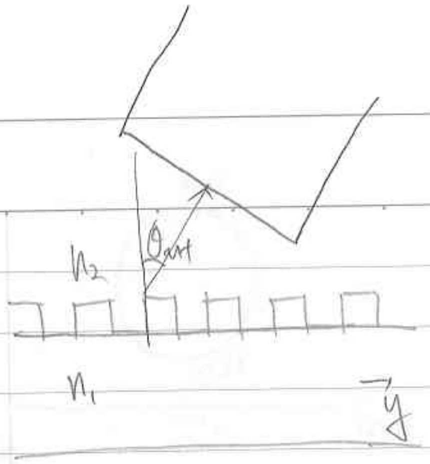


$K_{out,y} = K_{in,y} + m\frac{2\pi}{d}$

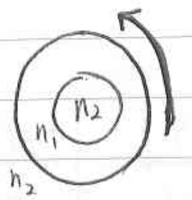
$K\cdot \sin\theta_{out} = K + m\frac{2\pi}{d}$

$n_2 k_0 \sin\theta_{out} = n_{eff}\cdot k_0 + m\cdot\frac{2\pi}{d}\lambda$

$(n_2\sin\theta_{out} - n_{eff}) = m\frac{\lambda}{d}$

$$d = \frac{m\lambda}{(n_{eff} - n_2\sin\theta_{out})}$$

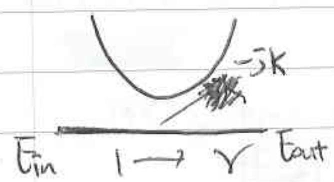
Lect 8. Ring Resonator.



즉 링 resonator이기 위해선 1바퀴후 in-phase여야 한다.

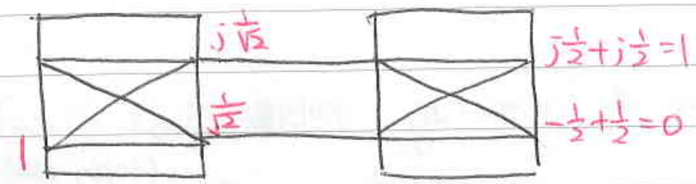
$e^{-j\beta L}=1$ $\quad \beta L = 2\pi m$

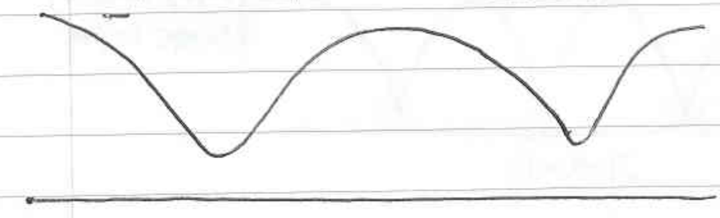
$L=2\pi R.$ $\quad \frac{2\pi}{\lambda}L = 2m\pi$, $L = m\lambda/n_{eff}$



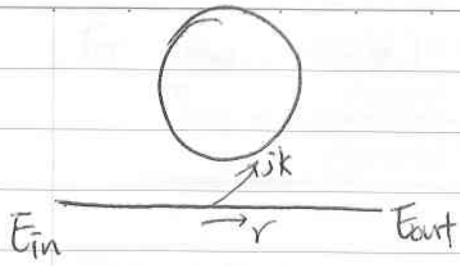
r, k는 E-field ratio. 항상 파워가 나누어지면

$|r|^2, |k|^2$ power ratio. field는 90° phase가 바뀐다.





