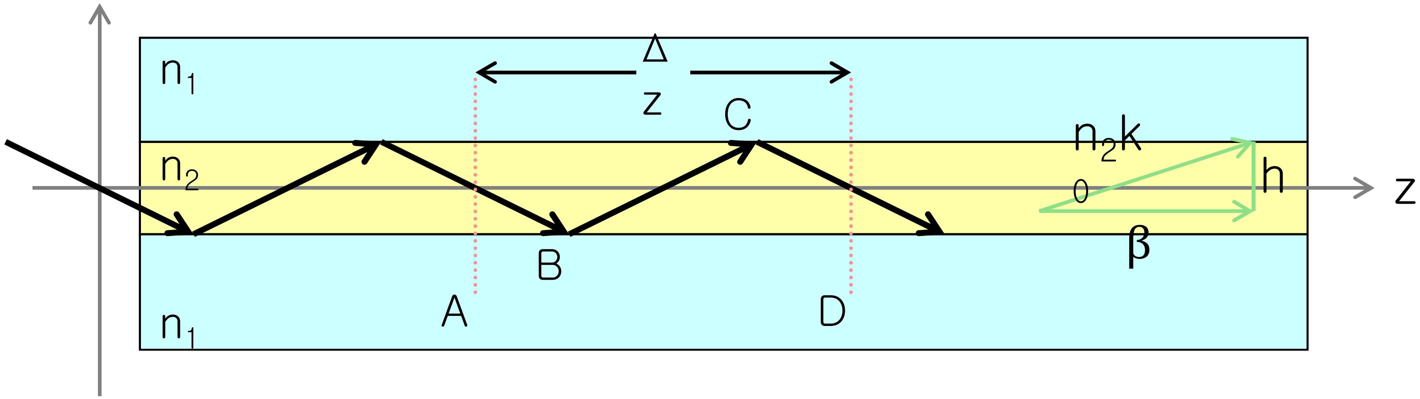
Lect. 4 Waveguides (1)

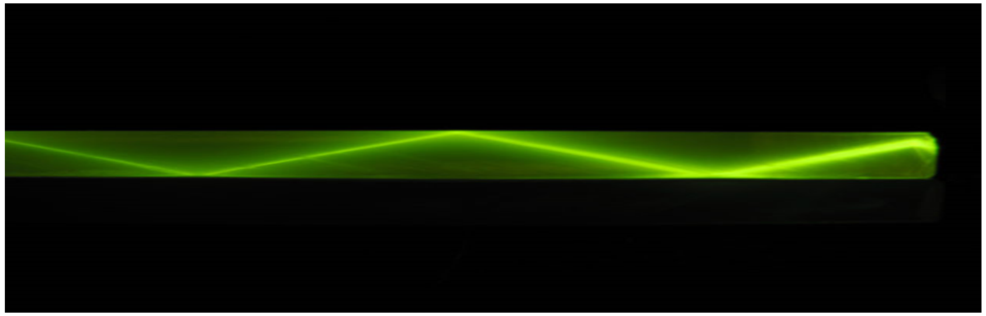
- Waveguide: Confines and guides EM waves

➔ Metallic, Dielectric, Plasmonic

- We are interested in ***dielectric waveguide***

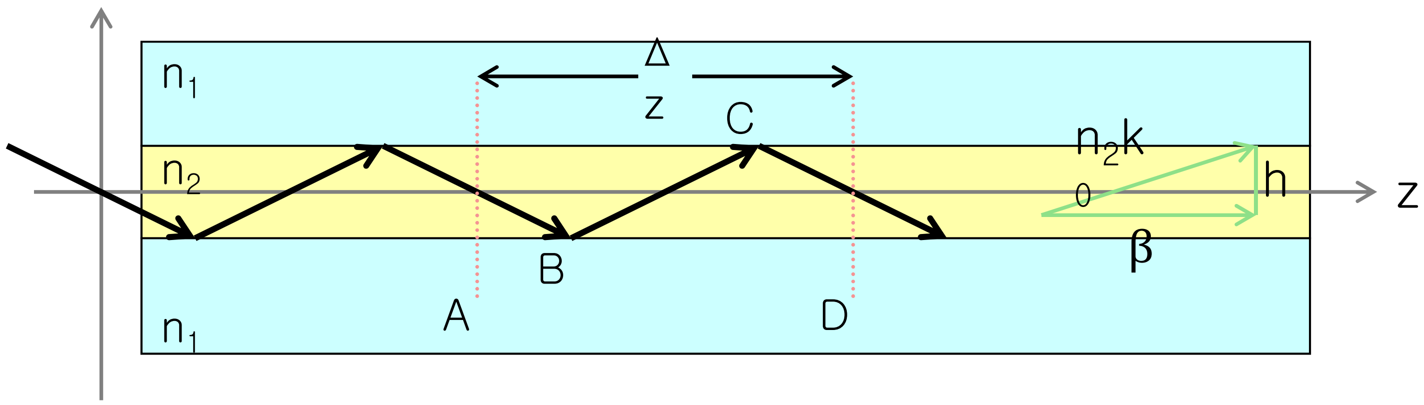
➔ Total internal reflection by refractive index differences





Lect. 4 Waveguides (1)

Total internal reflection by refractive index differences



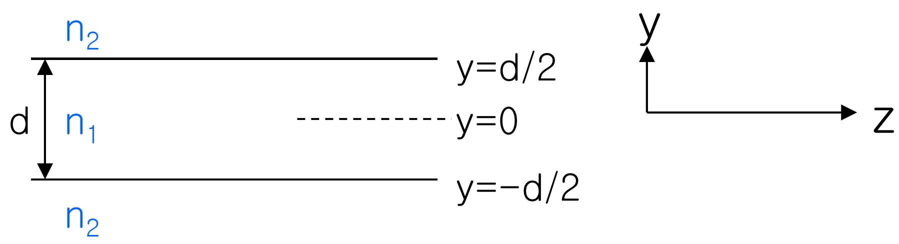
- Conditions for guidance

- Characteristics of guided light in a dielectric waveguide:
  Mode, Effective index, Confinement factor

