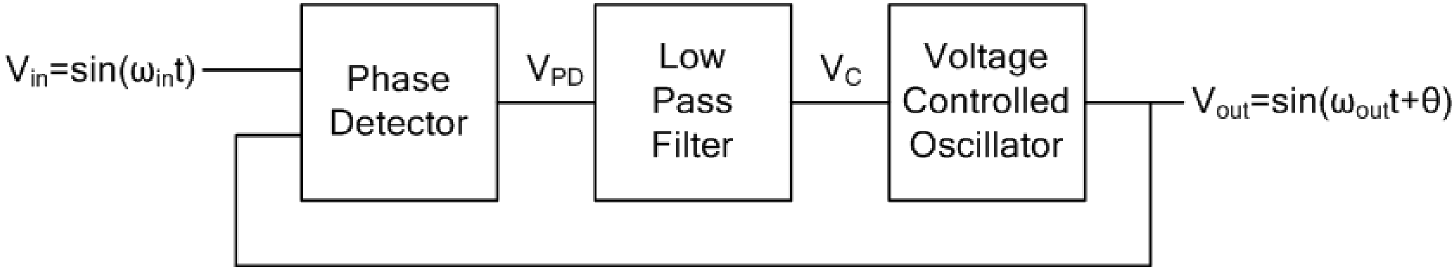
Lect. 26: PLL Dynamics

PLL Block Diagram



ϕ_{out}/ϕ_{in} in s-domain?

ω_{out}/ω_{in} in s-domain?

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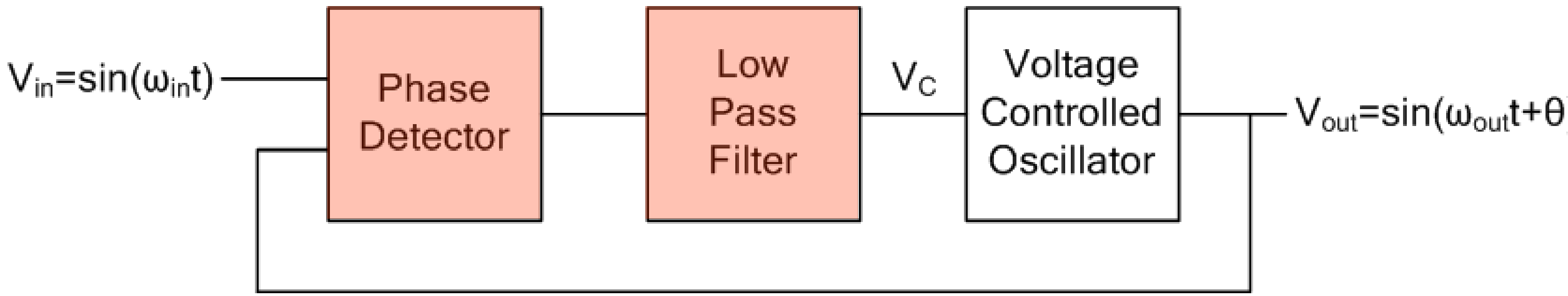
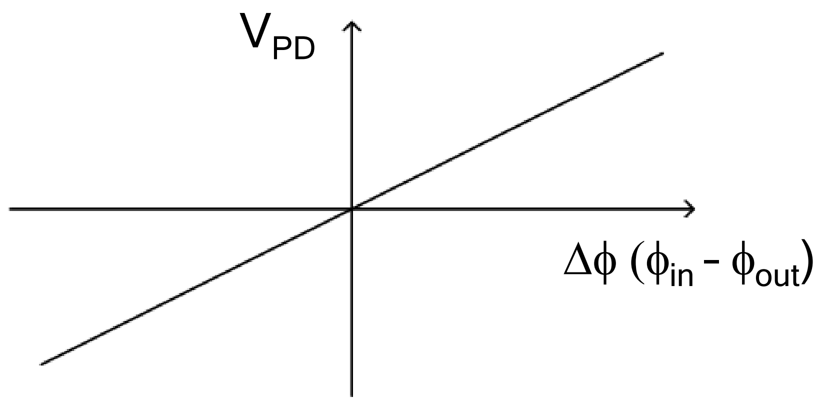


Linear approximation for PD characteristics



$V_{PD} = K_{PD} \times \Delta\phi$

LPF: First order with pole at $-\omega_p$

$$T(s) = \frac{\omega_p}{s+\omega_p}$$

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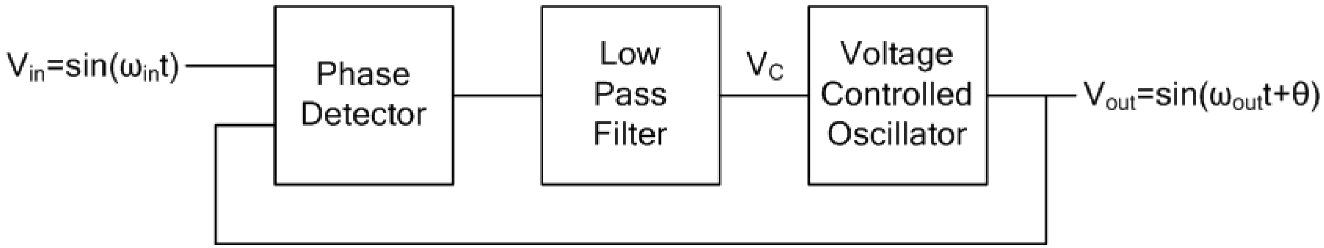

