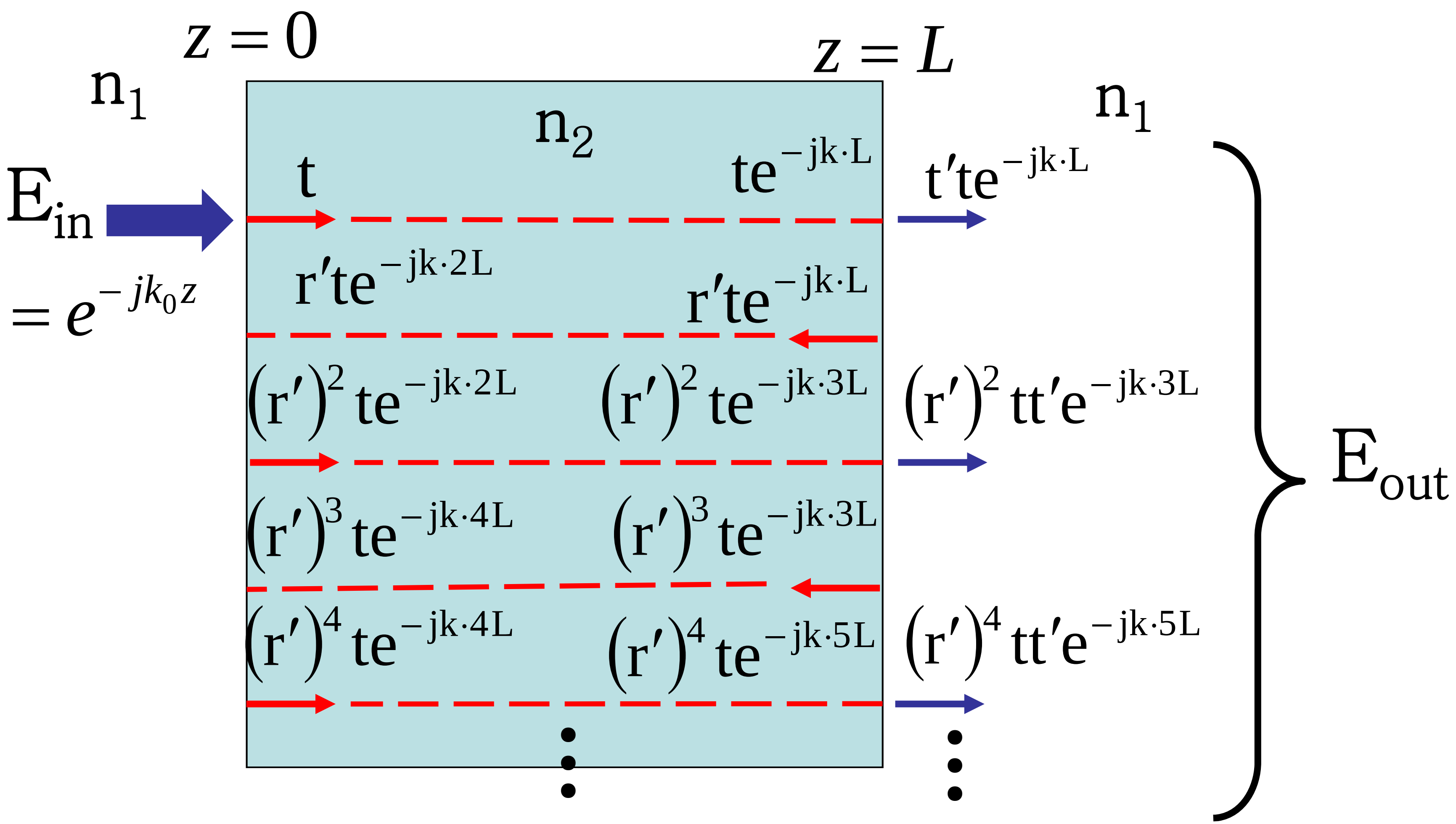
Lect. 8: Interferometers



Lect. 8: Interferometers

$$E_{out} = E_{t,total} = tt'e^{-jk\cdot L} + (r')^2\, tt'e^{-jk\cdot 3L} + (r')^4\, tt'e^{-jk\cdot 5L} + \bullet\bullet\bullet = \frac{tt'e^{-jk\cdot L}}{1-(r')^2\, e^{-j2kL}}$$

$$T = \frac{|E_t|^2}{|E_i|^2} = \frac{(tt')^2}{[1-(r')^2\, e^{-j2kL}][1-(r')^2\, e^{j2kL}]} = \frac{(tt')^2}{[1-(r')^2]^2 + 4r'^2 \sin^2(kL)}$$

$$[1-(r')^2 e^{-j2kL}]\,[1-(r')^2 e^{j2kL}] = 1-(r')^2 e^{j2kL} - (r')^2 e^{-j2kL} + (r')^4$$