- How to make dielectric waveguides

Lect. 4 Waveguides (1)

- Conditions for guided EM waves



- Governing equations for EM waves

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

$$\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t}$$

$$\nabla \bullet \overline{D} = \rho \qquad \begin{cases} \overline{D} = \varepsilon \overline{E} \\ \overline{B} = \mu \overline{H} \end{cases}$$

$$\nabla \bullet \overline{B} = 0$$

$$\nabla^2 \overline{E} = \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2} \qquad \nabla^2 \overline{H} = \mu\varepsilon \frac{\partial^2 \overline{H}}{\partial t^2}$$

EM Wave Equations

➔ Governs light propagation

- Solutions for Wave Equations

$$\nabla^2 \overline{E} = \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2}$$

Plane-wave solution: $\overline{E} = \overline{x} E_0 e^{j(\omega t - kz)}$

$$\nabla^2 \overline{E} = \overline{x}(-k^2) E_0 e^{j(\omega t - kz)} \qquad \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2} = \overline{x}\mu\varepsilon(-\omega^2) E_0 e^{j(\omega t - kz)}$$

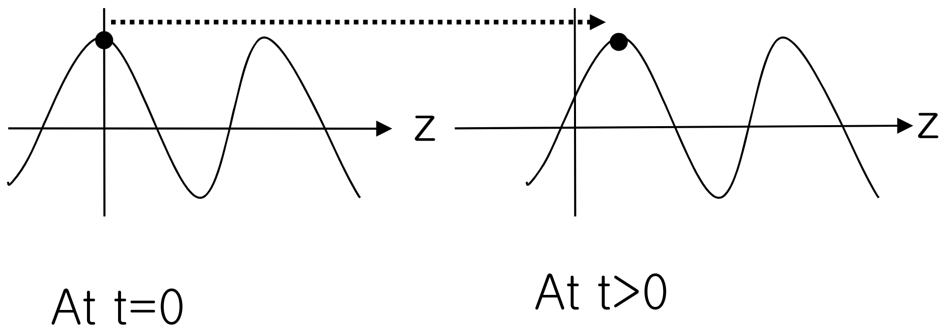
$$k^2 = \mu\varepsilon\omega^2 \qquad \therefore k = \omega\sqrt{\mu\varepsilon}$$

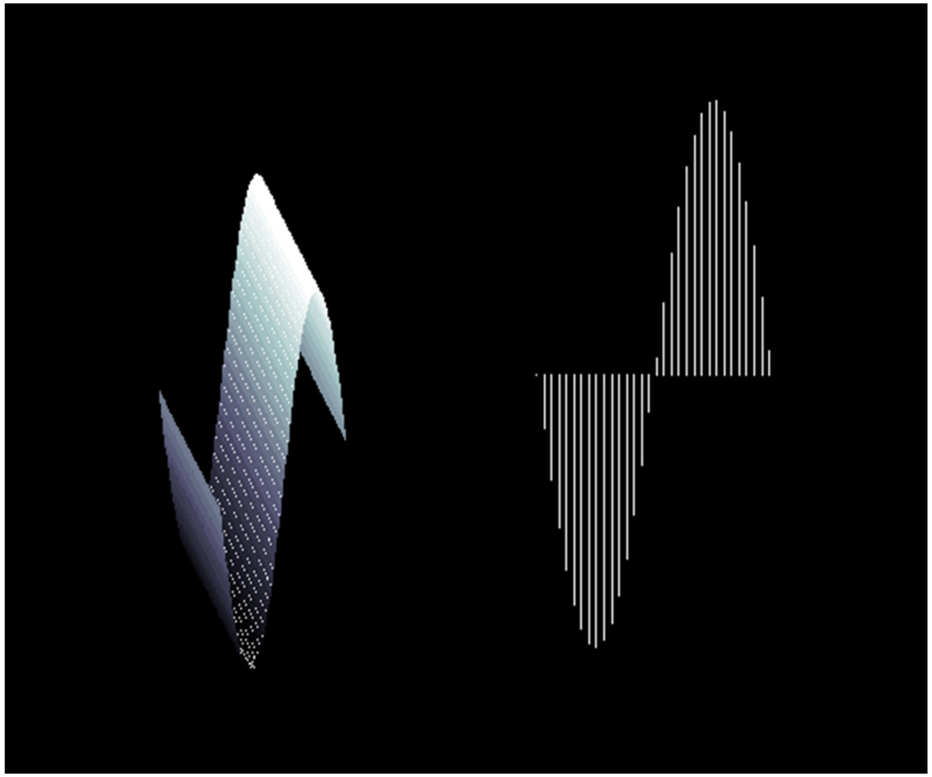
$$\omega = \frac{2\pi}{T} \qquad k = \frac{2\pi}{\lambda} \qquad \frac{\lambda}{T} = \frac{1}{\sqrt{\mu\varepsilon}}$$

Speed of light!

How does the plane-wave solution look like?

For physical representation, $\mathrm{Re}\left[\bar{x}E_0 e^{j(\omega t - kz)}\right] = \bar{x}E_0 \cos(\omega t - kz)$





Lect. 4 Waveguides (1)

How about H-field?

$$\nabla^2 \overline{E} = \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2} \qquad \overline{E} = \overline{x} E_0 e^{j(\omega t - kz)}$$

From Maxwell's Equations, $\overline{H} = \overline{y} \frac{\varepsilon}{\mu} E_0 e^{j(\omega t - kz)}$

