Lect. 18: Second-Order Passive Filters

(14.3 in Razavi)

2nd-order filter $$T(s)=\frac{a_2s^2+a_1s^1+a_0}{s^2+b_1s^1+b_0}=\frac{a_2(s-z_1)(s-z_2)}{(s-p_1)(s-p_2)}$$

$$T(s)=a_2\frac{(s-z_1)}{(s-p_1)}\bullet\frac{(s-z_2)}{(s-p_2)}$$

➔ If Cascading of two 1st-order filters

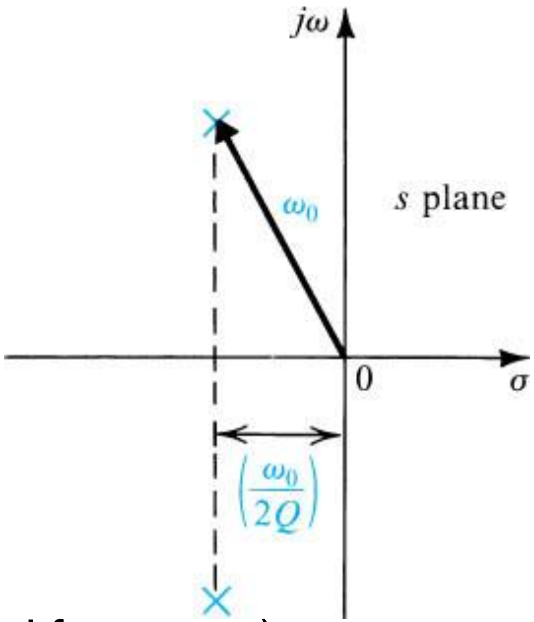
(Assuming p_1, p_2, z_1, z_2 are real)

For example,

$$T(s)=\frac{1}{(s+\frac{1}{R_1C_1})(s+\frac{1}{R_2C_2})}$$

Lect. 18: Second-Order Passive Filters

Consider T(s) with complex poles



$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + (\omega_0 / Q)s + \omega_0^2}$$

$$p_1, p_2 = -\frac{\omega_0}{2Q} \pm j\omega_0 \sqrt{1-(1/4Q^2)}$$

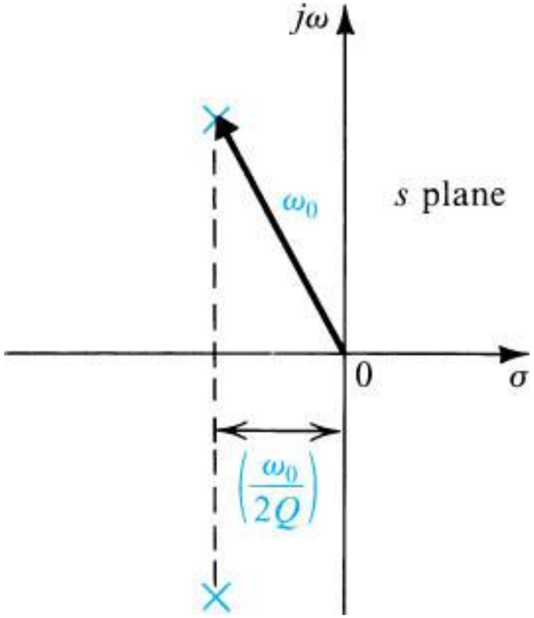
ω_o (ω_n): resonant angular frequency (natural frequency)

Q (Quality factor): Sharpness of the response

Q > ½, (underdamped)

Lect. 18: Second-Order Passive Filters

Consider T(s) with complex poles



$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + (\omega_0 / Q)s + \omega_0^2}$$

