Electro-optical effect

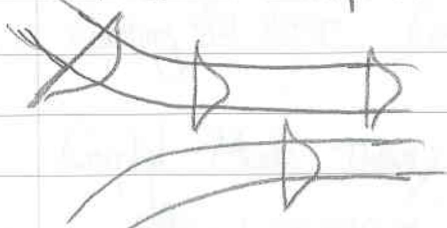
Si, SiO_2의 굴절율 차이가 크므로 Waveguide의 높이와 폭이 작아야 singlemode로 만들수 있다

Strip : etching해서 표면이 rough하므로 scattering loss가 rib보다 훨씬 더 크다.

Waveguide의 Solution들은 모두 orthogonal하다 즉 임의의 신호를 넣으면 여러 mode들의 합으로 표현된다 (Fourier Series 같이).
Singlemode waveguide면 그것만 남고 나머지는 다 radiation wave로 사라진다

Waveguide Devices

1. Directional Coupler

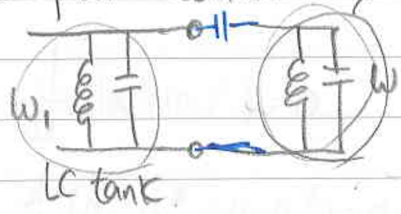


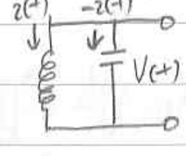
길이에 따라 Power를 모두 전달할 수 있고, 반씩 나누어서 가질수도 있다.

Coupled Oscillator

$V(t) = L\frac{di(t)}{dt}$

Coupling은 작은 Cap을 통해서 연결할 수 있다.





$-i(t) = C\frac{V(t)}{dt}$

$\omega = \frac{1}{\sqrt{LC}}$

$V(t) = -LC\frac{d^2V(t)}{dt^2}$

$\frac{d^2V(t)}{dt^2} + \frac{1}{LC}V(t) = 0$

$V(t), i(t)$로 사용하면 복잡하므로 $a(t)$라는 변수로 해석가능

$a_{\pm}(t) = \sqrt{\frac{C}{2}}V(t) \pm j\sqrt{\frac{L}{2}}i(t)$

$\frac{da_{+}(t)}{dt} = \sqrt{\frac{C}{2}}\frac{dV}{dt} + j\sqrt{\frac{L}{2}}\frac{di(t)}{dt} = \sqrt{\frac{C}{2}}\left(-\frac{i}{C}\right) + j\sqrt{\frac{L}{2}}\cdot\frac{V}{L} = -\frac{i}{\sqrt{2C}} + j\frac{V}{\sqrt{2L}} = j\omega$

$(\omega_0 =$

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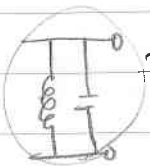
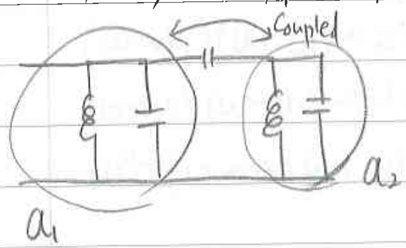
$$\frac{da_-(t)}{dt} = -j\omega_0 a_-(t)$$



$\rightarrow$ a라고 $|a|^2 = a \cdot a^* = a_+ \cdot a_- = \underbrace{\frac{C}{2} \cdot |V|^2}_{\text{Cap의 E}} + \underbrace{\frac{L}{2} |i|^2}_{\text{Inductor의 E.}}$

$\therefore a_\pm = \sqrt{\text{Energy}} \cdot \underbrace{e^{\pm j\omega_0 t}}$ 로 나타낼수 있다.
↳ 미분하면 나오므로

즉 V, i는 계속 변하고 왔다갔다하므로 계산 불편 ∴ Power 측면의 a 사용



Coupling 양이 적으면 Coupled Mode Theory 사용하는

Coupled-Mode Theory

$$\frac{da_1}{dt} = j\omega_1 a_1 + K_{12} a_2$$

$$\frac{da_2}{dt} = j\omega_2 a_2 + K_{21} a_1$$

uncoupled. Coupled.

K_{12}, K_{21} : Coupling coefficient.

$$\frac{d}{dt}(|a_1|^2 + |a_2|^2) = 0$$

$$\frac{d}{dt}(a_1 \cdot a_1^* + a_2 \cdot a_2^*) = a_1 \frac{da_1^*}{dt} + \frac{da_1}{dt} \cdot a_1^* + \quad +$$

$$= a_1(-j\omega_1 a_1^* + K_{12}^* a_2^*) + a_1^*(j\omega_1 a_1 + K_{12} a_2) + \bigcirc + \bigcirc$$

$$\frac{d}{dt}(|a_1|^2 + |a_2|^2) = a_1 K_{12}^* a_2^* + a_1^* K_{12} a_2 + a_2 K_{21}^* a_1^* + a_2^* K_{21} a_1 = 0$$

$$= a_1 a_2^* (K_{12}^* + K_{21}) + a_1^* a_2 (K_{12} + K_{21}^*) = 0$$

즉 initial condition인 a_1, a_2는 다른값이 될수 있으므로 위식에서 영향이 없어야 므

$$\therefore K_{12}^* + K_{21} = 0$$

$K_{12}^* = K_{12,r} - jK_{12,i}$ $\qquad K_{12}, K_{21} \Rightarrow$ imaginary

$K_{21} = K_{21,r} + jK_{21,i}$ $\qquad K_{12} = K_{21} \Rightarrow K_{12} = K_{21} = jK.$

'0'가 되어(더블 자기일 있다)

