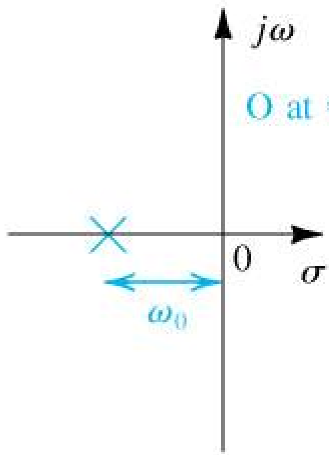
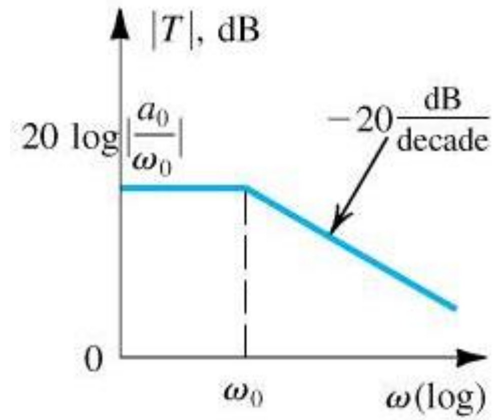
Lect. 21: Higher-Order Low-Pass Filters

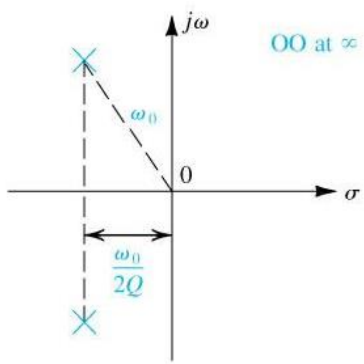
(Razavi 14.5.1)

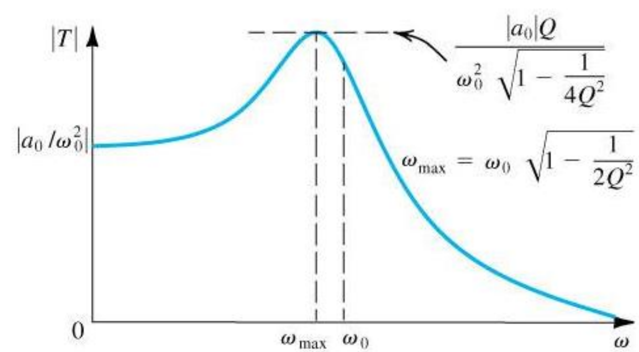
First-Order LP Filter





Second-Order LP Filter





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Various high-order LP filters

- Butterworth
- Chebyshev
- Elliptic
- Bessel
- Gaussian

...

Higher-order filters provide more flexibility in filter characteristics

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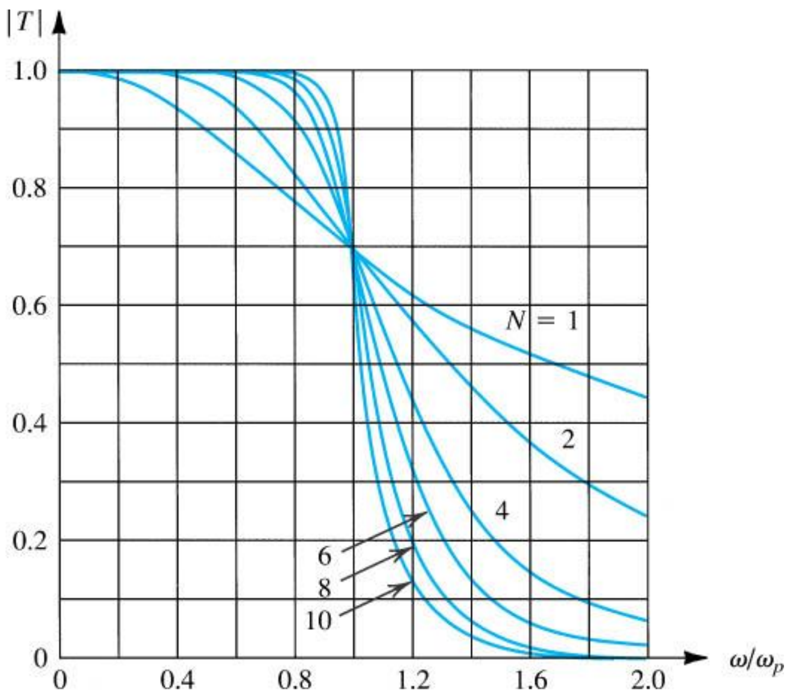

$$\left|T(j\omega)\right| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_p}\right)^{2N}}}$$

Butterworth LP Filter

Maximally flat response (no ripples) in pass band

Maximum variation in pass band?