$$p_1, p_2 = -\frac{\omega_0}{2Q} \pm j\omega_0 \sqrt{1-(1/4Q^2)}$$

2nd order system can be also described with damping ratio $\zeta = 1 / 2Q$

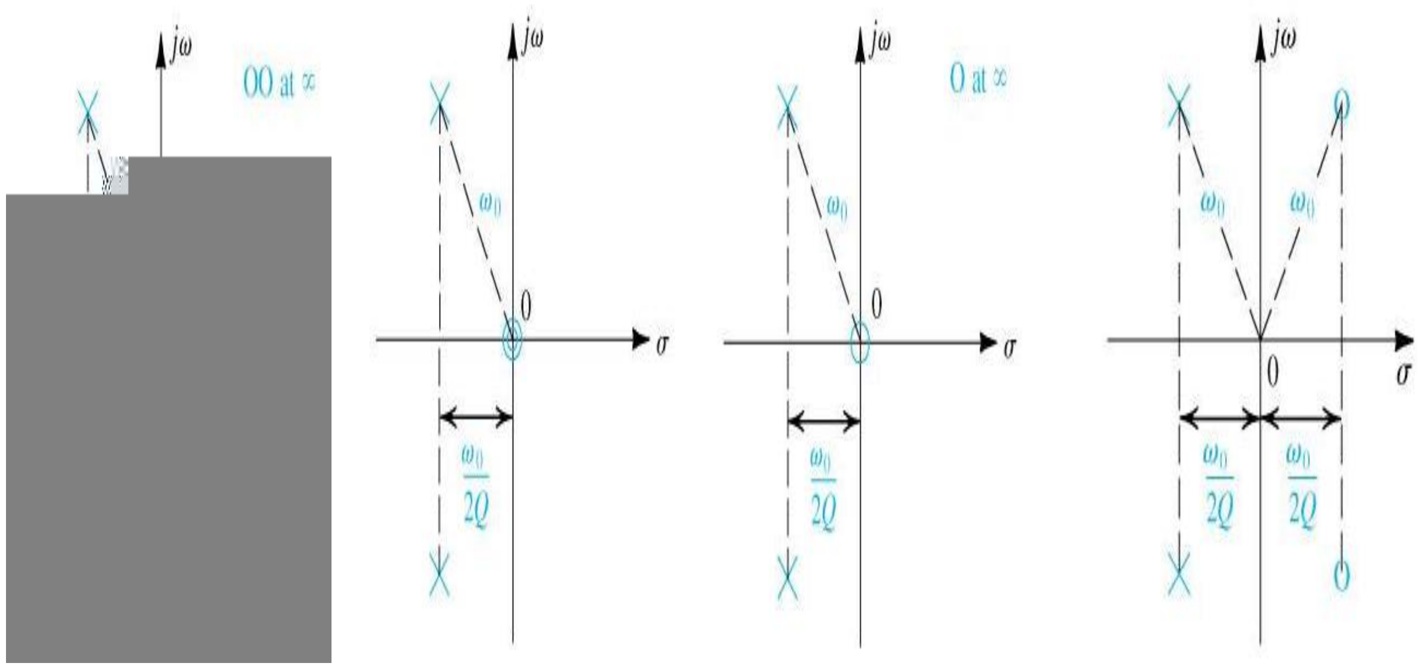
$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + 2\xi\omega_0 s + \omega_0^2} \qquad p_1, p_2 = -\xi\omega_0 \pm j\omega_0 \sqrt{1-\xi^2}$$

$\zeta < 1$ (underdamped)

Depending on zero locations, various filter characteristics!