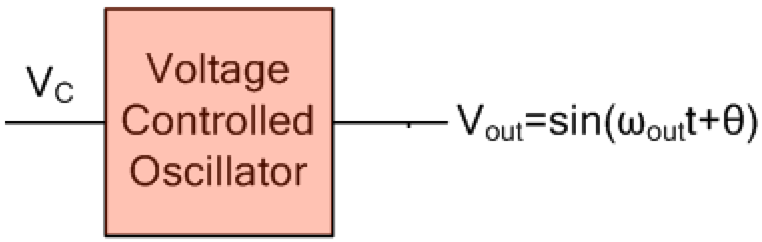

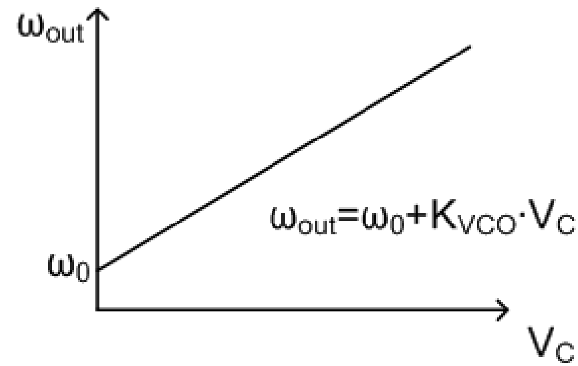

For dynamics, we are interested in the change of ϕ_{out}

$$\phi_{out} = (1/s)\, K_{VCO} \times V_C$$

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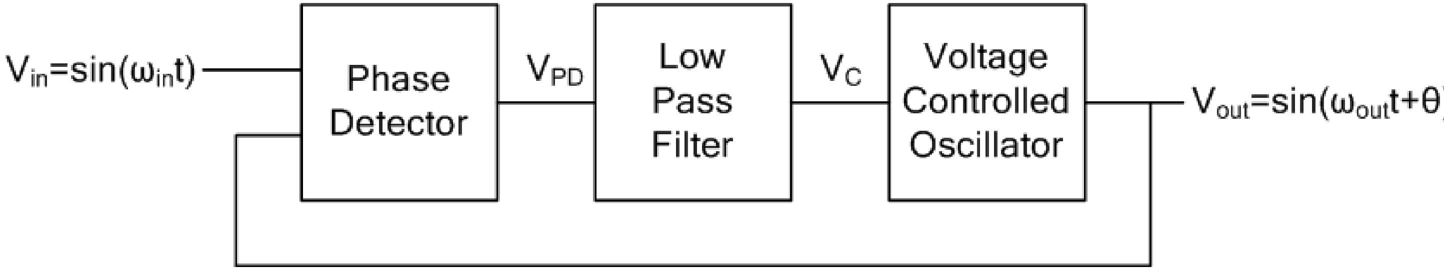
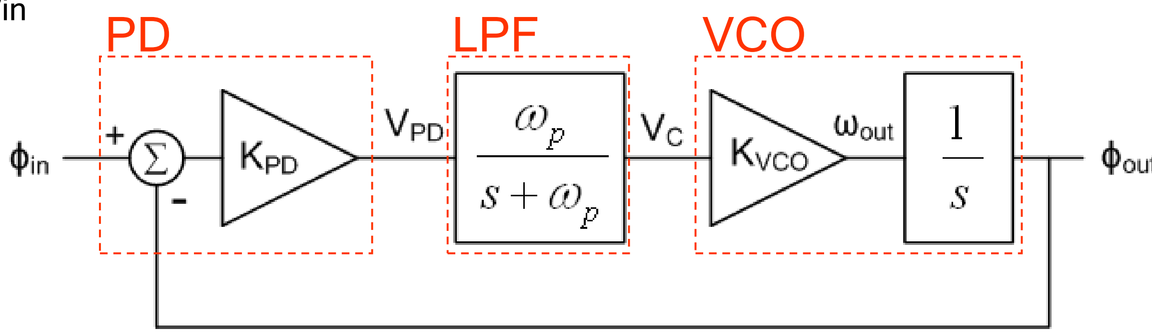
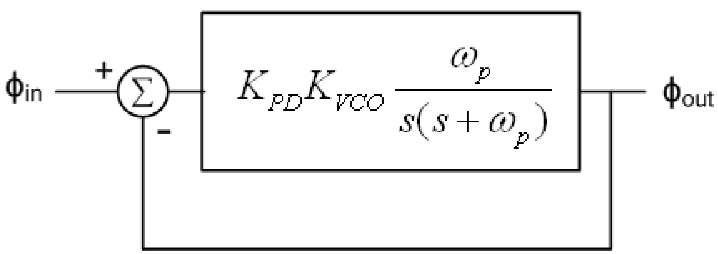
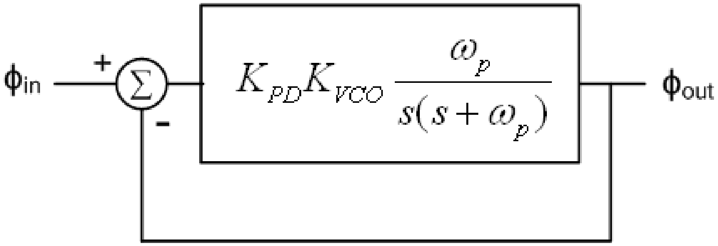