What determines stop-band attenuation?

How many poles and zeros?

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Butterworth Filter

$$\left|T(j\omega)\right| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_p}\right)^{2N}}}$$

Determine locations of poles

$$1+\left(\frac{-s_p^{\,2}}{\omega_p^{\,2}}\right)^N = 0 \qquad \left(\frac{-s_p^{\,2}}{\omega_p^{\,2}}\right)^N = -1$$

$$\frac{-s_p^{\,2}}{\omega_p^{\,2}} = (-1)^{\frac{1}{N}} \qquad -s_p^{\,2} = (-1)^{\frac{1}{N}}\,\omega_p^{\,2}$$

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$$-s_p^2 = (-1)^{\frac{1}{N}} \omega_p^2$$

$$(-1)^{\frac{1}{N}} = \exp\left[j\frac{(2k-1)\pi}{N} \right], \; k = 1,2,3\ldots N \qquad -s_p^2 = \omega_p^2 \left[j\frac{(2k-1)\pi}{N} \right], \; k = 1,2,3\ldots N$$

$$s_p^2 = \omega_p^2 \exp\left[j\left(\pi + \frac{(2k-1)\pi}{N} \right) \right] = \omega_p^2 \exp\left[j\left(\frac{(2k+N-1)\pi}{N} \right) \right], \; k = 1,2,3\ldots N$$

$$\therefore \; s_p = \pm \omega_p \exp\left[j\frac{(2k+N-1)\pi}{2N} \right], \; k = 1,2,3\ldots N$$

Which are left half-plane poles?

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Taking only left half plane poles $s_p = \omega_p \exp\left[j\frac{(2k+N-1)\pi}{2N} \right],\ k = 1,2,3 \ldots N$

For N = 1,

$$s_p = \omega_p \exp[j\pi]$$

$$T(s) = \frac{\omega_p}{(s - s_p)}$$

With normalization: T(s=0) = 1

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$$s_p = \omega_p \exp\left[j \frac{(2k + N - 1)\pi}{2N} \right], \quad k = 1, 2, 3 \ldots N$$

For N=2,

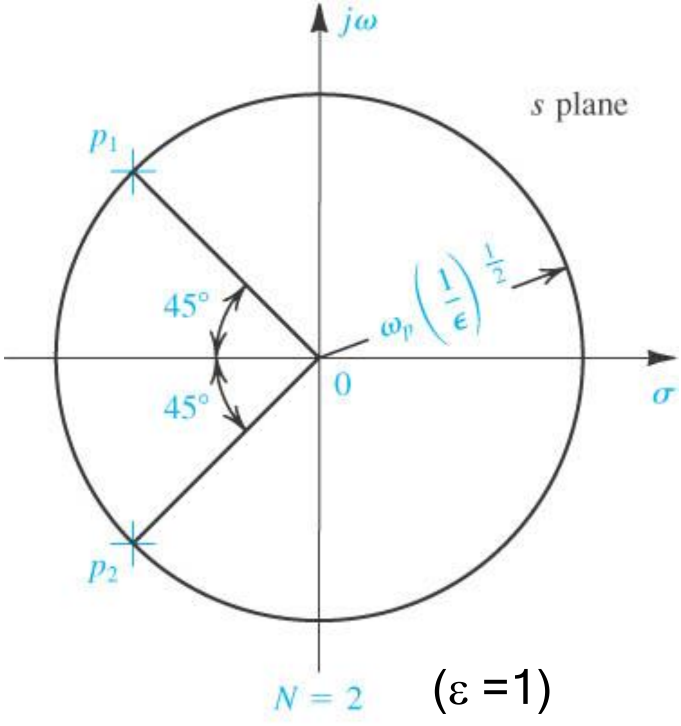
$$s_p = \omega_p \exp\left[j \frac{(2k + 1)\pi}{4} \right], \quad k = 1, 2$$

$$\text{For } k = 1, \quad s_{p1} = \omega_p \exp\left(j \frac{3\pi}{4} \right)$$

$$\text{For } k = 2, \quad s_{p2} = \omega_p \exp\left(j \frac{5\pi}{4} \right)$$

