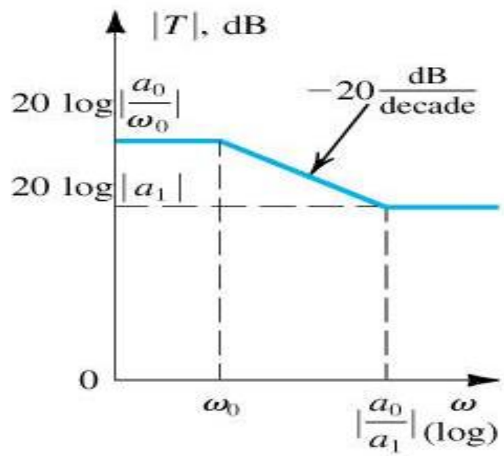


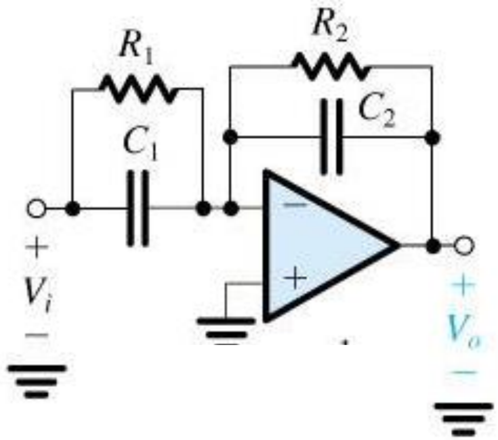
$$\frac{V_o(s)}{V_i(s)} = -\frac{sR_2C}{sR_1C+1}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{sRC}{sRC+1}$$

Lect. 17: First-Order Filters







$$\frac{V_o(s)}{V_i(s)} = \frac{sR_1R_2C_1 + R_2}{sR_1R_2\left(C_1 + C_2\right) + R_1 + R_2}$$

$$\frac{V_o(s)}{V_i(s)} = -\frac{sR_1R_2C_1 + R_2}{sR_1R_2C_2 + R_1}$$

$$\omega_0 = \frac{R_1 + R_2}{R_1R_2\left(C_1 + C_2\right)}$$

$$\omega_0 =$$

$$\frac{a_0}{a_1} = \frac{1}{R_1C_1}$$

$$\text{DC gain} = \frac{R_2}{R_1 + R_2}$$

$$\text{HF gain} = \frac{C_1}{C_1 + C_2}$$

$$\frac{a_0}{a_1} =$$

W.-Y. Choi

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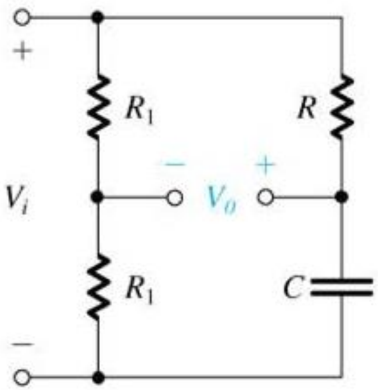
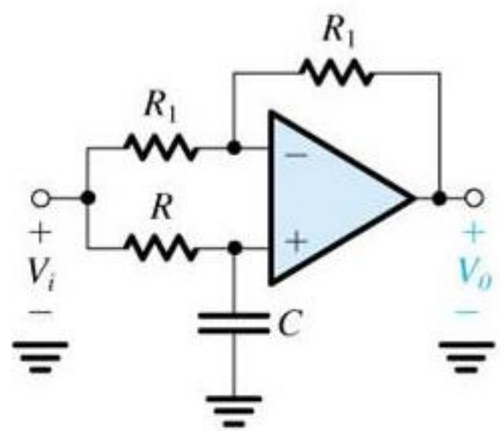


$CR = 1/\omega_0$
Flat gain $(a_1) = 0.5$



$$V^{+} = V_i \frac{1/sC}{1/sC + R} = V_i \frac{1}{sRC+1}$$

$$V^{-} = \frac{V_i + V_o}{2}$$

$$V_i\left(\frac{1}{sRC+1} - \frac{1}{2}\right) = \frac{V_o}{2}$$

$$V_i\left(\frac{2-(sRC+1)}{2(sRC+1)}\right) = \frac{V_o}{2}$$

$$\frac{V_o}{V_i} = \frac{1-sRC}{1+sRC} = -\frac{sRC-1}{sRC+1}$$

$$= -\frac{s - \frac{1}{RC}}{s + -\frac{1}{RC}}$$

$$\frac{V_o(s)}{V_i(s)} = -\frac{1}{2}\,\frac{s - \frac{1}{RC}}{s + \frac{1}{RC}}$$

W.-Y. Choi