Linear Model for ϕ_{out}/ϕ_{in}





Lect. 26: PLL Dynamics



Open loop gain:

$$G(s) = K_{PD}K_{VCO}\frac{\omega_p}{s(s+\omega_p)}$$

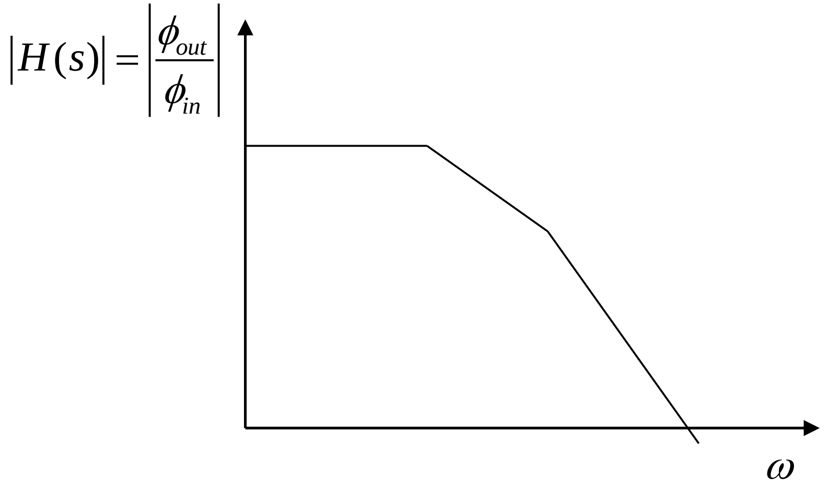
Closed loop gain

$$H(s) = \frac{\phi_{out}}{\phi_{in}} = \frac{G(s)}{1+G(s)} = \frac{K_{PD}K_{VCO}\dfrac{\omega_p}{s(s+\omega_p)}}{1+K_{PD}K_{VCO}\dfrac{\omega_p}{s(s+\omega_p)}} = \frac{K_{PD}K_{VCO}\omega_p}{s^2+\omega_p s+K_{PD}K_{VCO}\omega_p}$$

➔ 2nd order LPF!

Lect. 26: PLL Dynamics

$$H(s) = \frac{\phi_{out}}{\phi_{out}} = \frac{K_{PD} K_{VCO} \omega_p}{s^2 + \omega_p s + K_{PD} K_{VCO} \omega_p}$$