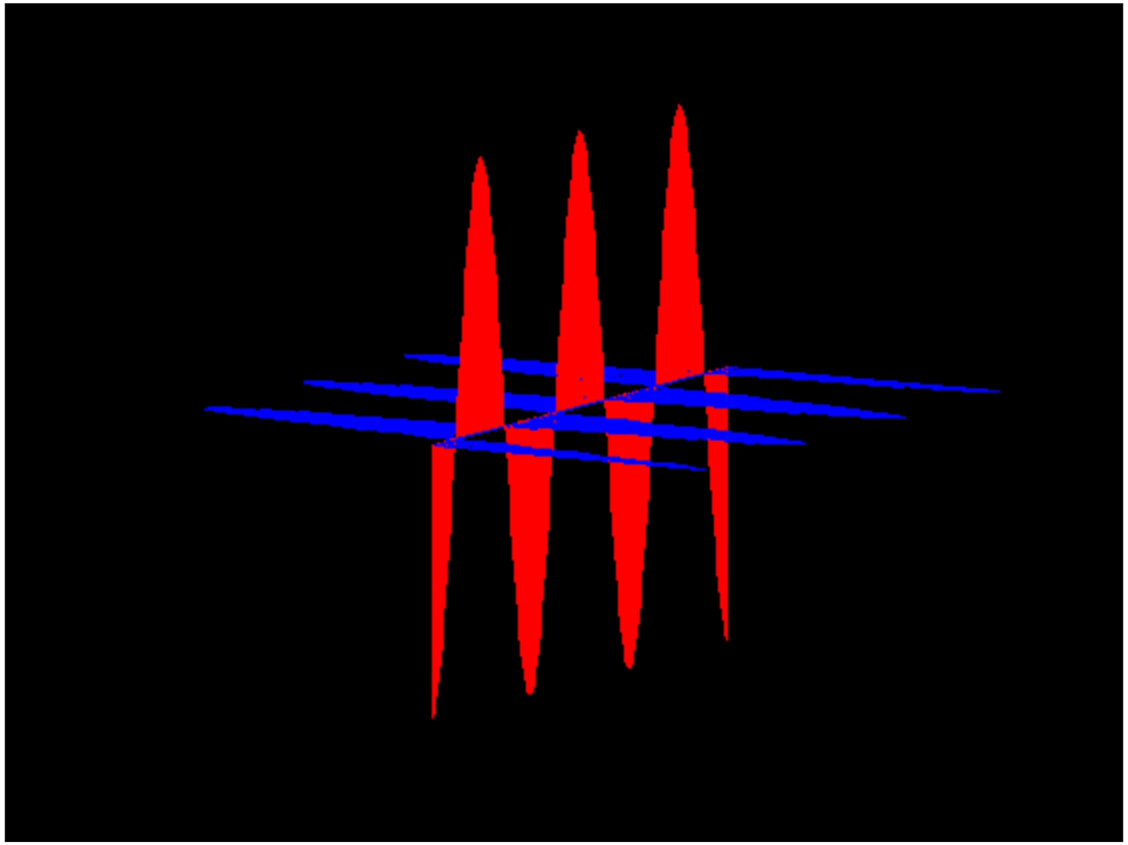
Direction of propagation?

Direction of E, H fields?

Speed of propagation?

$$\frac{E_0}{H_0} = \sqrt{\frac{\mu}{\varepsilon}} = \eta \quad (377\Omega \text{ for vacuum})$$

How does the plane-wave solution look like?



Lect. 4 Waveguides (1)

When a wave is propagating into   Plane wave solutions

+z direction: $e^{j(\omega t - kz)}$

-z direction: $e^{j(\omega t + kz)}$

+y direction: $e^{j(\omega t - ky)}$

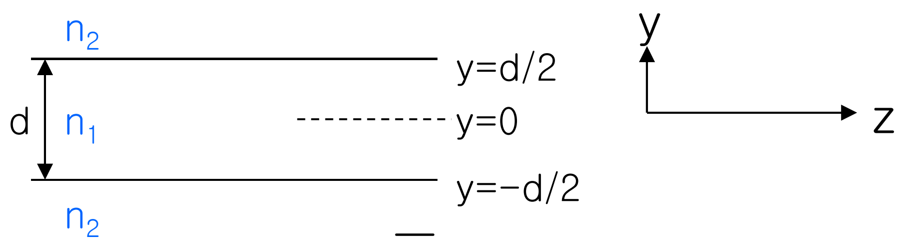
Any direction? $e^{j\omega t} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z} = e^{j(\omega t - \bar{k} \bullet \bar{R})}$

$$\bar{k} = \bar{x} k_x + \bar{y} k_y + \bar{z} k_z \qquad \bar{R} = \bar{x} x + \bar{y} y + \bar{z} z$$

$$\left|\bar{k}\right| = \frac{2\pi}{\lambda}, \ \angle \bar{k}: \text{ direction of propagation}$$

Lect. 4 Waveguides (1)

- Mathematical solutions for guided EM waves



$$\nabla^2 \overline{E}(y,z,t) = \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2}(y,z,t)$$

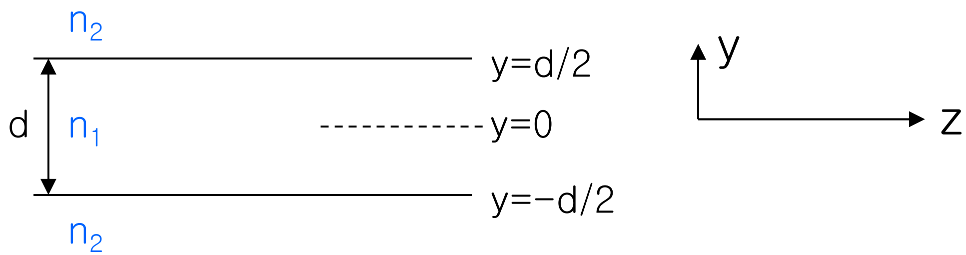
Assuming $\overline{E}(y,z,t) = \overline{E}(y,z) \cdot e^{j\omega t}$,

$$\nabla^2 \overline{E} + k^2(y)\overline{E} = 0, \text{ where } k^2(y) = \mu\varepsilon(y)\omega^2$$

$$k(y) = n_2 k_0 \text{ for } |\mathrm{y}| > \frac{d}{2}; \text{ cladding}$$

$$k(y) = n_1 k_0 \text{ for } |\mathrm{y}| < \frac{d}{2}; \text{ core}$$

Lect. 4 Waveguides (1)



Consider TE Solution (or E having only x-component)