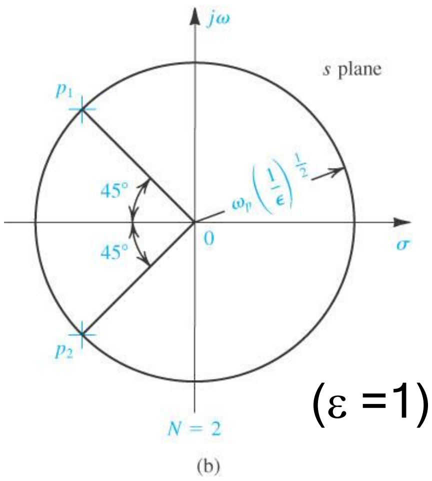
$$T(s) = \frac{\omega_p^2}{(s - s_{p1})(s - s_{p2})}$$

With normalization: T(s=0) = 1



(ε =1)

(b)



(ε =1)

$$T(s) = \frac{\omega_p^2}{(s - s_{p1})(s - s_{p2})}$$

$$s_{p1} = \omega_p \exp\left(j\frac{3\pi}{4}\right)$$

$$s_{p2} = \omega_p \exp\left(j\frac{5\pi}{4}\right)$$

Two poles that are complex conjugates

➔ Underdamped 2nd order filter (Lect. 18)

ω_o, Q (for ε = 1)?

2nd-order Butterworth filter: 2nd-order LPF with Q=1/sqrt(2)

Peaking behaviour?

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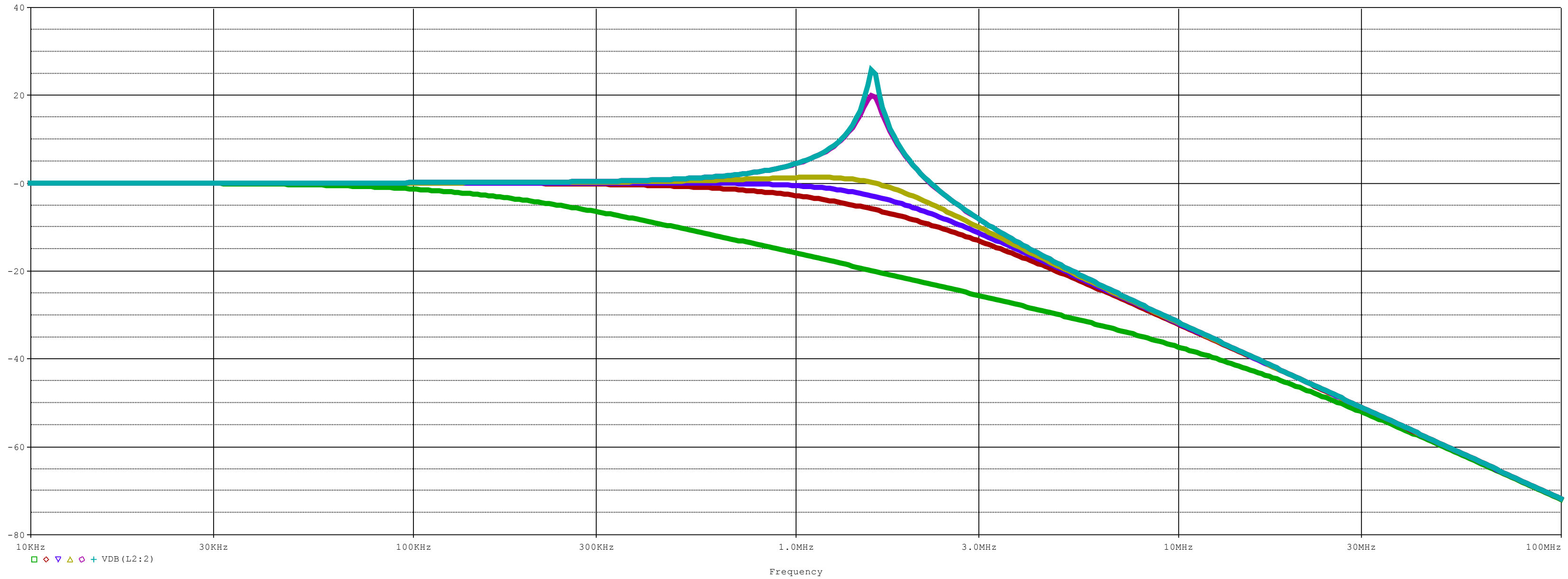
Frequency response with different Q values (From Lect. 18)



Q=0.1, 0.5, 0.7, 1, 10, 20

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$$s_p = \omega_p \exp\left[j \frac{(2k + N - 1)\pi}{2N} \right], \quad k = 1, 2, 3 \ldots N$$