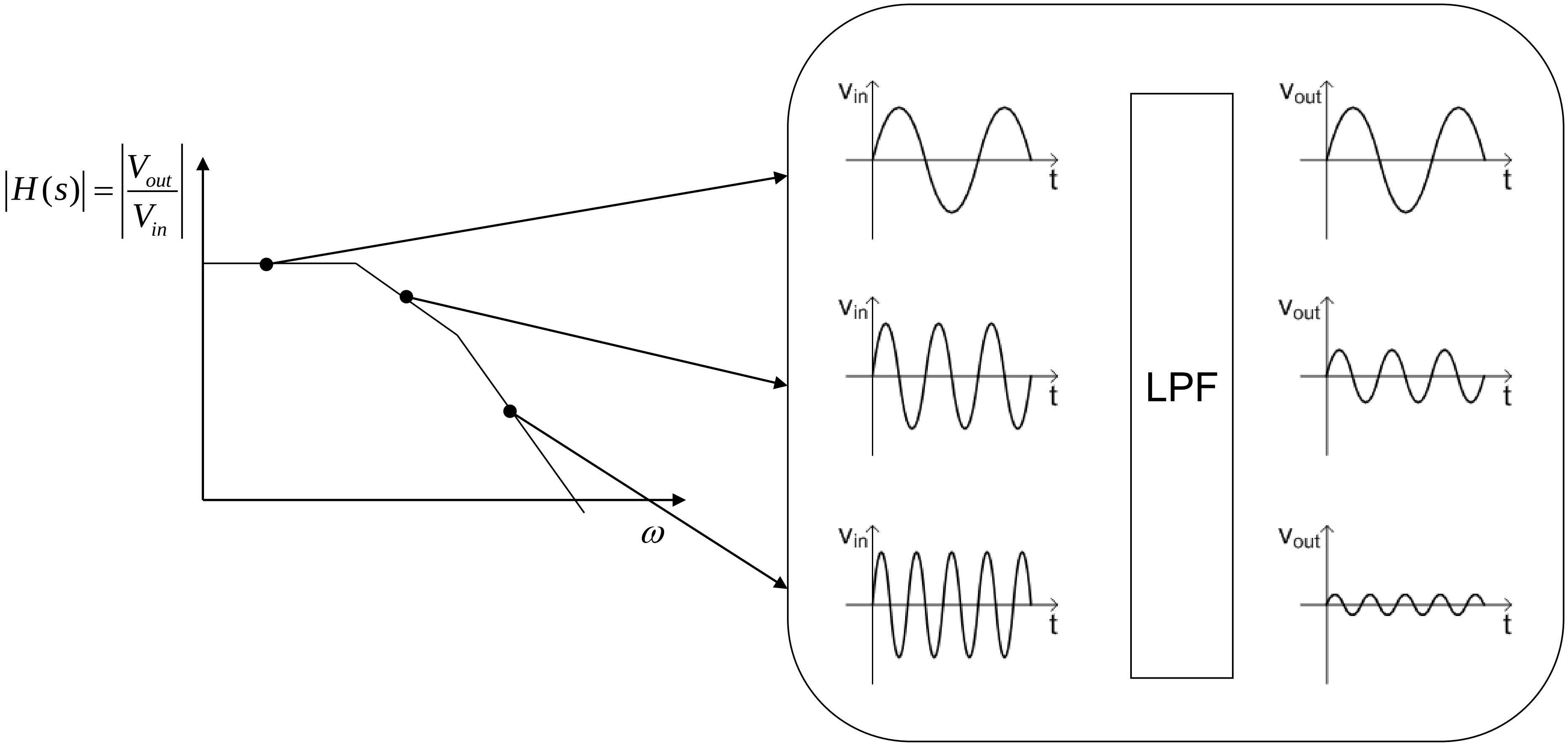
(Assuming two real poles)

Note that input and output are *'phase'*.

What does ω mean in x-axis?