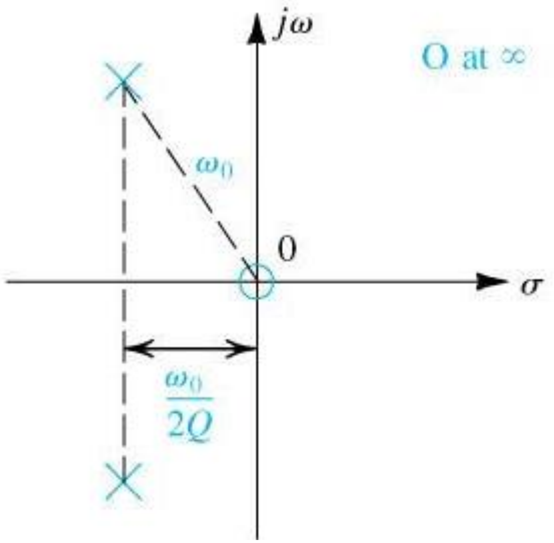
Lect. 18: Second-Order Passive Filters

Second-order filter pole-zero diagrams



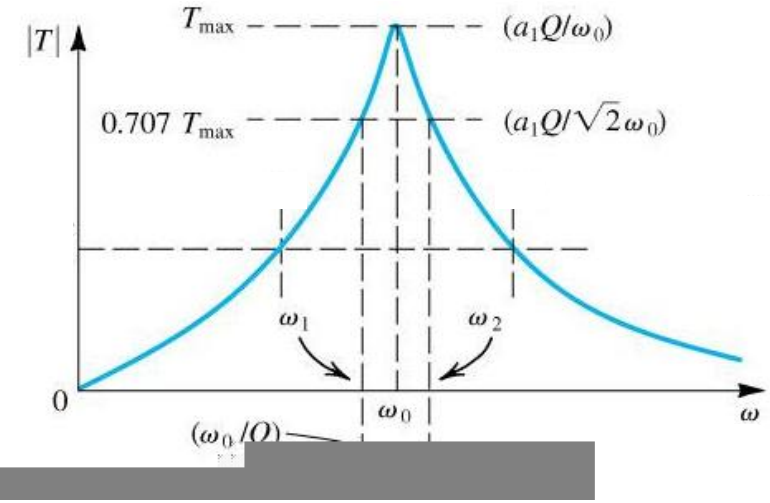
Filter characteristics for each case

Lect. 18: Second-Order Passive Filters



$$T(s) = \frac{a_1 s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

BP Filter



Peak

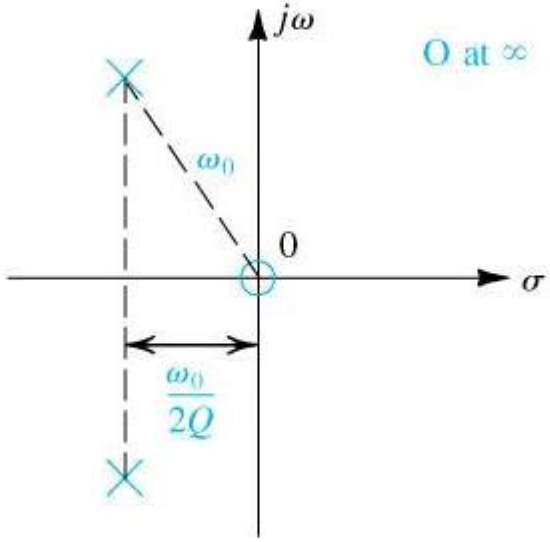
Bandwidth

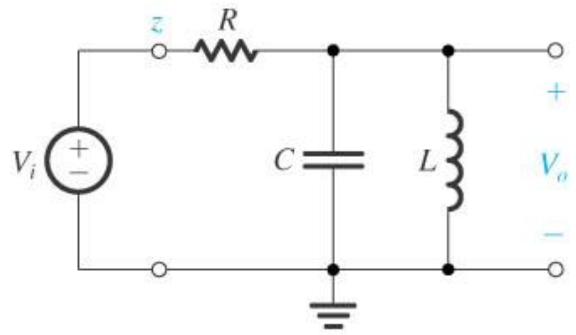
Larger Q ➔ Sharper response

Comparison with 2nd-order BP filter with two real poles?

In general, sharper response when ω is nearer to a pole

Lect. 18: Second-Order Passive Filters





(d) BP

$$T(s) = \frac{a_1 s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

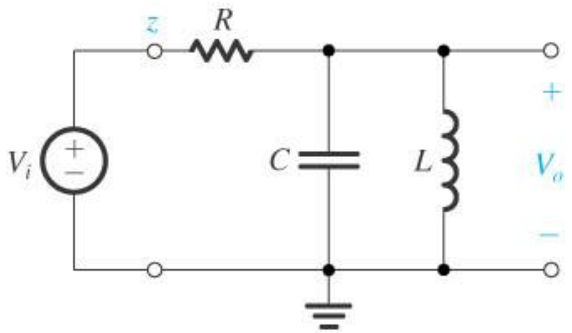
$$T(s) = \frac{\mathbf{V_o}}{\mathbf{V_i}} = \frac{Y_1}{Y_1 + Y_2} = \frac{1/R}{(1/R) + sC + (1/sL)}$$

$$= \frac{s(1/RC)}{s^2 + s(1/RC) + (1/LC)}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad \frac{\omega_0}{Q} = \frac{1}{RC} \qquad Q = \omega_0 CR = \sqrt{\frac{C}{L}}R$$

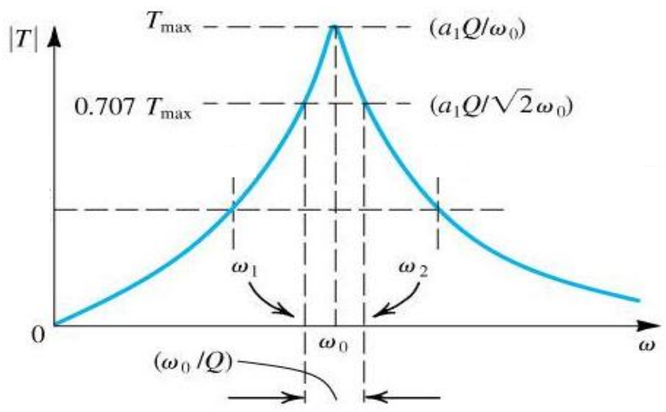
Lect. 18: Second-Order Passive Filters

What happens at resonance?

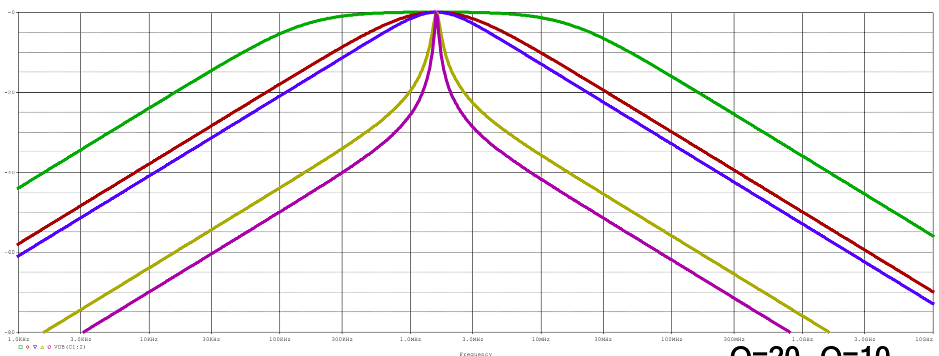


(d) BP

$$T(s) = \frac{a_1 s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$



$$T(s) = \frac{s(1/RC)}{s^2 + s(1/RC) + (1/LC)}$$



$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad Q = \sqrt{\frac{C}{L}}R$$