For N=3,

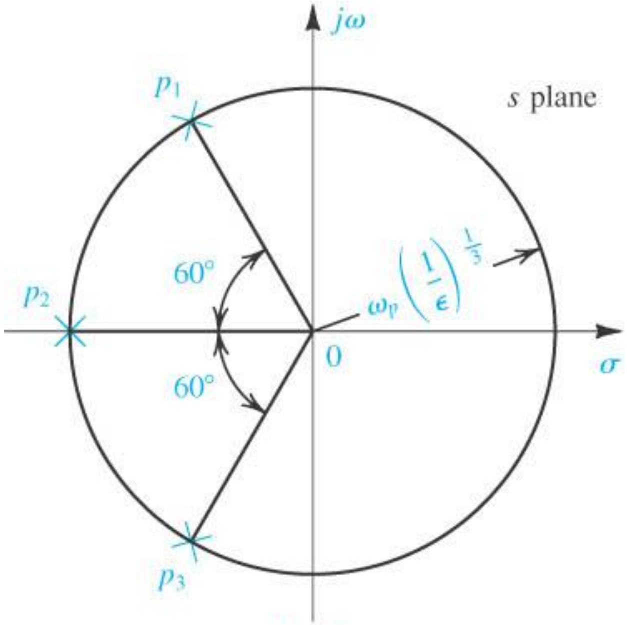
$$s_p = \omega_p \exp\left[j \frac{(2k + 2)\pi}{6} \right], \quad k = 1, 2, 3$$

$$s_{p1} = \omega_p \exp\left(j \frac{2\pi}{3} \right), \quad k = 1$$

$$s_{p2} = \omega_p \exp(j\pi), \quad k = 2$$

$$s_{p3} = \omega_p \exp\left(j \frac{4\pi}{3} \right), \quad k = 3$$

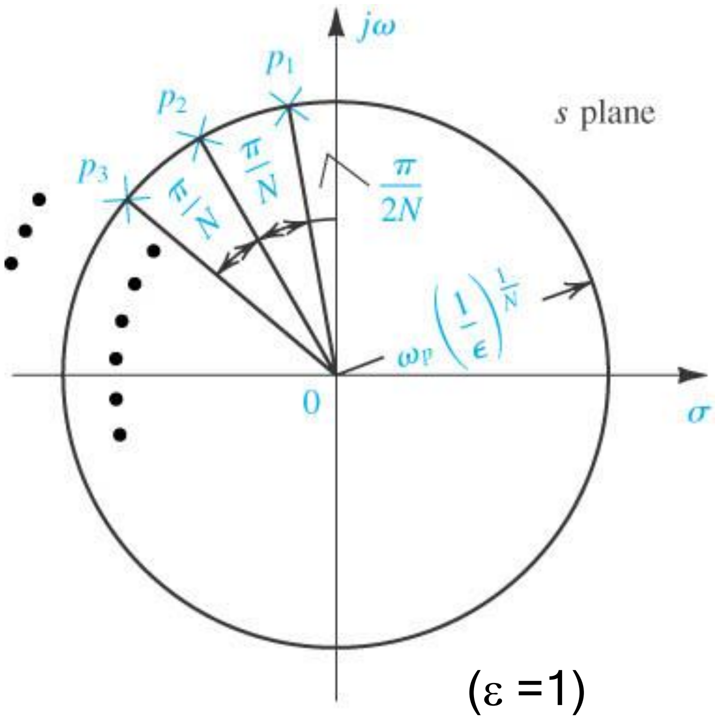
$$T(s) = \frac{\omega_p^3}{(s - s_{p1})(s - s_{p2})(s - s_{p3})}$$



(ε =1)

(c)

Butterworth LP Filter



(a)

$$T(s) = \frac{\omega_p^N}{(s - s_{p1})(s - s_{p2}) \cdots (s - s_{pN})}$$

$$s_{p,k} = \omega_p \exp\left[j \frac{(2k + N - 1)\pi}{2N} \right]$$

Lect. 21: Higher-Order Low-Pass Filters

How to design higher-order filters with electronic circuits

1. Select the filter type

2. Determine the required transfer function that satisfies the requirement

   In the case of Butterworth LP filter, N and ω_p

3. Design electronic circuits for the required transfer function

   Passive, active, biquad ...

➔ Proejct #2: Butterworth LP filter, biquad, switched capacitor

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Homework:
Determine N and ω_p for a Butterworth LP filter that satisfies the filter specification shown below.