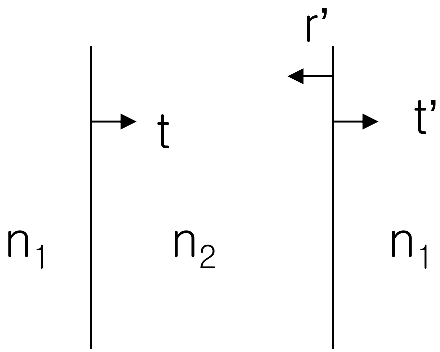
$$= 1 - 2(r')^2 \cos(2kL) + (r')^4$$

$$= 1 - 2(r')^2 (1 - 2\sin^2 kL) + (r')^4$$

$$= [1-(r')^2]^2 + 4(r')^2 \sin^2(kL)$$

Lect. 8: Interferometers

$$T = \frac{|E_t|^2}{|E_i|^2} = \frac{(tt')^2}{[1-(r')^2]^2 + 4r'^2 \sin^2(kL)}$$



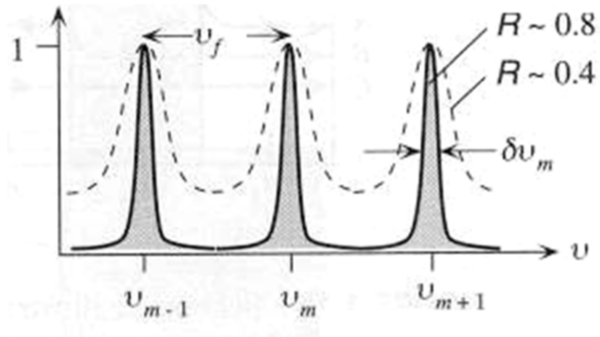
$$t = \frac{2n_1}{n_1+n_2}, \quad r' = \frac{n_2-n_1}{n_2+n_1}, \quad t' = \frac{2n_2}{n_1+n_2}$$

$$\therefore tt' = \frac{4n_1n_2}{(n_1+n_2)^2}, \quad 1 - r'^2 = \frac{4n_1n_2}{(n_2+n_1)^2}$$

Let $R = r'^2$ $\qquad T = \dfrac{(1-R)^2}{(1-R)^2 + 4R\sin^2(kL)}$

Lect. 8: Interferometers

$$T = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(kL)}$$



Max. Transmission: $\sin(kL) = 0 \Rightarrow T = 1$

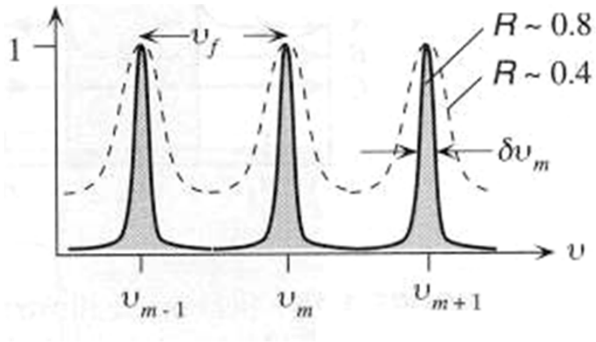
$$kL = m\pi;\ \ n_2 \frac{2\pi}{\lambda} L = m\pi \ \Rightarrow L = m\frac{\lambda}{2n_2} \ \text{(half wavelength)}$$

Min. Transmission: $\sin(kL) = 1$

$$kL = (m+\frac{1}{2})\pi;\ \ n_2 \frac{2\pi}{\lambda} L = (m+\frac{1}{2})\pi \quad \Rightarrow L = \frac{\lambda}{2n_2}(m+\frac{1}{2}) = \frac{\lambda}{n_2}(\frac{2m+1}{4})$$

(quarter wavelength)