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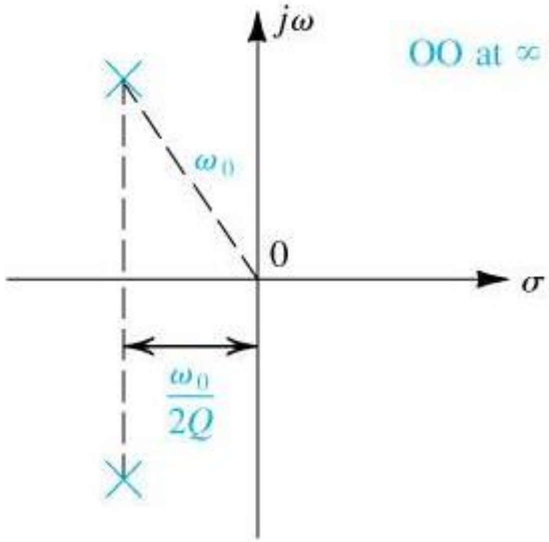
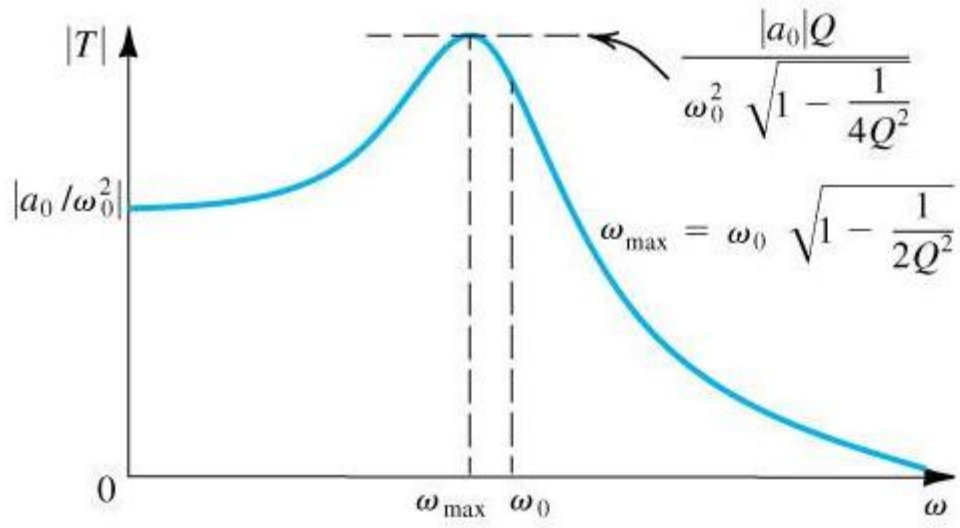
Lect. 18: Second-Order Passive Filters





LP Filter

$$T(s) = \frac{a_0}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

Comparison with two real pole filters?

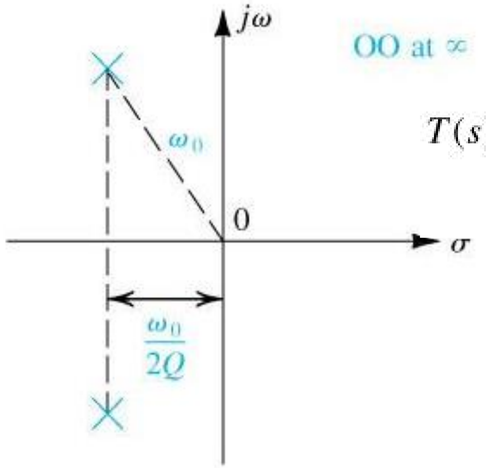
Peak slightly less than ω_0

Larger Q ➔ More and sharper peaking

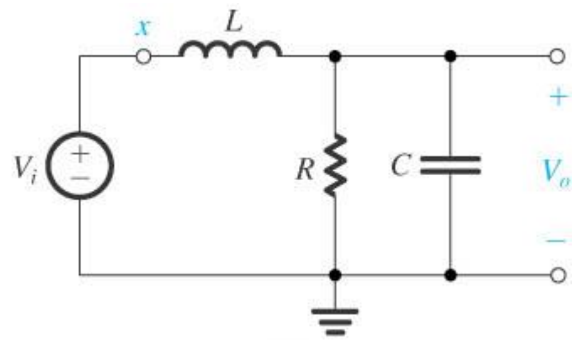
Phase?

Lect. 18: Second-Order Passive Filters

How to realize passive second-order LP filter

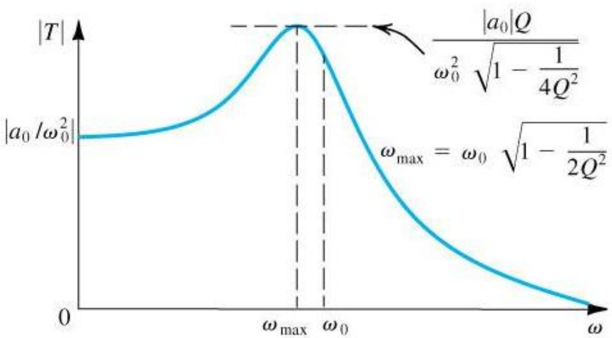


$$T(s) = \frac{a_0}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$



(b) LP

$$T(s) \equiv \frac{V_o}{V_i} = \frac{Z_2}{Z_1 + Z_2} = \frac{Y_1}{Y_1 + Y_2} = \frac{1/sL}{(1/sL) + sC + (1/R)}$$

