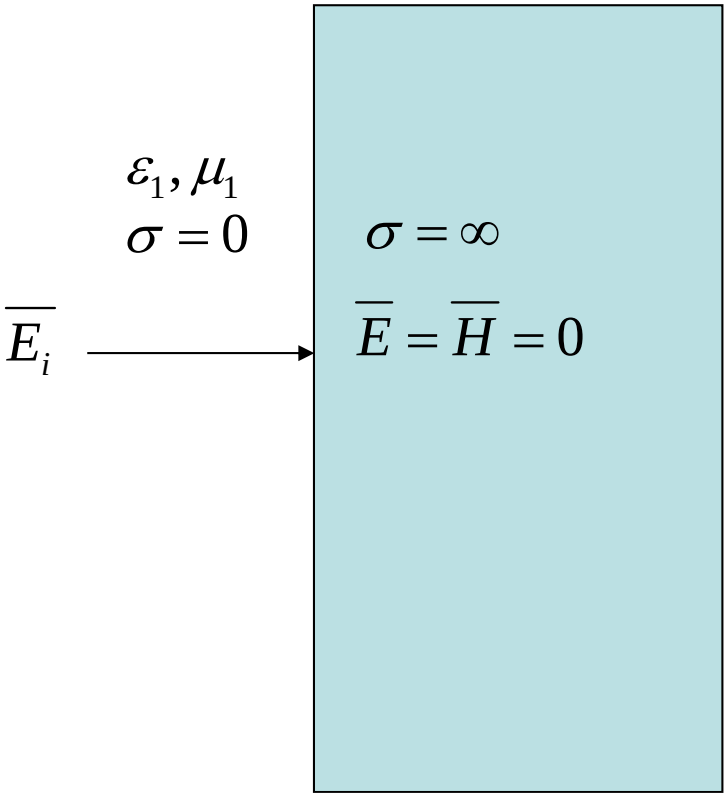
Lect. 10: EM Waves Incident at Conductor

(Cheng 8-6)

What happens to EM waves normally incident at a perfect conductor?



No penetration (skin depth = 0)

➔ Reflection

Determine other fields if

$$\overline{E}_i = \overline{x} E_0 \exp(-j\beta z)$$

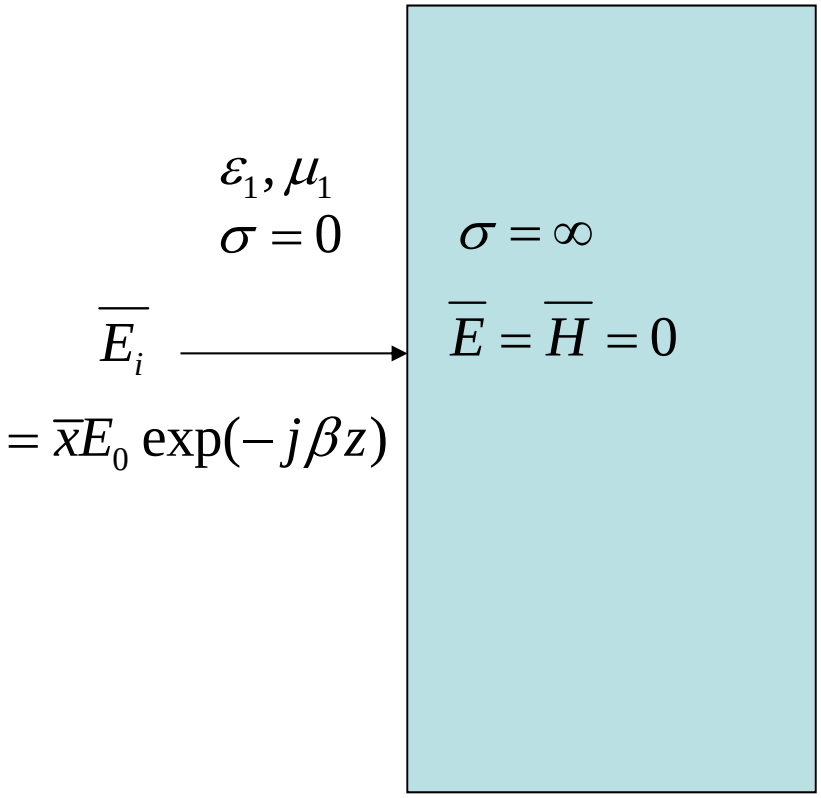
(1) $\overline{H}_i$? $\quad \overline{H}_i = \overline{y} \dfrac{E_0}{\eta} \exp(-j\beta z)$