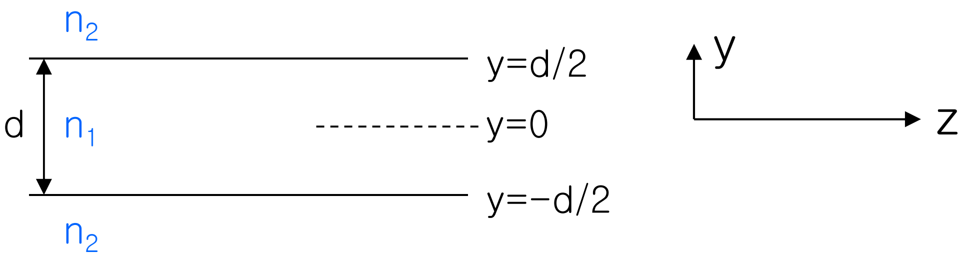
$$\overline{E}(y,z) = \overline{x}\, E(y)\, e^{-j\beta z}$$

Then, $$\frac{d^2 E(y)}{dy^2} + (k^2(y) - \beta^2)E(y) = 0$$

=> Eigen value equation. Solve for β and $E(y)$

$k^2(y) - \beta^2 > 0$ in core => $E(y) \sim \sin(k_y y)$ or $\cos(k_y y)$ with $k_y = \sqrt{(n_1 k_0)^2 - \beta^2}$

$k^2(y) - \beta^2 < 0$ in cladding => $E(y) \sim \exp(\alpha y)$ or $\exp(-\alpha y)$ with $\alpha = \sqrt{\beta^2 - (n_2 k_0)^2}$



Solutions

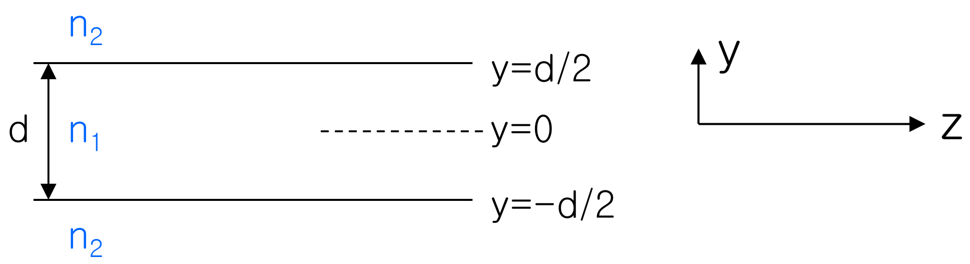
$$y > \frac{d}{2} : E(y) = A\exp(\alpha y) + B\exp(-\alpha y)$$

$$|y| < \frac{d}{2} : E(y) = C\sin(k_y y) + D\cos(k_y y)$$

$$y < -\frac{d}{2} : E(y) = E\exp(\alpha y) + F\exp(-\alpha y)$$

Here, A=0 and F=0.
For easy analysis, divide the solutions into even and odd solutions

Lect. 4 Waveguides (1)



Even Solutions

$$y > \frac{d}{2} : E(y) = B\exp(-\alpha y)$$

$$|y| < \frac{d}{2} : E(y) = D\cos(k_y y)$$

$$y < -\frac{d}{2} : E(y) = B\exp(\alpha y)$$

$(E = B)$

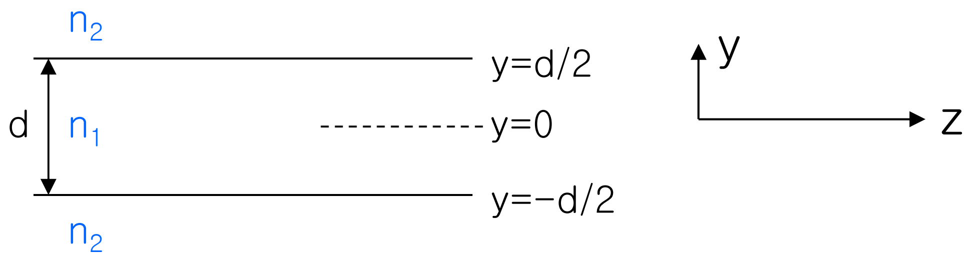
Apply boundary conditions:

$E(y)$ and $\dfrac{dE(y)}{dy}$ are continuous at $y = \pm\dfrac{d}{2}$

$$B\exp(-\alpha\frac{d}{2}) = D\cos(k_y\frac{d}{2}) \quad \text{------- (1)}$$

$$-\alpha B\exp(-\alpha\frac{d}{2}) = -k_y D\sin(k_y\frac{d}{2}) \quad \text{----- (2)}$$

$$\frac{(2)}{(1)} \Longrightarrow \alpha = k_y\tan(k_y\frac{d}{2})$$



Odd Solutions

$$y > \frac{d}{2} : E(y) = B\exp(-\alpha y)$$

$$|y| < \frac{d}{2} : E(y) = D\sin(k_y y)$$

$$y < -\frac{d}{2} : E(y) = -B\exp(\alpha y)$$

$$(E = -B)$$

Apply boundary conditions.

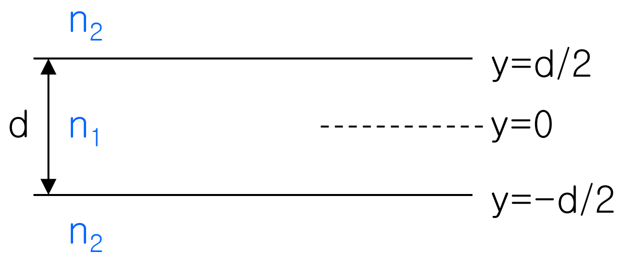
$E(y)$ and $\frac{dE(y)}{dy}$ are continuous at $y = \pm\frac{d}{2}$

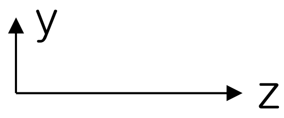
$$B\exp(-\alpha\frac{d}{2}) = D\sin(k_y\frac{d}{2}) \quad \text{-------- (1)}$$

$$-\alpha B\exp(-\alpha\frac{d}{2}) = k_y D\cos(k_y\frac{d}{2}) \quad \text{------ (2)}$$

$$\frac{(2)}{(1)} \Longrightarrow \alpha = -k_y\cot(k_y\frac{d}{2}) = k_y\tan(k_y\frac{d}{2} - \frac{\pi}{2})$$

Lect. 4 Waveguides (1)





Even: $\alpha = k_y \tan(k_y \frac{d}{2})$

What do these mean?