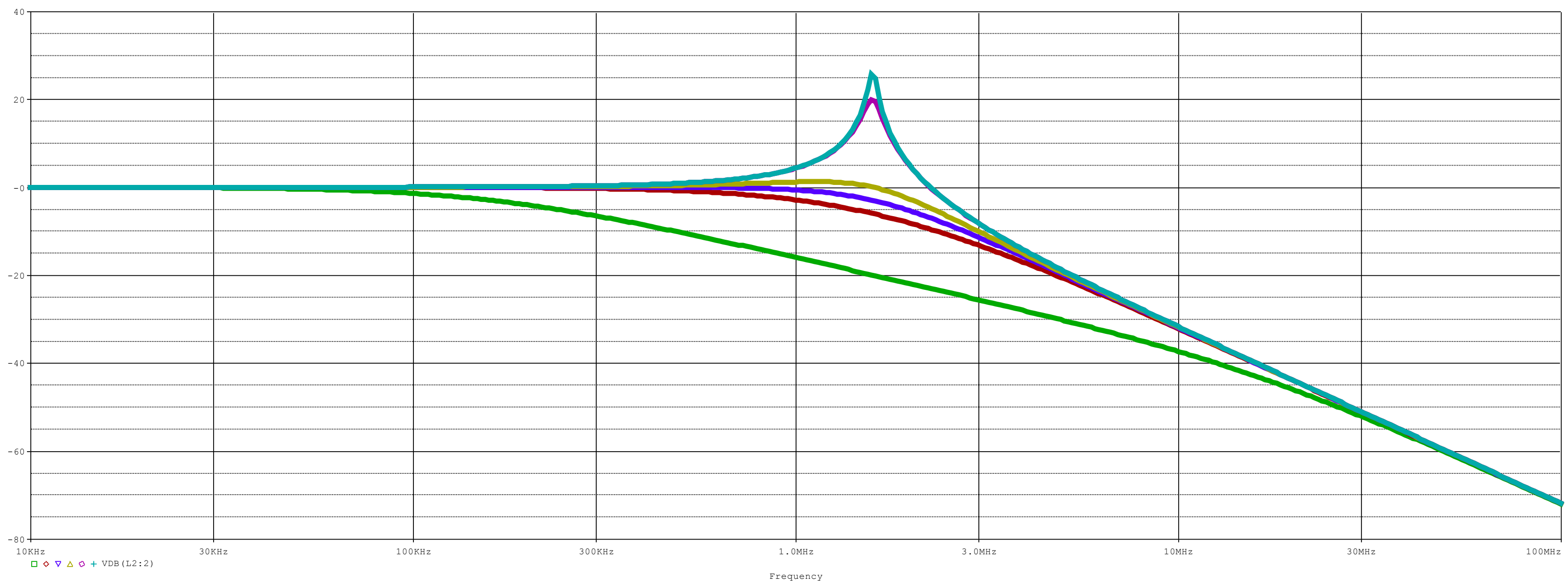


$$= \frac{1}{1 + s^2LC + (1/R)sL} = \frac{1/LC}{s^2 + s(1/RC) + 1/LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad \frac{\omega_0}{Q} = \frac{1}{RC} \qquad Q = \omega_0 CR = \sqrt{\frac{C}{L}}R$$

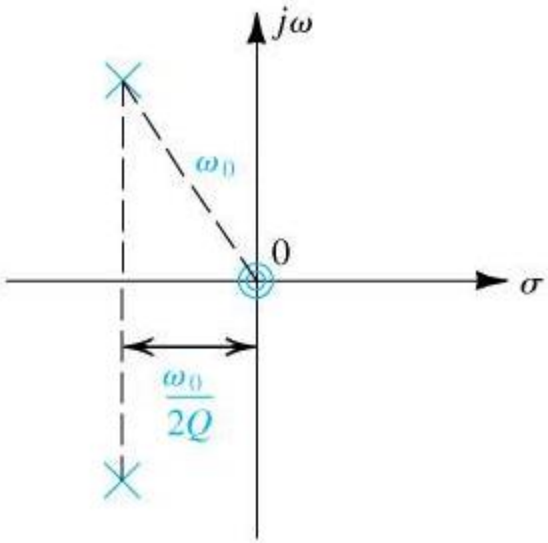
Lect. 18: Second-Order Passive Filters

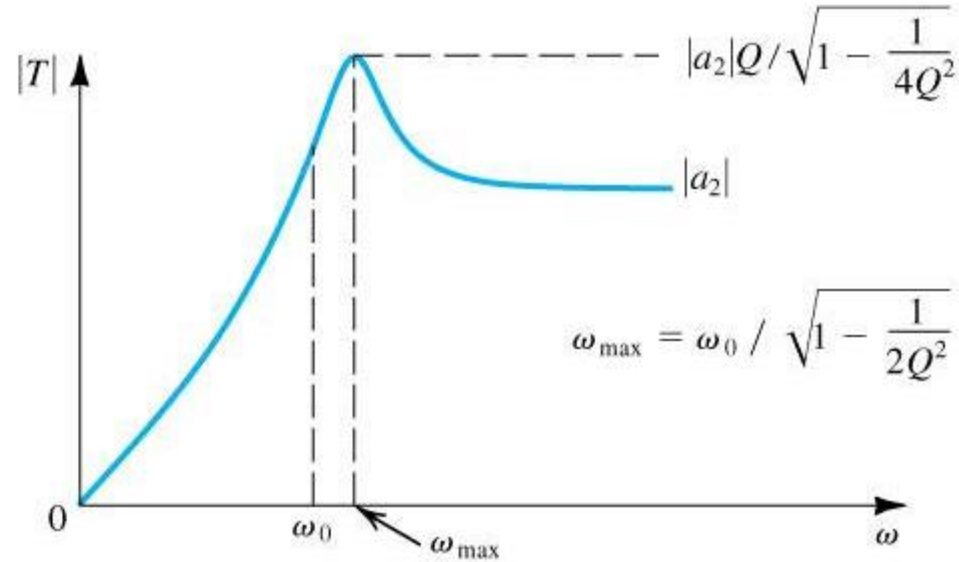
Frequency response with different Q values