Lect. 8: Interferometers



$$T = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(kL)}$$

Period? ➔ Free Spectral Range

$$\Delta kL = \pi \quad => \quad \Delta k = \frac{\pi}{L}$$

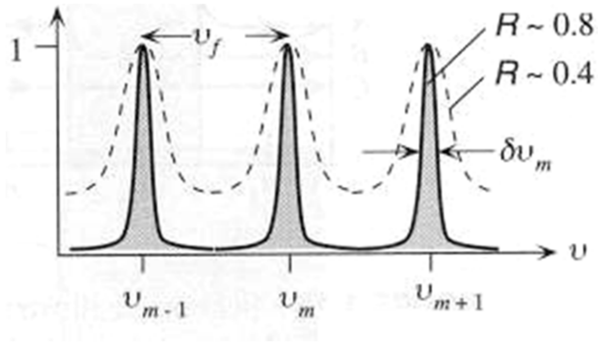
$$\Delta\omega = ? \quad \text{Since } k = n_2\frac{\omega}{c}, \quad \Delta\omega = \frac{c}{n_2}\Delta k = \frac{c}{n_2}\frac{\pi}{L} \qquad \Delta f = \frac{c}{2n_2L} = \frac{1}{T};$$

$$T = \frac{2L}{c/n_2}; \text{ round-trip time}$$

$$\Delta\lambda = ? \quad \lambda = n_2\frac{2\pi}{k} \qquad \Delta\lambda = \frac{d\lambda}{dk}\Delta k = -n_2\frac{2\pi}{k^2}\Delta k = -\frac{\lambda^2}{2n_2L}$$

Lect. 8: Interferometers

$$T = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(kL)}$$



Sharpness?

Determine k where $T = 0.5$

$$\frac{1}{2} = \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2(kL)} \quad \text{or} \quad \frac{4R}{(1-R)^2}\sin^2(kL) = 1$$

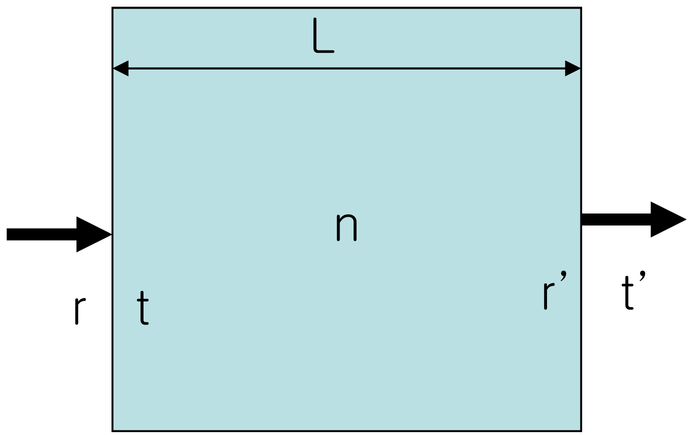
$$kL = \sin^{-1}\sqrt{\frac{(1-R)^2}{4R}} = \sin^{-1}\frac{(1-R)}{2\sqrt{R}}$$

$$\text{FWHM (Full Width at Half Maximum)} = 2\sin^{-1}\frac{(1-R)}{2\sqrt{R}} \sim \frac{(1-R)}{\sqrt{R}} \quad \text{(If FWHM} \ll 1\text{)}$$

As R increases, FWHM decreases => sharper response

Lect. 8: Interferometers

Fabry-Perot Interferometer

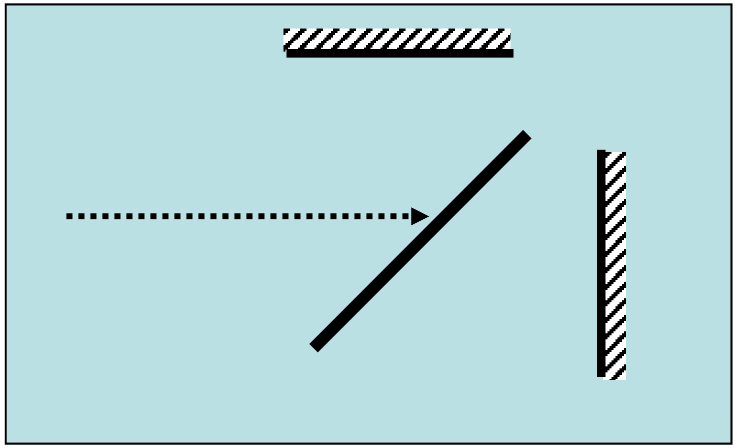


$$T = \frac{I_{out}}{I_{in}} = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(kL)} \quad (R = r'^2)$$