(2) $\overline{E}_r$?

(3) $\overline{H}_r$?

W.-Y. Choi

Lect. 10: EM Waves Incident at Conductor



(2) $\overline{E}_r = ?$

Apply B.C. at $z = 0$,

$$E_{2,t} - E_{1,t} = 0$$

$$\overline{E}_i(z=0) + \overline{E}_r(z=0) = 0$$

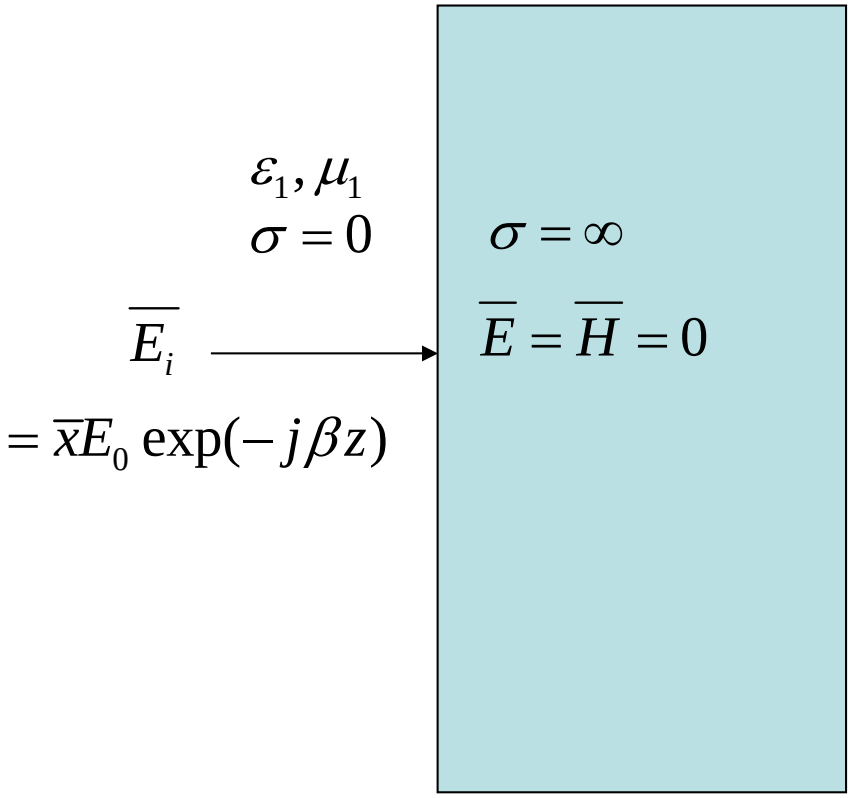
$$\therefore \overline{E}_r = -\overline{x} E_0 \exp(j\beta z)$$

Other BC?

$$D_{2,n} - D_{1,n} = \rho_s \Leftrightarrow \varepsilon_2 E_{2,n} - \varepsilon_1 E_{1,n} = \rho_s$$