Q=0.1, 0.5, 0.7, 1, 10, 20

Lect. 18: Second-Order Passive Filters



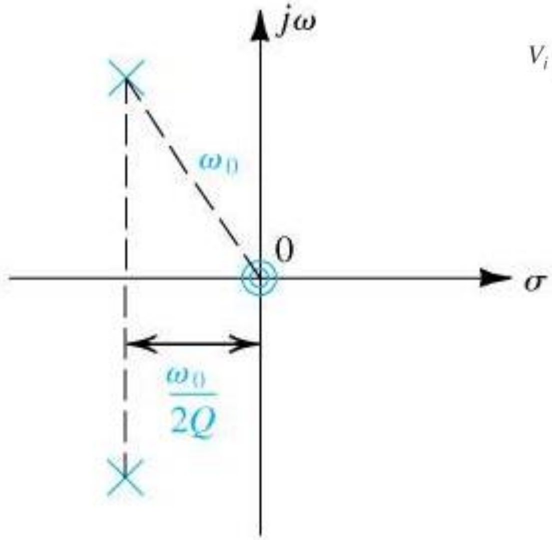


$$T(s) = \frac{a_2 s^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

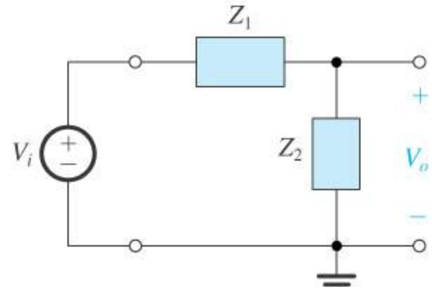
HP Filter

Phase?

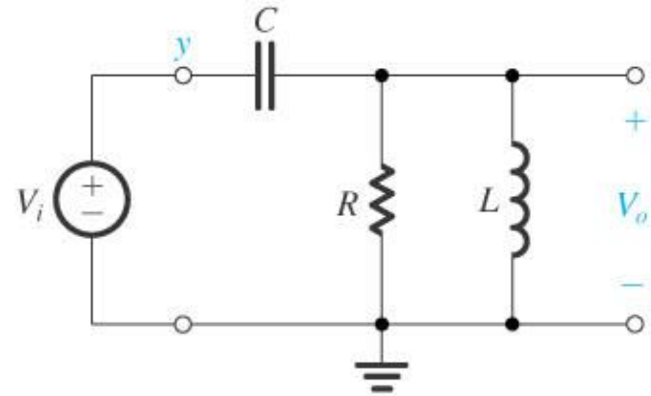
Lect. 18: Second-Order Passive Filters



$$T(s) = \frac{a_2 s^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$



Two zeros at s=0



(c) HP

$$T(s) \equiv \frac{V_o}{V_i} = \frac{Z_2}{Z_1 + Z_2} = \frac{Y_1}{Y_1 + Y_2} = \frac{sC}{sC + (1/R) + (1/sL)}$$

$$= \frac{s^2}{s^2 + s(1/RC) + (1/LC)}$$

Same pole characteristics as BP, LP

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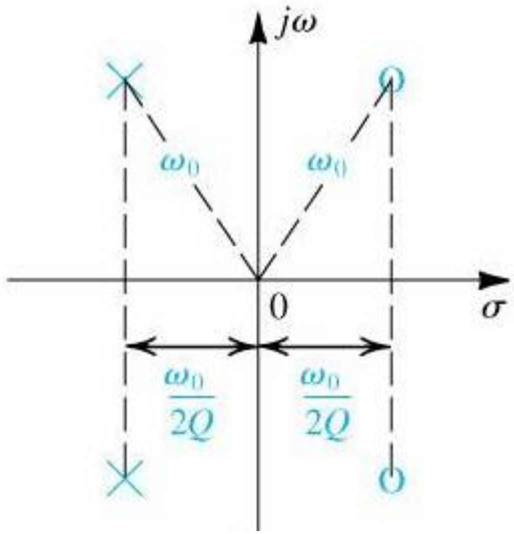
Lect. 18: Second-Order Passive Filters



$$T(s) = a_2 \frac{s^2 - s\frac{\omega_0}{Q} + \omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$p_1, p_2 = -\frac{\omega_0}{2Q} \pm j\omega_0\sqrt{1-(1/4Q^2)}$$

$$z_1, z_2 = \frac{\omega_0}{2Q} \pm j\omega_0\sqrt{1-(1/4Q^2)}$$

$$T(j\omega) = \frac{-\omega^2 - j\omega\frac{\omega_0}{Q} + \omega_0{}^2}{-\omega^2 + j\omega\frac{\omega_0}{Q} + \omega_0{}^2} = \frac{A - jB}{A + jB}$$