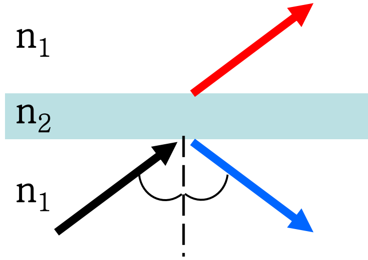
FP Interferometer can be also realized with two parallel partially reflecting mirrors

Lect. 8: Interferometers

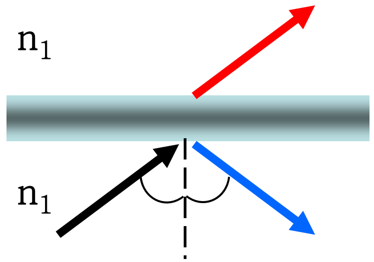
Michelson Interferometer: Two mirrors and one beam splitter



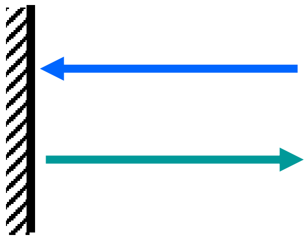
Frustrated TIR



Partially Coated Mirror

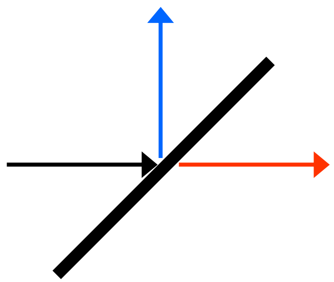


- Mirror (perfect conductor)



R=1, r=-1

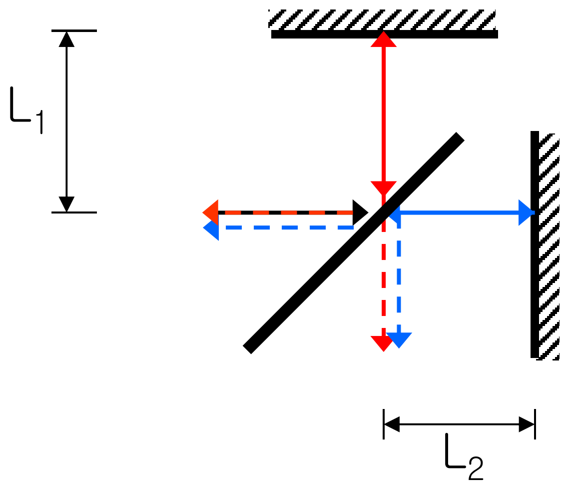
- Beam Splitter



$$R=\frac{1}{2},\ T=\frac{1}{2}$$

$$r=?,\ t=?$$

Michelson Interferometer:



Four outputs:

Side	1:	r	r
	2:	t	t
Bottom	1:	r	t
	2:	t	r

Assuming $E_{in} = 1$,

$$E_{out,\,side} = re^{-jkl_1}(-1)e^{-jkl_1}r + te^{-jkl_2}(-1)e^{-jkl_2}t = -r^2e^{-j2kl_1} - t^2e^{-j2kl_2}$$

$$E_{out,\,bottom} = re^{-jkl_1}(-1)e^{-jkl_1}t + te^{-jkl_2}(-1)e^{-jkl_2}r = -rte^{-j2kl_1} - rte^{-j2kl_2}$$

Lect. 8: Interferometers

With $r = t = \dfrac{1}{\sqrt{2}}$,

$$E_{out,\,side} = -r^2 e^{-j2kl_1} - t^2 e^{-j2kl_2}$$

$$= -\frac{1}{2}\left(e^{-j2kl_1} + e^{-j2kl_2}\right) = -\frac{1}{2}e^{-jk(l_1+l_2)}\left(\mathrm{e}^{-jk(l_1-l_2)} + \mathrm{e}^{jk(l_1-l_2)}\right)$$

$$I_{out,\,side} = \left|E_{out,\,side}\right|^2 = \cos^2[k(l_1 - l_2)]$$

$$E_{out,\,bottom} = -rte^{-j2kl_1} - rte^{-j2kl_2}$$

$$= -\frac{1}{2}\left(e^{-j2kl_1} + e^{-j2kl_2}\right)$$

$$I_{out,\,bottom} = \left|E_{out,\,bottom}\right|^2 = \cos^2[k(l_1 - l_2)]$$

Against energy conservation!