lossless $|r|^2 + |k|^2 = 1$



$\alpha e^{-j\beta L}$ α: Loss in the ring $(\alpha<1)$

$$\left|\frac{E_{out}}{E_{in}}\right|^2 = ?$$

$$E_{out} = rE_{in} + jkE_{in}\cdot\alpha\cdot e^{-j\beta L}\cdot jk + jk\cdot E_{in}\cdot(\alpha\cdot e^{-j\beta L})^2 r\cdot jk + \cdots$$

$$\frac{E_{out}}{E_{in}} = r - k^2\alpha e^{-j\beta L}\left(1 + r\alpha e^{-j\beta L} + (r\alpha e^{-j\beta L})^2 + (\quad)^3 + \cdots\right)$$

$$= r - k^2\alpha e^{-j\beta L}/(1 - r\alpha e^{-j\beta L}) = \frac{r - r^2\alpha e^{-j\beta L} - \alpha e^{-j\beta L} + r^2\alpha e^{-j\beta L}}{1 - r\alpha e^{-j\beta L}}$$

$$= \frac{r - \alpha e^{-j\beta L}}{1 - r\alpha e^{-j\beta L}}$$

$$\left|\frac{E_{out}}{E_{in}}\right|^2 = \frac{r - \alpha e^{-j\beta L}}{1 - r\alpha e^{-j\beta L}} \times \frac{r - \alpha e^{j\beta L}}{1 - r\alpha e^{j\beta L}} = \underline{\underline{\frac{r^2 + \alpha^2 - 2r\alpha\cos\beta L}{1 + r^2\alpha^2 - 2r\alpha\cos\beta L}}}$$

If) $\alpha = 1 \Rightarrow T = 1$

~~T_{min}~~

$\alpha < 1$, how much loss?

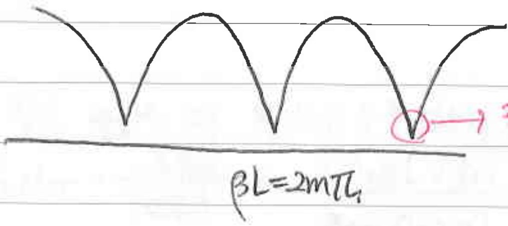
$$|E_{out}|^2 - |E_{in}|^2 = 1 + r^2\alpha^2 - r^2 - \alpha^2 = \underbrace{(1-r^2)(1-\alpha^2)}$$

처음 들어가서 한바퀴 돌 때 loss.

$T_{min} \Rightarrow \cos\beta L = 1$일 때 $\beta L = 2m\pi$, 즉 resonance 조건이다.

(미분해서 구하기)

파장에 대한 미분



→ 즉 나머지 power는 다 loss로 작용한다.

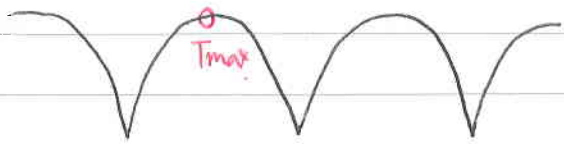
$\therefore$ $\beta L = 2m\pi$인 경우 $T_{min} = \frac{(r-\alpha)}{(1-r\alpha)^2}$

이 중 $r = \alpha$인 경우 $T_{min} = 0$. Critical Coupling.

Date . . No.

For T_{max} $\cos(\beta L) = -1$

$$T_{max} = \frac{(r+a)^2}{(1+ra)^2}$$



특정한 주파수만 걸러내는 notch filter 다 (IIR)

FSR : $\Delta\beta = \frac{2\pi}{L}$ or $\Delta\lambda = ?$

neff를 상수로 취급.

$$\beta = \frac{2\pi}{\lambda} \cdot n_{eff} \Rightarrow \Delta\lambda = \frac{d\lambda}{d\beta}\Delta\beta$$

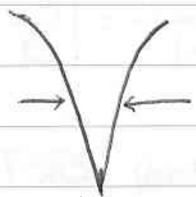
$$\frac{d\lambda}{d\beta} = -\frac{2\pi}{\beta^2} n_{eff} = -\frac{2\pi}{\left(\frac{2\pi}{\lambda} n_{eff}\right)^2} n_{eff} = -\frac{\lambda}{2\pi n_{eff}}$$

$$\Delta\lambda = \frac{\partial\lambda}{\partial\beta}\Delta\beta = -\frac{\lambda^2}{2\pi n_{eff}}\frac{2\pi}{L} = -\frac{\lambda^2}{n_{eff} L}$$