Odd: $\alpha = k_y \tan(k_y \frac{d}{2} - \frac{\pi}{2})$

Determine k_y and α that satisfy above conditions.

For graphical analysis, do following normalization.

Let $X = k_y \frac{d}{2}$, $Y = \alpha \frac{d}{2}$

Then, $Y = X \tan X$ for even

$Y = X \tan(X - \frac{\pi}{2})$ for odd

$$\text{But } X^2 + Y^2 = \left(\frac{d}{2}\right)^2 (k_y{}^2 + \alpha^2) = \left(\frac{d}{2}\right)^2 [(n_1 k_0)^2 - \beta^2 + \beta^2 - (n_2 k_0)^2]$$

$$= \left(\frac{d}{2}\right)^2 k_0{}^2 (n_1{}^2 - n_2{}^2) = r^2$$

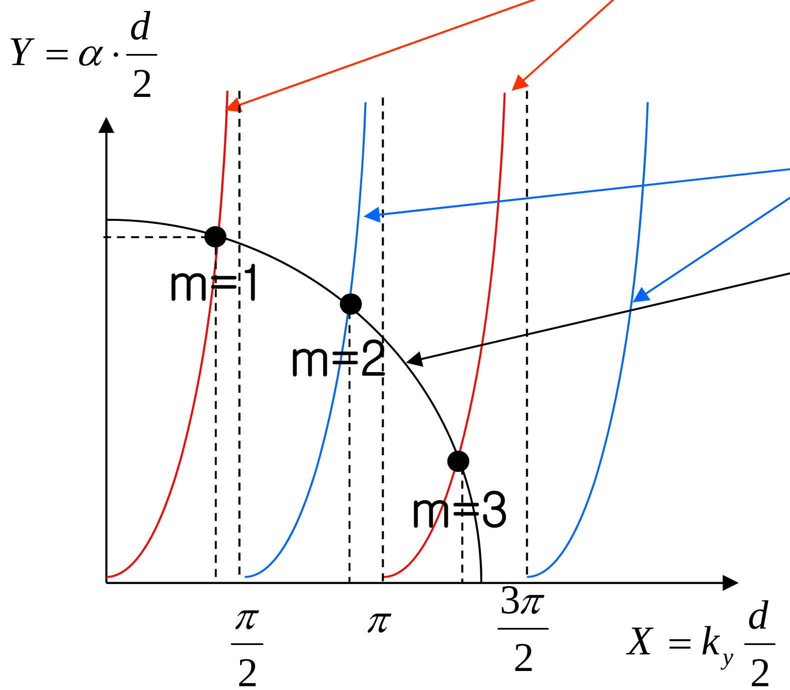
Plot these on X-Y plane.

Lect. 4 Waveguides (1)

$Y = X \tan X$: even mode

$Y = X \tan(X - \pi/2)$: odd mode

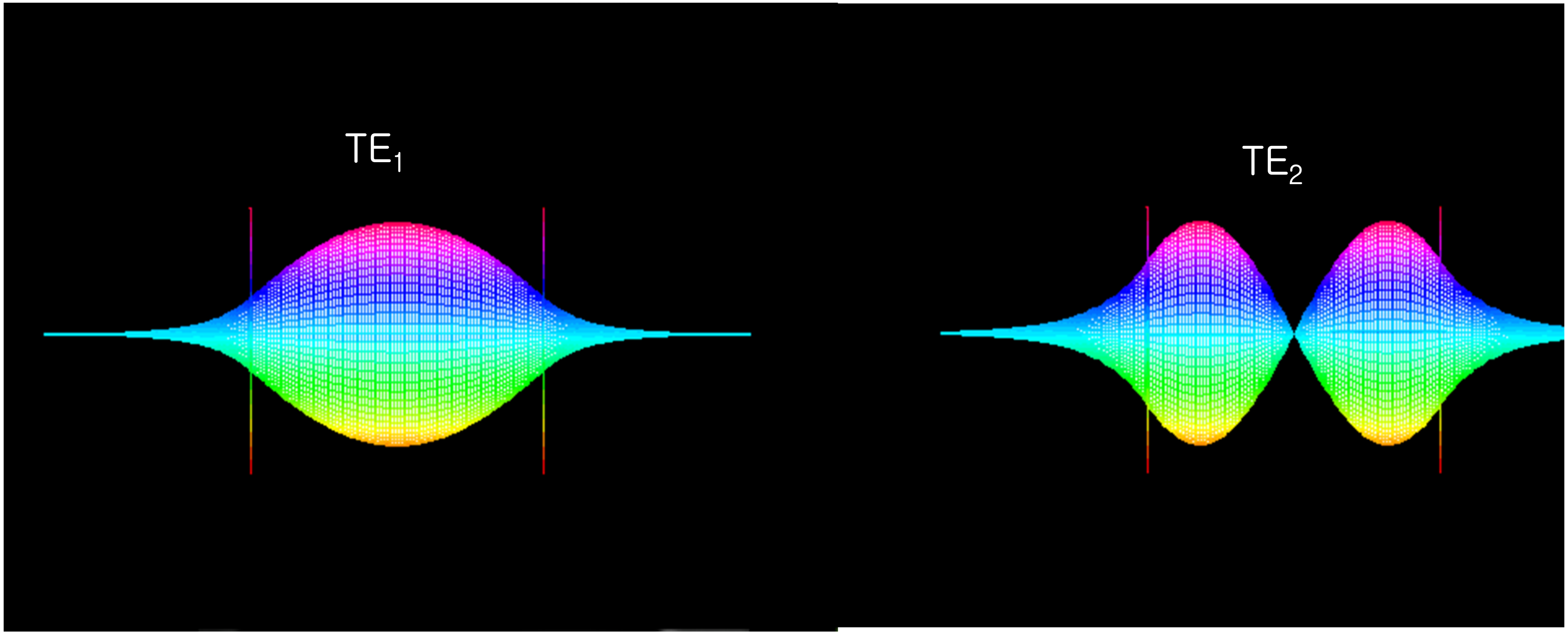
$$X^2 + Y^2 = \left(\frac{d}{2}\right)^2 k_0^2 (n_1^2 - n_2^2) = r^2$$