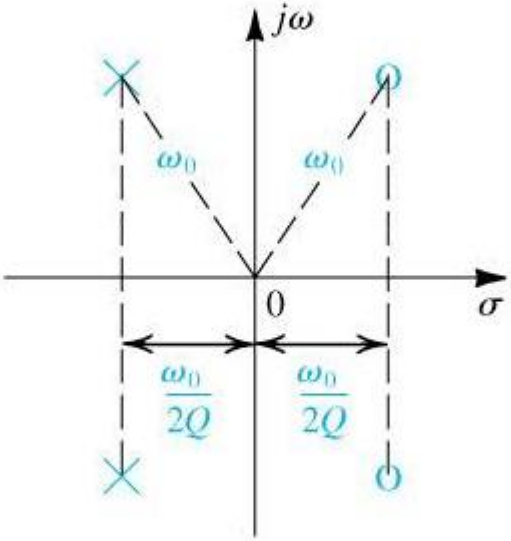
$$|T(j\omega)| = 1$$

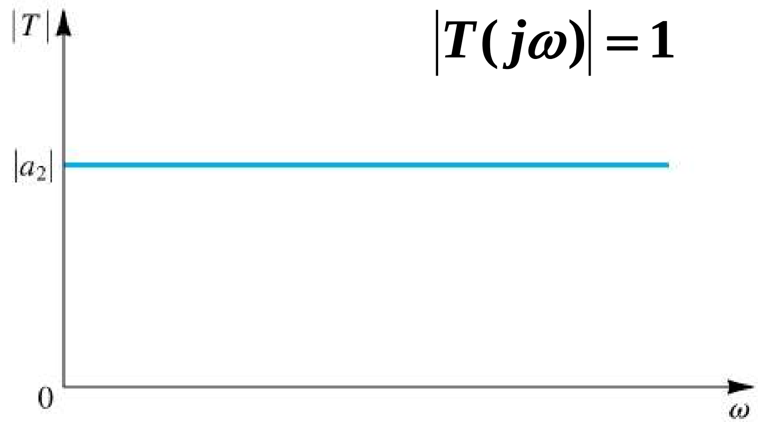
$$\angle T(j\omega) = \tan^{-1}(\frac{-B}{A}) - \tan^{-1}(\frac{B}{A})$$

$$= -2\tan^{-1}(\frac{B}{A})$$

$$= -2\tan^{-1}\left(\frac{\omega\frac{\omega_0}{Q}}{-\omega^2 + \omega_0{}^2}\right)$$

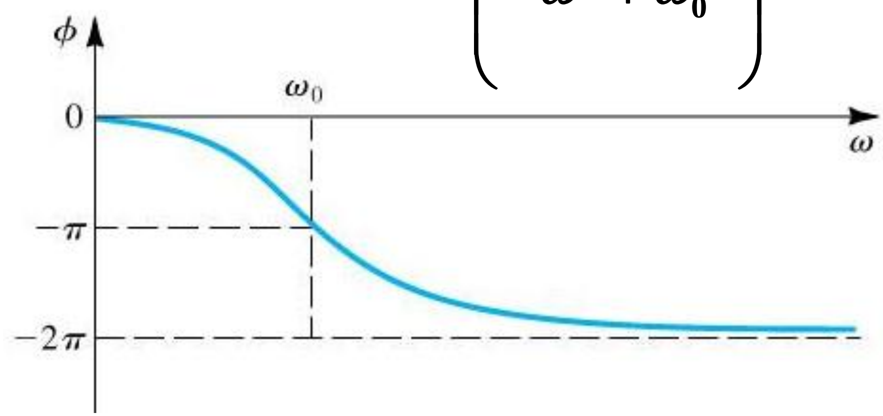
Lect. 18: Second-Order Passive Filters





$$\left|T(j\omega)\right| = 1$$

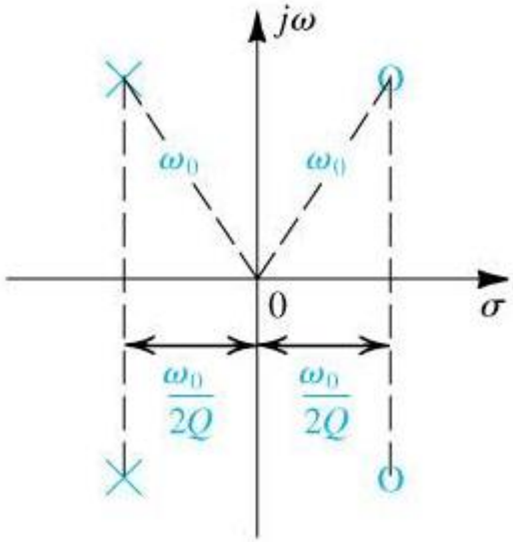
$$\angle T(j\omega) = -2\tan^{-1}\left(\frac{\omega\dfrac{\omega_0}{Q}}{-\omega^2+\omega_0^{\;2}}\right)$$



$$T(s) = a_2\frac{s^2 - s\dfrac{\omega_0}{Q} + \omega_0^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$$