$n_{eff}(\lambda)^{v} \rightarrow \beta = \frac{2\pi}{\lambda} n_{eff}$

$$\frac{d\beta}{d\lambda} = -\frac{2\pi}{\lambda^2} n_{eff} + \frac{2\pi}{\lambda} \cdot \frac{dn_{eff}}{d\lambda} = -\frac{2\pi}{\lambda}\Big(\underbrace{n_{eff} - \lambda\frac{dn_{eff}}{d\lambda}}_{n_g}\Big)$$

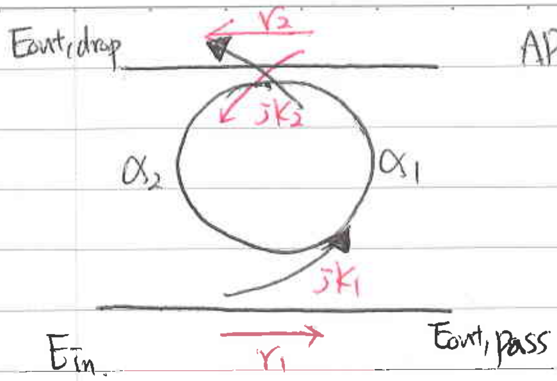
$$\therefore \Delta\lambda = -\frac{\lambda^2}{n_g L}$$



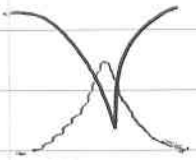
$$FWHM = \frac{(1-ra)\lambda_{res}^2}{\pi n_{eff} L \sqrt{ra}}$$

Q-factor : 빛을 넣었을 때 몇바퀴 돌수 있는지

$$Q\text{-factor} = \frac{\lambda_{res}}{FWHM} = \frac{\pi n_{eff} L \sqrt{ra}}{\lambda_{res}(1-ra)}$$



APP-Drop



Tpass?

Tdrop?

$\alpha_1 \alpha_2 = \alpha$

$$\frac{E_{pass}}{E_{in}} = r_1 + jk_1\alpha_1 \cdot e^{-j\frac{\beta L}{2}} r_2 \cdot \alpha_2 \cdot e^{-j\frac{\beta L}{2}} \cdot jk_1 + \cdots$$

$$= r_1 - (k_1)^2 \alpha r_2 e^{-j\beta L} \left[1 + \alpha r_1 r_2 e^{-j\beta L} + \cdots\right]$$

$$= r_1 - \frac{(1-r_1)^2 \alpha r_2 e^{-j\beta L}}{1 - \alpha r_2 r_1 e^{-j\beta L}}$$

즉 add-drop은 all-pass에 비해 loss가 더 커진 형태가 된다.

$$\frac{E_{drop}}{E_{in}} = jk_1\alpha_1 e^{-j\frac{\beta L}{2}} jk_2 \left(1 + r_2 r_1 \alpha e^{-j\beta L} + \cdots\right)$$

$$= \frac{-k_1 k_2 \alpha_1 e^{-j\beta L/2}}{1 - r_1 r_2 \alpha e^{-j\beta L}}$$

$$\left|\frac{E_{drop}}{E_{in}}\right|^2 = \frac{(1-r_1^2)(1-r_2^2)\,\alpha}{1 + r_1^2 r_2^2 \alpha - 2 r_1 r_2 \alpha \cos\beta L}$$

$T_{max} \Rightarrow \beta L = 2m\pi$

$T_{min} \Rightarrow \beta L = (2m+1)\pi$

FSR도 pass와 drop은 같다.

ring에서 온도가 바뀌면 resonance wavelength가 바뀌는데 그 때의 n_{eff}를 구할 수 있다.