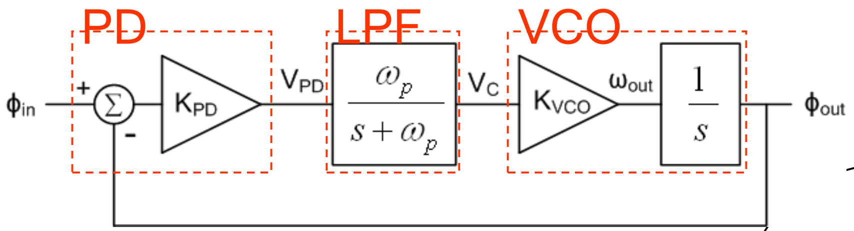
Lect. 26: PLL Dynamics

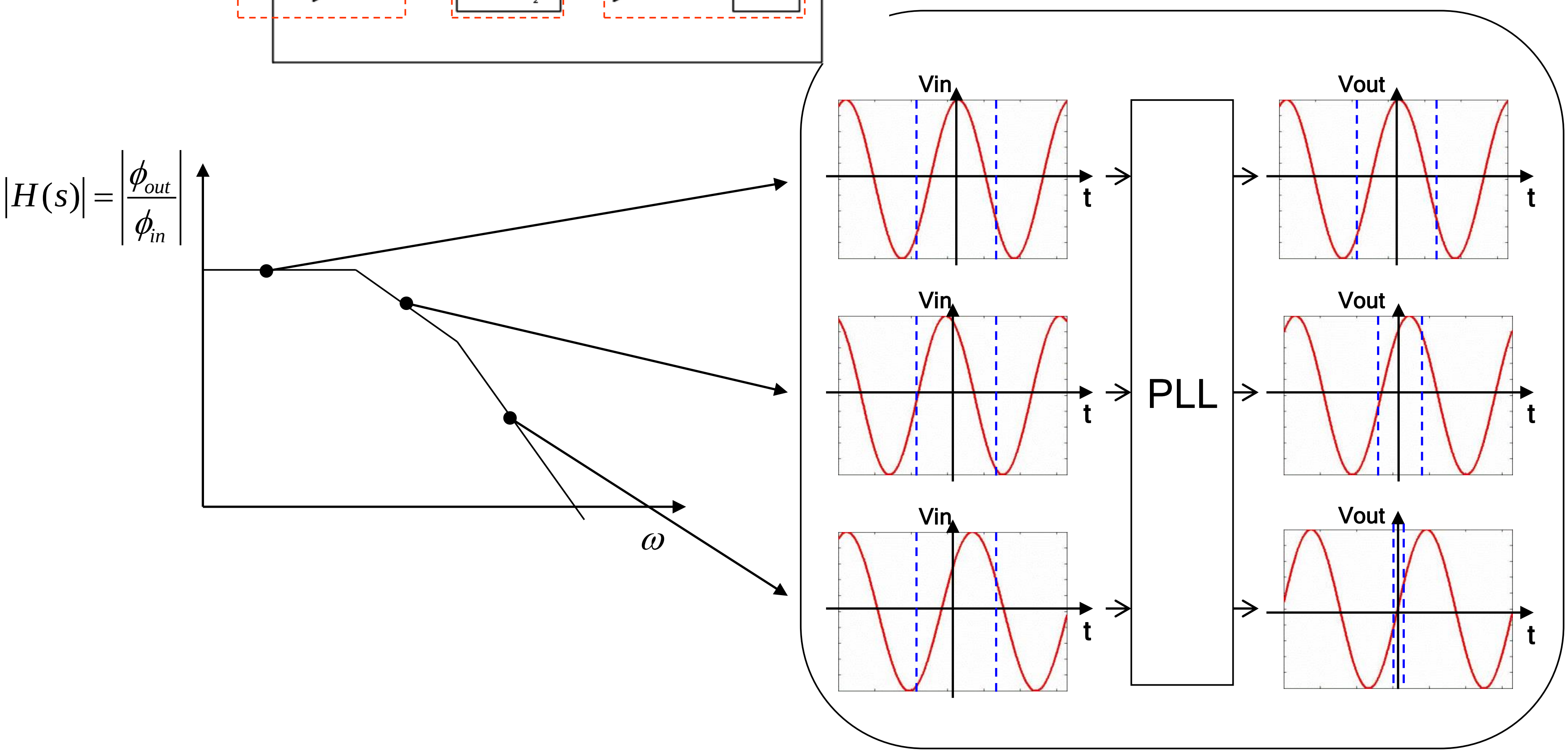
In LPF,



Lect. 26: PLL Dynamics

In PLL,





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Lect. 26: PLL Dynamics

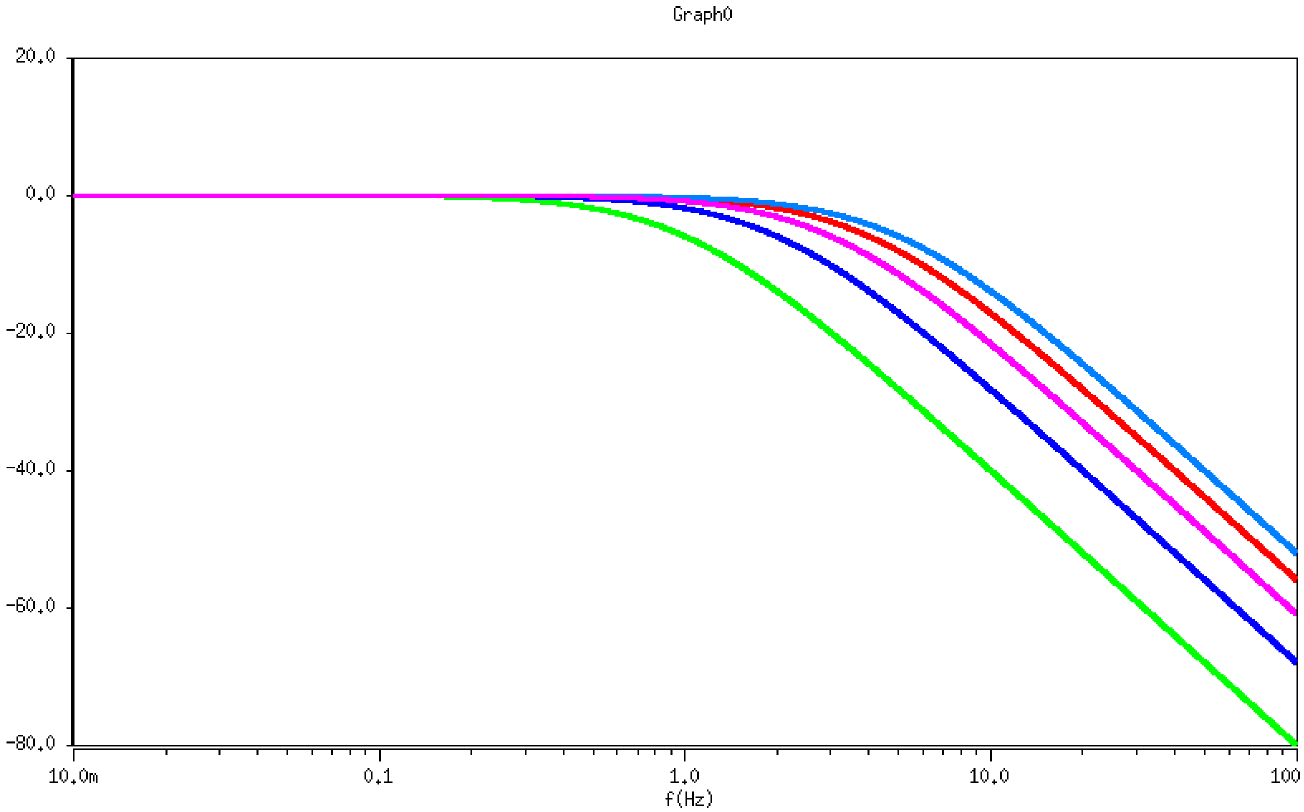
$$H(s) = \frac{\phi_{out}}{\phi_{in}} = \frac{K_{PD}K_{VCO}\omega_p}{s^2 + \omega_p s + K_{PD}K_{VCO}\omega_p}$$