⇒ Coupled Mode Theory는

$$\frac{da_1}{dt} = j\omega_1 a_1 + jka_2$$

$$\frac{da_2}{dt} = j\omega_2 a_2 + jka_1$$

$a_1(t)$, $a_2(t) \sim e^{j\omega t}$ (즉 a_1, a_2가 Coupling되었을 때 갖는 고유주파수)

$j\omega a_1(t) = j\omega_1 a_1(t) + ka_2(t)$ ⇒ $a_1(t)[\omega-\omega_1] - ka_2(t) = 0$

$j\omega a_2(t) = j\omega_2 a_2(t) + ka_1(t)$ $\quad a_1(t)[k] + a_2(t)[\omega_2-\omega] = 0$

$(\omega-\omega_1)(\omega_2-\omega) + k^2 = 0$

$(\omega-\omega_1)(\omega-\omega_2) - k^2 = 0$

$\omega^2 - (\omega_1+\omega_2)\omega + \omega_1\omega_2 - k^2 = 0 \Rightarrow \omega = \frac{\omega_1+\omega_2}{2} \pm \frac{\sqrt{(\omega_1+\omega_2)^2 - 4(\omega_1\omega_2 - k^2)}}{2}$

Case i) $\omega_1 = \omega_2 = \omega_0$

$\omega = \omega_0 \pm \frac{\sqrt{4\omega_0^2 - 4\omega_0^2 + 4k^2}}{2} = \omega_0 \pm k$ 즉 eigen value가 2개 나온다

if) $\omega = \omega_0 + k$, $a_1, a_2 \sim e^{j(\omega_0+k)t}$, a_1과 a_2의 관계는?

$j(\omega_0+k)a_1 = j\omega_0 a_1 + jka_2$

$j(\omega_0+k)a_2 = j\omega_0 a_2 + jka_1$

$\Rightarrow ka_1 = ka_2 \Rightarrow a_1 = a_2$ (Even mode)

if) $\omega = \omega_0 - k$ 〃

같은 이유로 $\Rightarrow -ka_1 = ka_2 \Rightarrow a_1(t) = -a_2(t)$ (Odd mode)

General Solution

$a_1(t) = Ae^{j(\omega_0+k)t} + Be^{j(\omega_0-k)t}$

$a_2(t) = Ae^{j(\omega_0+k)t} - Be^{j(\omega_0-k)t}$ odd

Even

if) $t=0$일때 $a_1(t), a_2(t) = 0$이면 (초기값)

$A - B = 0$

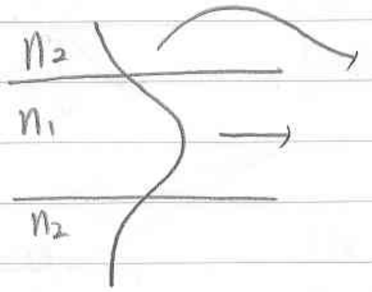
$a_1(t) = Ae^{j(\omega_0+k)t} + Ae^{j(\omega_0-k)t}$ 즉 $t=0$일때 모든 Energy는 a_1에 있다

$a_2(t) = Ae^{j(\omega_0+k)t} - Ae^{j(\omega_0-k)t}$

$Kt=\frac{\pi}{2} \sim t=\pi/2K$

$a_1(t)=A\cdot e^{j\omega_0 t} j + A e^{j\omega_0 t}(-j)=0$

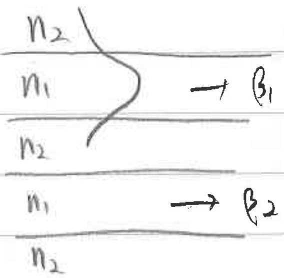
$a_2(t)=A e^{j\omega_0 t} j - A e^{j\omega_0 t}(-j)\neq 0$ 즉 Energy가 a_2로 넘어갔다



$E(z) \sim e^{-j\beta z}$ $\beta = n_{eff}\cdot k_0$

$a(z) \sim e^{-j\beta z}$

$\begin{bmatrix}a_1\\a_2\end{bmatrix} = A\begin{bmatrix}1\\1\end{bmatrix}e^{-j(\beta+k)z}+B\begin{bmatrix}1\\-1\end{bmatrix}e^{-j(\beta}$



$a_1(z)\simeq e^{-j\beta_1 z}$ $\frac{da_1}{dz}=-j\beta_1 a_1+K_{12}a_2$

$a_2(z)\simeq e^{-j\beta_2 z}$ $\frac{da_2}{dz}=-j\beta_2 a_2+K_{21}a_1$

에너지 보존법칙에 의해 $K_{12}=K_{21}^* \Rightarrow K_{12}=K_{21}=-jK$

$k=\frac{2\pi}{\lambda}$

$$\Rightarrow \begin{cases}\frac{da_1}{dz}=-j\beta_1 a_1-jKa_2\\ \frac{da_2}{dz}=-j\beta_2 a_2-jKa_1\end{cases}$$

대입

즉 $a_1, a_2 \sim e^{-j\beta z}$

if) $\beta_1=\beta_2=\beta_0$ (즉 두 waveguide가 동일한 경우)

$\beta = \beta_0 \pm K$

$\beta_1 = n_{eff,1}\cdot k_0$

$\beta_2 = n_{eff,2}\cdot k_0$

$\beta = n_{eff}\cdot k_0$,

$n_{eff}=\frac{\beta}{k_0}=n_{eff,0}\pm\frac{K}{k_0}$

$(\beta_0+K)=n_{eff,1}k_0$ $(\beta_0-K)=n_{eff,2}k_0$

$(n_{eff,1}-n_{eff,2})=\frac{2K}{k_0}$ → kappa

∴ Coupling length: L $(\beta_+-\beta_-)L=\pi$ $L=\frac{\pi}{\beta_+-\beta_-}=\frac{\pi}{2K}$