Observations:

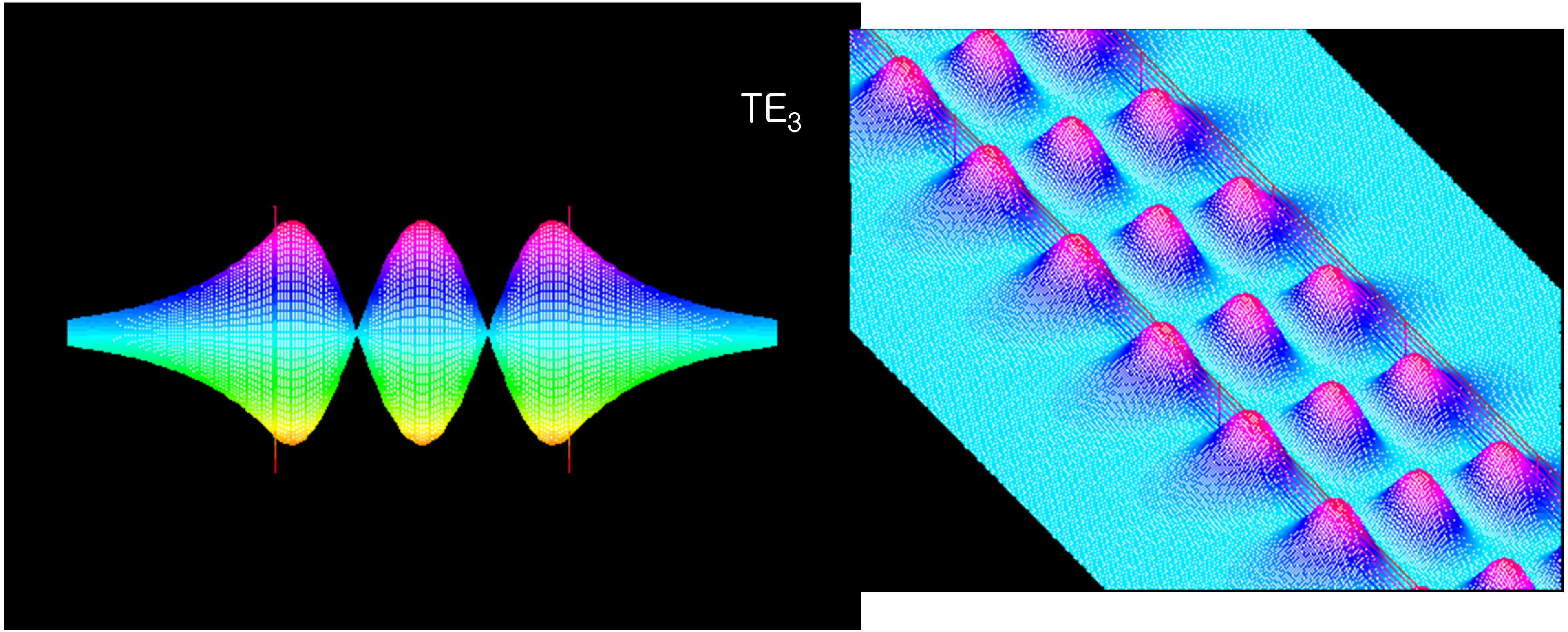
- Points where circle and tangent curves intersect are solutions ➔ mode
- With larger r (larger d, smaller λ, larger $n_1^2 - n_2^2$), more modes exist in the waveguide

E(y) profile: n_1=1.5, n_2=1.495, d=10μm, λ=1μm



Lect. 4 Waveguides (1)

E(y) profile: n_1=1.5, n_2=1.495, d=10μm, λ=1μm



Lect. 4 Waveguides (1)

- Effective Index: N = β/k_0

➔ Different modes have different effective indices: Modal dispersion

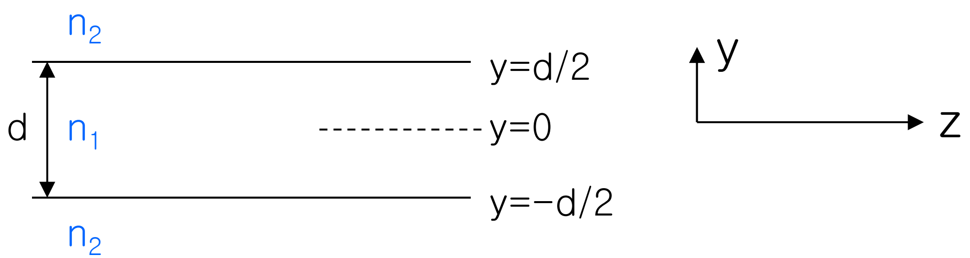
- Confinement factor:

$$\Gamma = \frac{\text{Power inside core}}{\text{Total Power}} = \frac{\int_{y=-\frac{d}{2}}^{y=\frac{d}{2}} |E(y)|^2 \, dy}{\int_{y=-\infty}^{y=\infty} |E(y)|^2 \, dy}$$

For higher modes, how does Γ change?

Lect. 4 Waveguides (1)

– Other polarization?