2nd order system $\boldsymbol{H(s) = \frac{a_0}{s^2 + (\omega_0 / Q)s + \omega_0^2}}$

$$\omega_0 = \sqrt{K_{PD}K_{VCO}\omega_P} \qquad Q = \sqrt{\frac{K_{PD}K_{VCO}}{\omega_P}} \qquad a_0 = \omega_0^2$$

$$\frac{\omega_{out}}{\omega_{in}} = \frac{s\phi_{out}}{s\phi_{in}} = \frac{\phi_{out}}{\phi_{in}} = \frac{\omega_0^2}{s^2 + (\omega_0 / Q)s + \omega_0^2}$$

ω_o dependence



(Q = 1/2)

$\omega_0 = 2\pi \times 1$

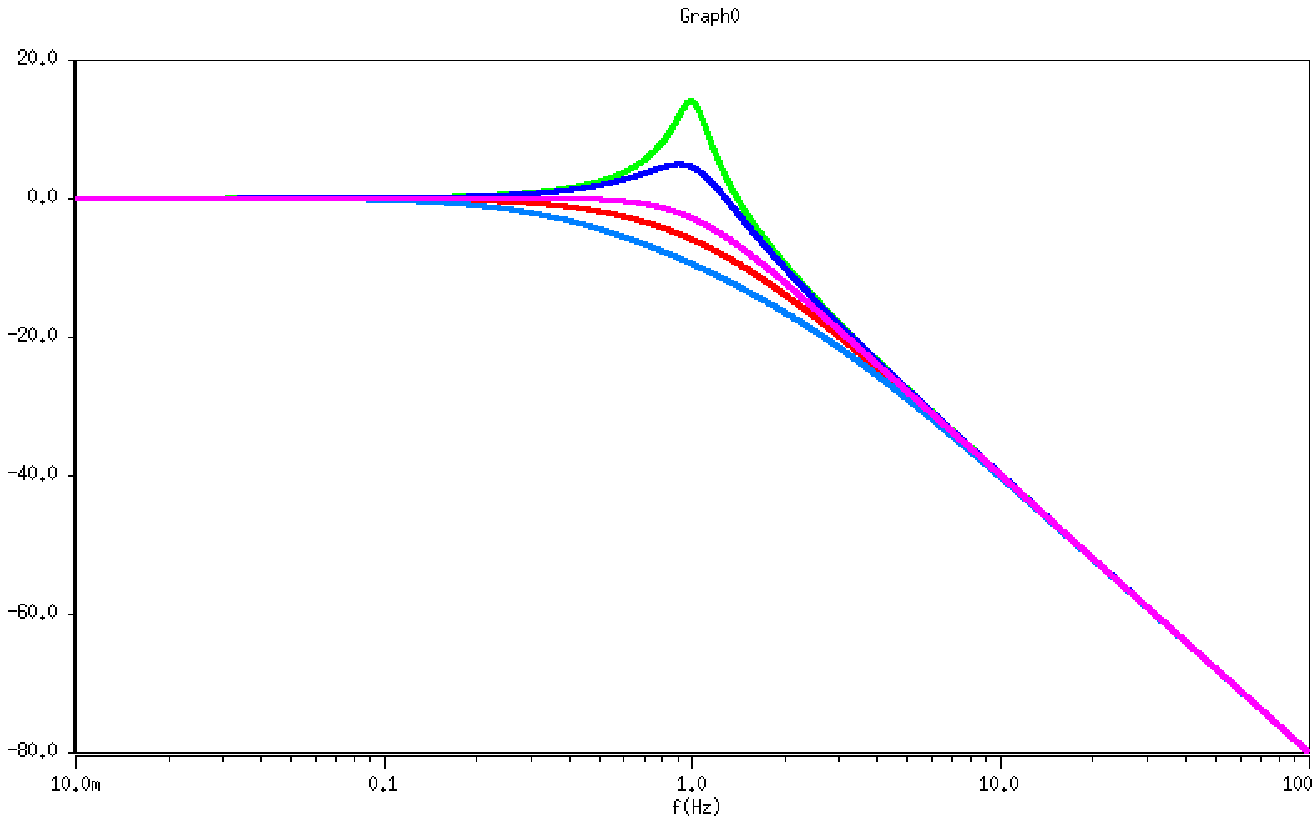
$\omega_0 = 2\pi \times 2$

$\omega_0 = 2\pi \times 3$