All-Pass Filter

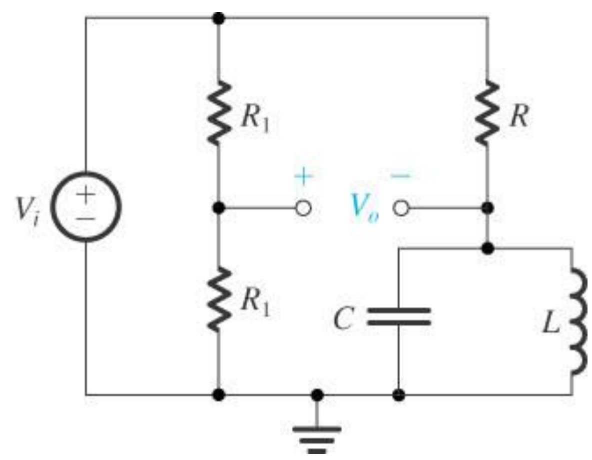
Lect. 18: Second-Order Passive Filters



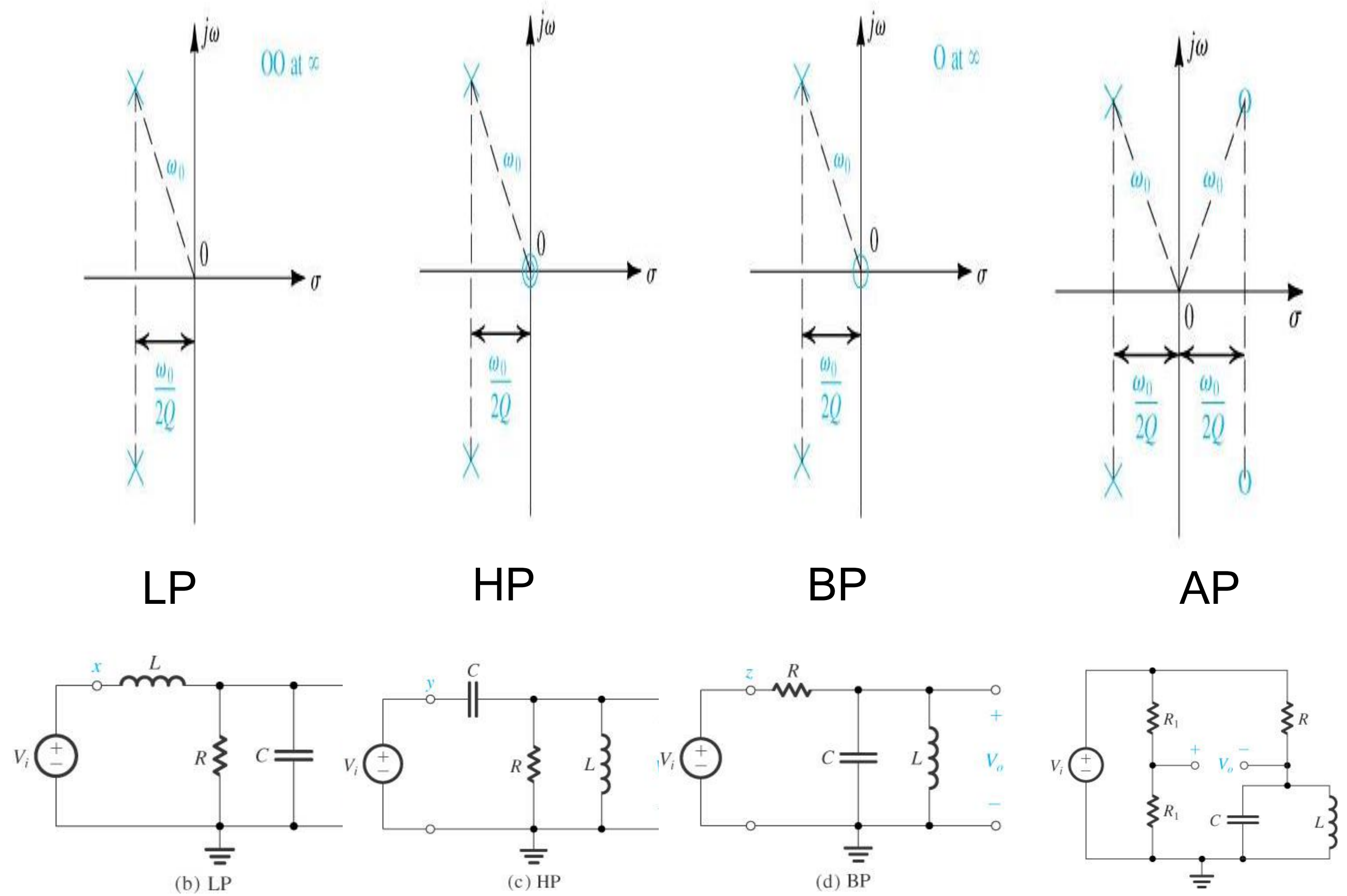
$$T(s)=\frac{s^2-s(\omega_0/Q)+\omega_0^2}{s^2+s(\omega_0/Q)+\omega_0^2}=1-\frac{s2(\omega_0/Q)}{s^2+s(\omega_0/Q)+\omega_0^2}$$

$$=2\left(\frac{1}{2}-\frac{s(\omega_0/Q)}{s^2+s(\omega_0/Q)+\omega_0^2}\right)$$

$$T(s)=a_2\frac{s^2-s\frac{\omega_0}{Q}+\omega_0^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}$$



Lect. 18: Second-Order Passive Filters



Lect. 18: Second-Order Passive Filters

Homework:

Determine $V_o(s)/V_i(s)$ the circuits shown below.
Sketch its pole, zero diagram (assume Q > 1/2)
What type of a filter is this?
Determine its filter characteristics (ω_o, Q, bandwidth)