TE solution (or E having only x-component) was assumed

$$\overline{E}(y,z) = \overline{x}\, E(y)\, e^{-j\beta z}$$

TM solution exists

$$\overline{H}(y,z) = \overline{x}\, H(y)\, e^{-j\beta z}$$

In general, TM solution has different guided-wave characteristics

Lect. 4 Waveguides (1)

Issues for practical waveguides

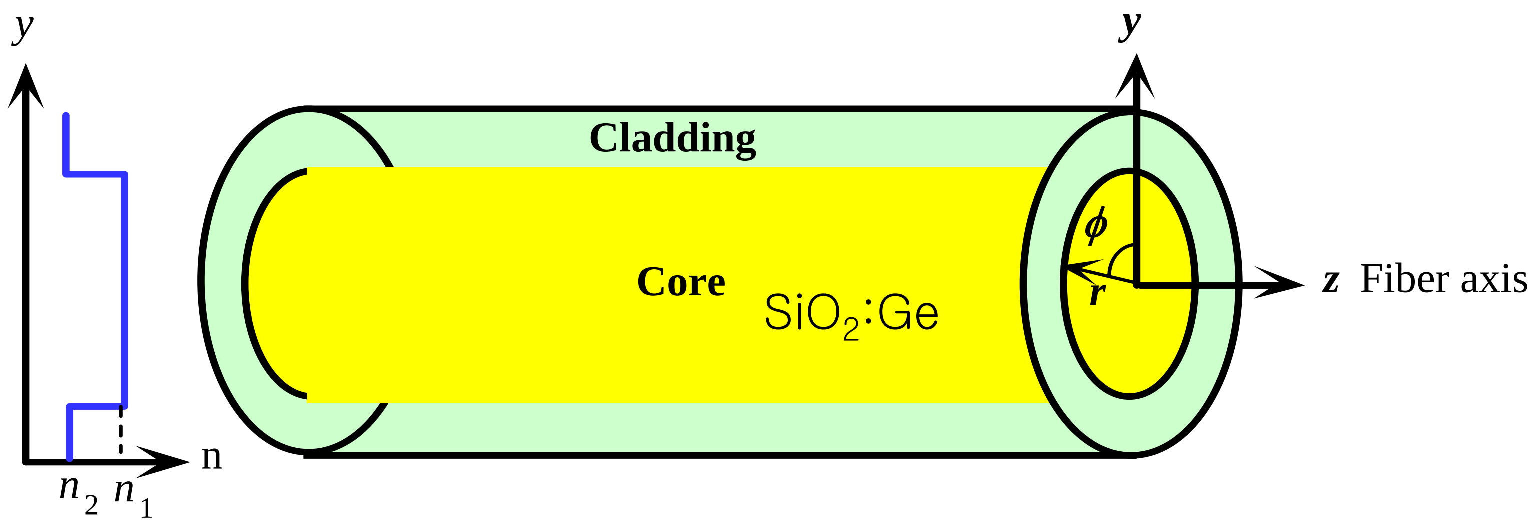
- Precise control of dimension and refractive index
- Low loss at desired λ
- Mass production possible
- Integration desirable
- Electrical control of refractive index and/or absorption

Materials used for waveguides

- Silica (SiO_2 with Ge doping) ➔ Optical fiber
- Dielectric materials: $LiNbO_3$ with Ti doping
- Semiconductors: GaAlAs, InGaAsP, Si/SiO_2

Lect. 4 Waveguides (1)

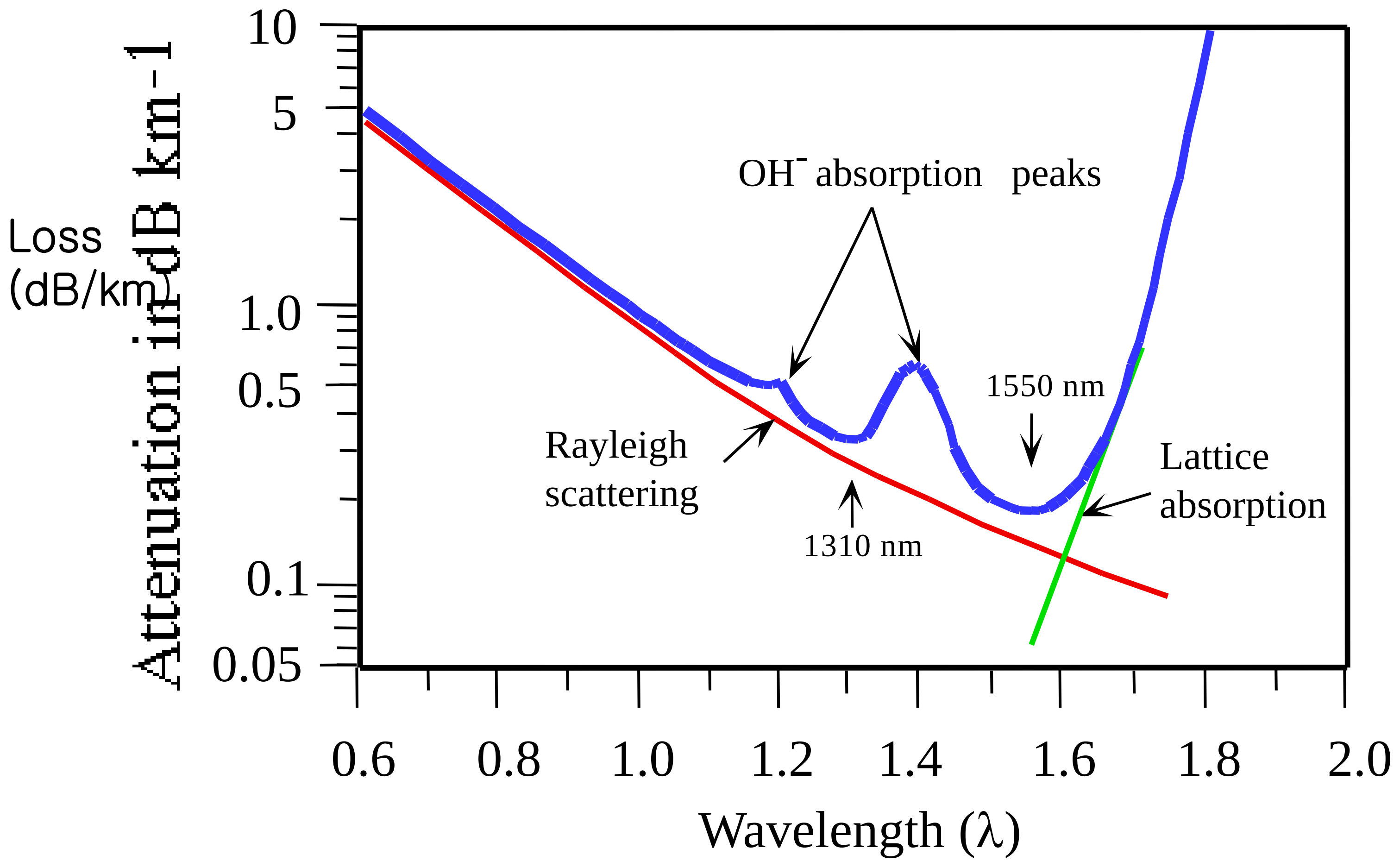
Optical Fiber: Circular dielectric waveguide made of silica (SiO_2)



What is special about fiber?