$\omega_0 = 2\pi \times 4$

$\omega_0 = 2\pi \times 5$

Q dependence



$\omega_n = 2\pi$

Q = 5

Q = 1.67

Q = 0.7

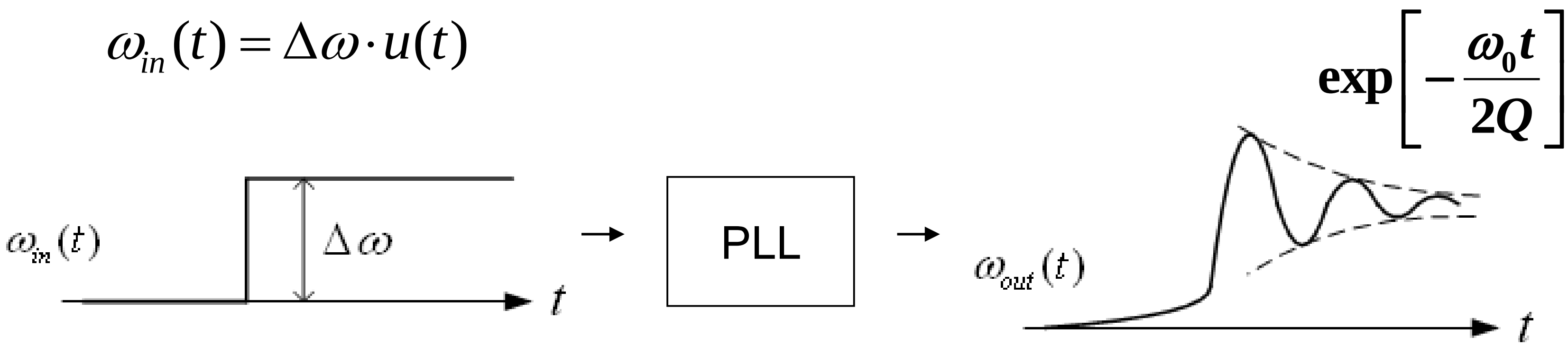
Q = 0.5

Q = 0.33

Lect. 26: PLL Dynamics

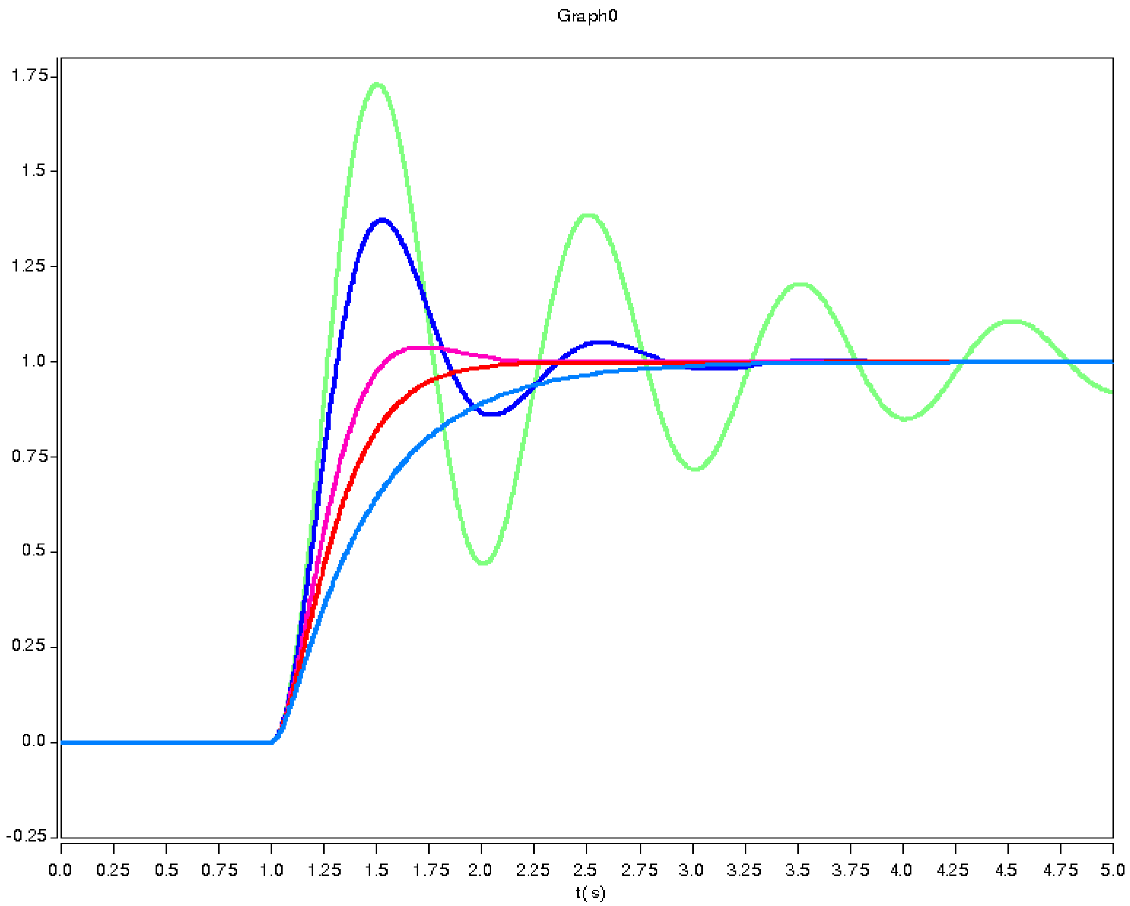
Step response

$$\omega_{in}(t) = \Delta\omega \cdot u(t)$$



$$\omega_{out}(t) = \left\{1 - e^{-\frac{\omega_0}{2Q}t}\left[\cos\left(\omega_0\sqrt{1-\frac{1}{4Q^2}}\cdot t\right) + \frac{1}{\sqrt{4Q^2}}\sin\left(\omega_0\sqrt{1-\frac{1}{4Q^2}}\cdot t\right)\right]\right\}\Delta\omega \cdot u(t)$$