Lect. 8: Interferometers

With $r = \frac{1}{\sqrt{2}},\ t = j\frac{1}{\sqrt{2}}$

(r and t should have $\pi/2$ phase difference in order to satisfy energy conservation)

$$E_{out,\,side} = -r^2 e^{-j2kl_1} - t^2 e^{-j2kl_2}$$

$$= -\frac{1}{2}\left(e^{-j2kl_1} - e^{-j2kl_2}\right) = -\frac{1}{2} e^{-jk(l_1+l_2)}\left(\mathrm{e}^{-jk(l_1-l_2)} - \mathrm{e}^{jk(l_1-l_2)}\right)$$

$$I_{out,\,side} = \left|E_{out,\,side}\right|^2 = \sin^2[k(l_1 - l_2)]$$

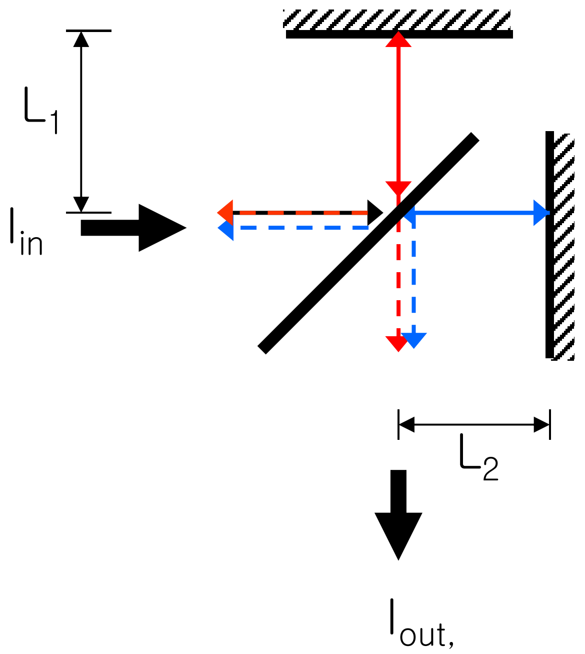
$$E_{out,\,bottom} = -rte^{-j2kl_1} - rte^{-j2kl_2}$$

$$= -\frac{j}{2}\left(e^{-j2kl_1} + e^{-j2kl_2}\right)$$

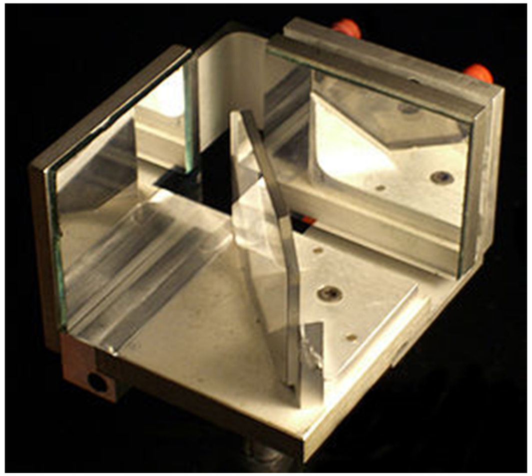
$$I_{out,\,bottom} = \left|E_{out,\,bottom}\right|^2 = \cos^2[k(l_1 - l_2)]$$

Lect. 8: Interferometers

Michelson Interferometer:



$$\frac{I_{out,bottom}}{I_{in}} = \cos^2[k(l_1 - l_2)]$$



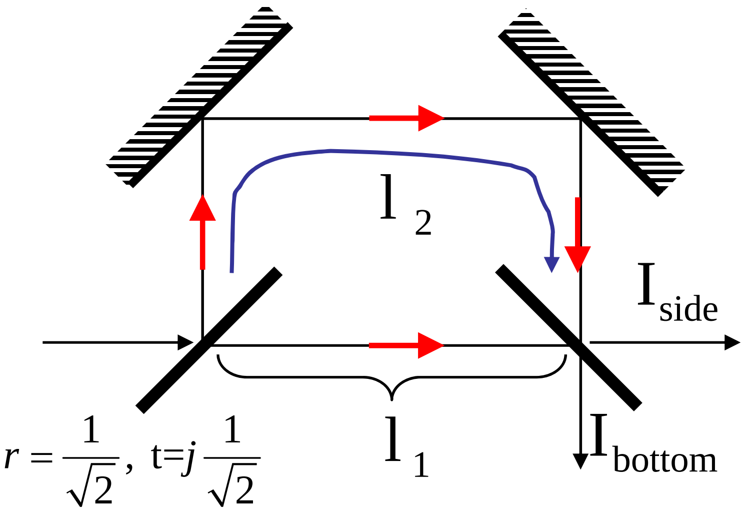
$k(l_1 - l_2)$ can be measured very precisely

- Experimental measurement of speed of light by Michelson in 1879.

➔ Provided a clue for Special Relativity: speed of light is same always (Michelson Morley experiment)

Lect. 8: Interferometers

Mach-Zehnder Interferometer (MZI):



Assuming $E_{in} = 1$,

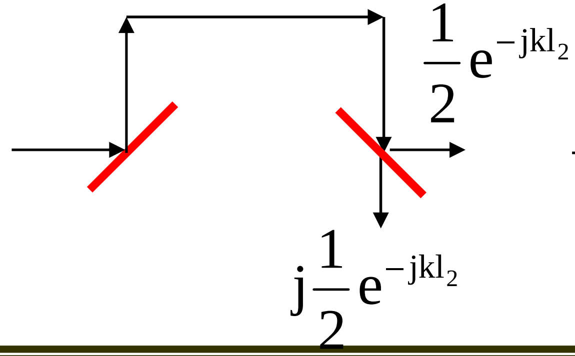
$$E_{out,\,side} = \frac{1}{2}\left(e^{-jkl_2} - e^{-jkl_1}\right) = \frac{1}{2}e^{-jk\frac{l_2+l_1}{2}}\left(e^{-jk\frac{l_2-l_1}{2}} - e^{jk\frac{l_2-l_1}{2}}\right)$$

$$I_{out,\,side} = \sin^2\left(k\frac{l_1-l_2}{2}\right)$$

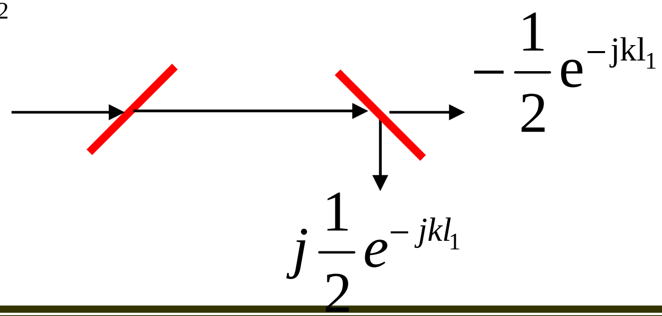
$$E_{out,\,bottom} = \frac{j}{2}\left(e^{-jkl_1} + e^{-jkl_2}\right) = \frac{j}{2}e^{-jk\frac{l_1+l_2}{2}}\left(e^{-jk\frac{l_1-l_2}{2}} + e^{jk\frac{l_1-l_2}{2}}\right)$$

$$I_{out,\,bottom} = \cos^2\left(k\frac{l_1-l_2}{2}\right)$$

Case#1



Case#2



Lect. 8: Interferometers

Can any EM wave cause interference?