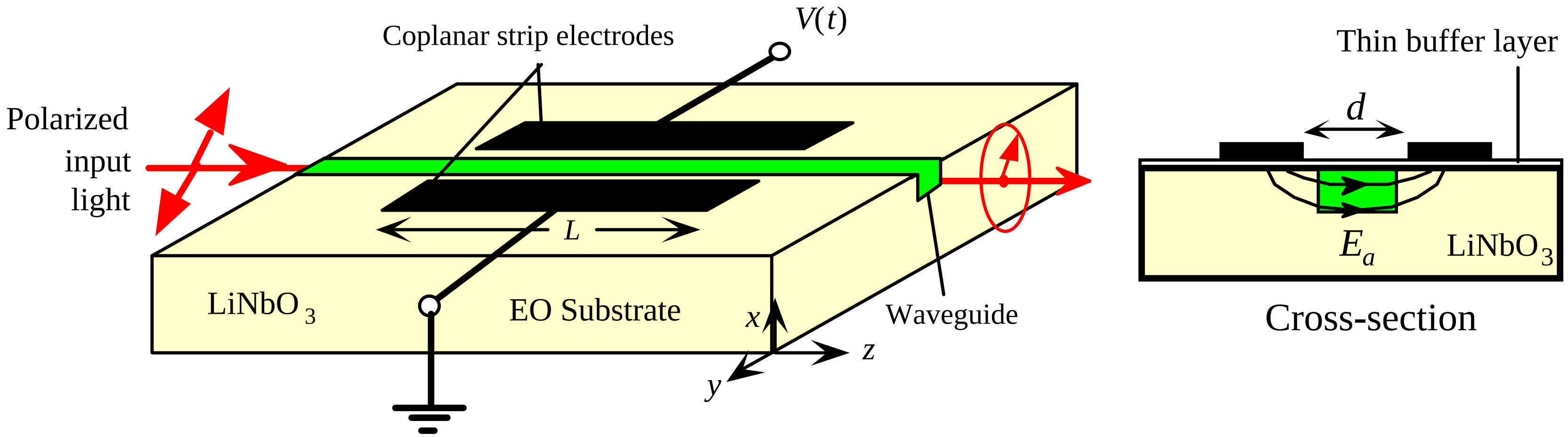
- Extremely low loss: 0.2dB/km
- Can be very long: 100's of km

Loss in fiber



Lect. 4 Waveguides (1)

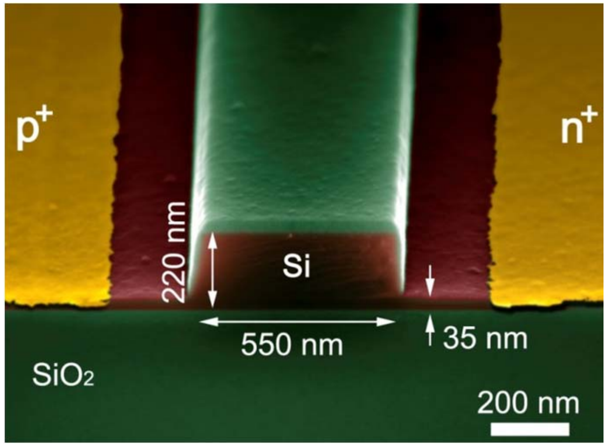
LiNbO$_3$ waveguide

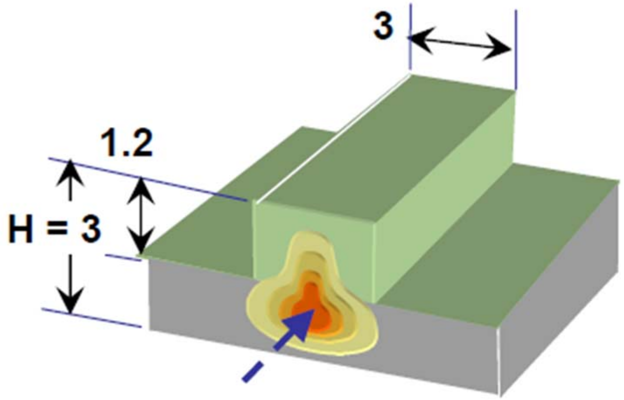


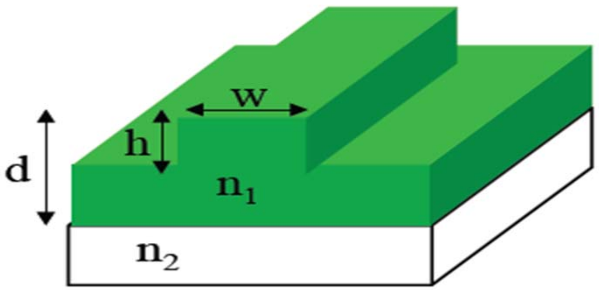
- Used for high-speed optical modulator

Lect. 4 Waveguides (1)

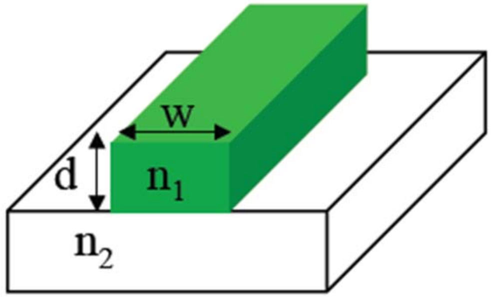
Example of Si/SiO_2 waveguide on SOI wafer fabricated with Si technology







(Rib/Ridge Waveguide)



(Strip/Channel Waveguide)

What affects the characteristics of Si/SiO2 waveguides? Size, scattering loss