Step response: Q dependence:



($\omega_n = 2\pi$)

Q = 5

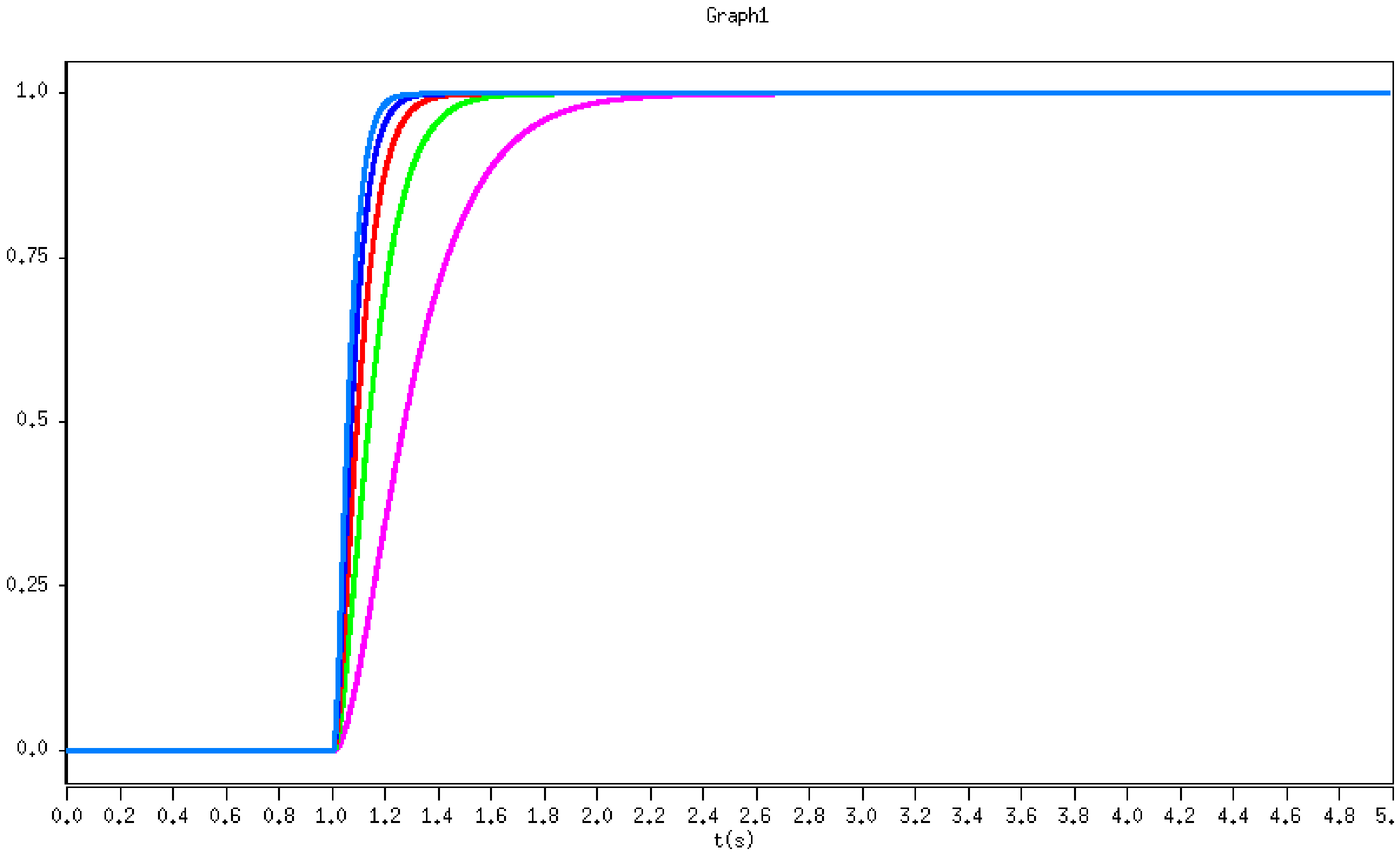
Q = 1.67

Q = 0.7

Q = 0.5

Q = 0.33

Natural frequency dependence: Step Response



(Q = 1/2)

$\omega_n = 2\pi \times 1$

$\omega_n = 2\pi \times 2$

$\omega_n = 2\pi \times 3$

$\omega_n = 2\pi \times 4$

$\omega_n = 2\pi \times 5$

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