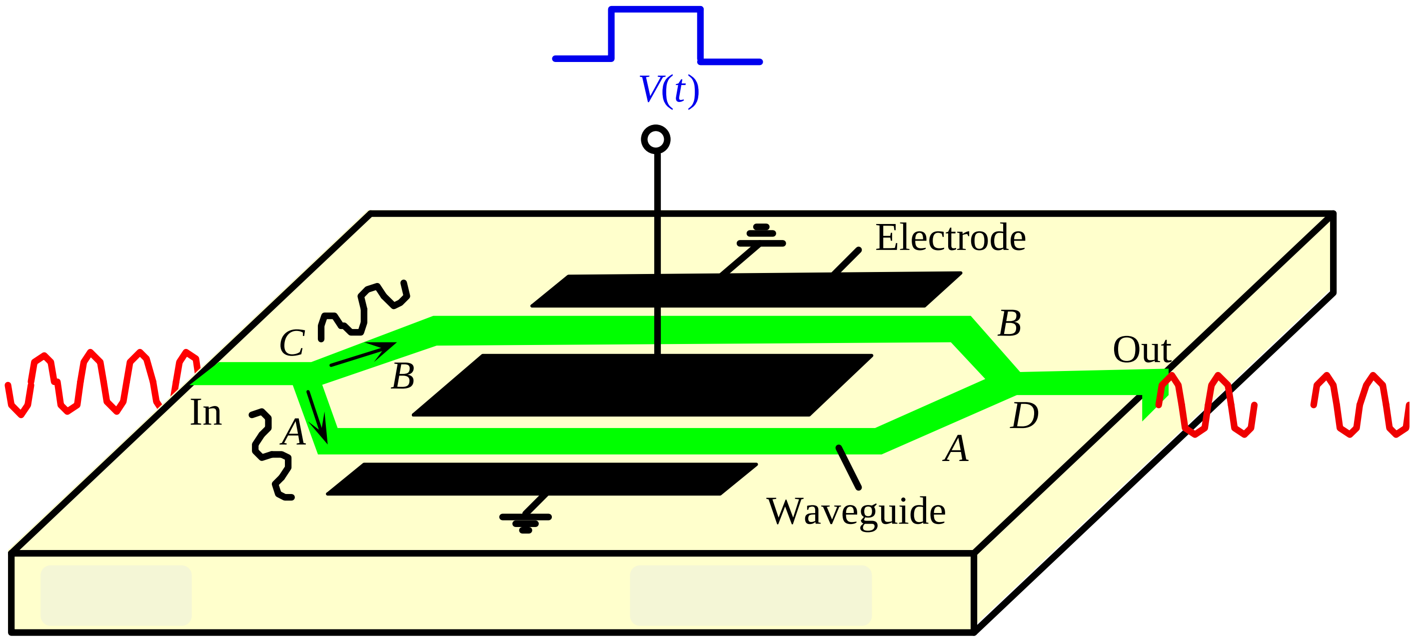
→ Only EM waves having clear phase relationship experience interference: coherent

How large $(l_1 - l_2)$ can be?

→ As long as two separated waves are coherent or within coherent length
Separated waves become incoherent due to intrinsic phase noises

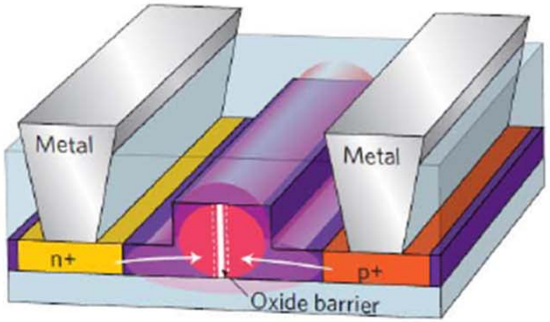
Lect. 8: Interferometers

Mach-Zender Interferometer Modulator

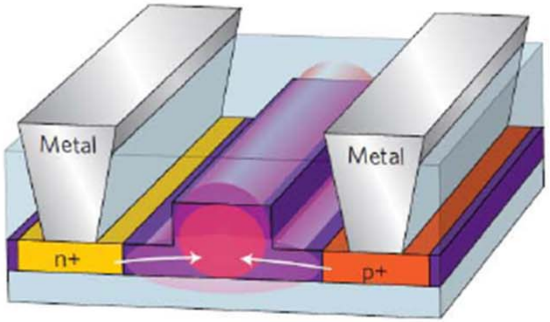


How to modulate Si refractive index with voltage?

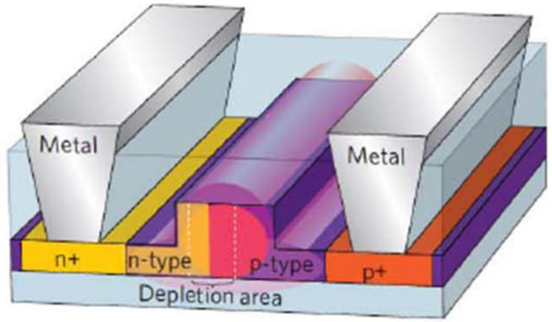
Lect. 8: Interferometers



Carrier accumulation



Carrier injection



Carrier depletion