W.-Y. Choi

Lect. 10: EM Waves Incident at Conductor



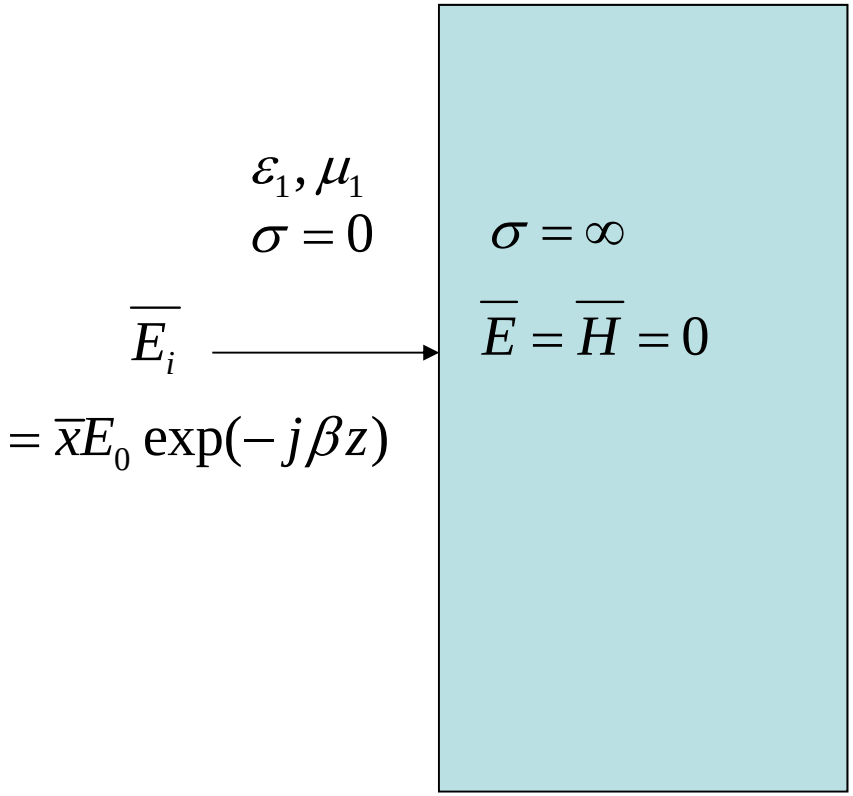
(3) $\overline{H}_r = ?$

From $\overline{E}_r = -\overline{x}E_0 \exp(j\beta z)$

$$\overline{H}_r = \overline{y}\frac{E_0}{\eta_1}\exp(j\beta z)$$

W.-Y. Choi

Lect. 10: EM Waves Incident at Conductor



$$\overline{H}_i = \overline{y}\frac{E_0}{\eta_1}\exp(-j\beta z)$$

$$\overline{H}_r = \overline{y}\frac{E_0}{\eta_1}\exp(j\beta z)$$

BC?

$$\overline{a_n} \times (\overline{H}_2 - \overline{H}_1) = \overline{J}_s$$

$$\overline{z} \times \left(0 - \overline{y}\frac{2E_0}{\eta_1}\right) = \overline{J}_s$$

$$\overline{J}_s = \overline{x}\frac{2E_0}{\eta}$$

W.-Y. Choi

Lect. 10: EM Waves Incident at Conductor

Total E-field for z < 0

$$\bar{E}_{total}(z) = \bar{E}_i + \bar{E}_r = \bar{x}E_0 \exp(-j\beta z) - \bar{x}E_0 \exp(j\beta z) = \bar{x}E_0(-2j)\sin(\beta z)$$

$$\bar{E}_{total}(z,t) = \bar{x}E_0(-2j)\sin(\beta z)\exp(j\omega t)$$

$$\mathrm{Re}\left[\bar{E}_{total}(z,t)\right] = \bar{x}E_0 2\sin(\beta z)\sin(\omega t)$$

Is this a wave?

➔ Standing Wave!

Lect. 10: EM Waves Incident at Conductor

Total H-field for z < 0

$$\bar{H}_{total}(z) = \bar{H}_i + \bar{H}_r = \bar{y}\frac{E_0}{\eta_1}\exp(-j\beta z) + \bar{y}\frac{E_0}{\eta_1}\exp(j\beta z) \quad = \bar{y}2\frac{E_0}{\eta_1}\cos(\beta z)$$

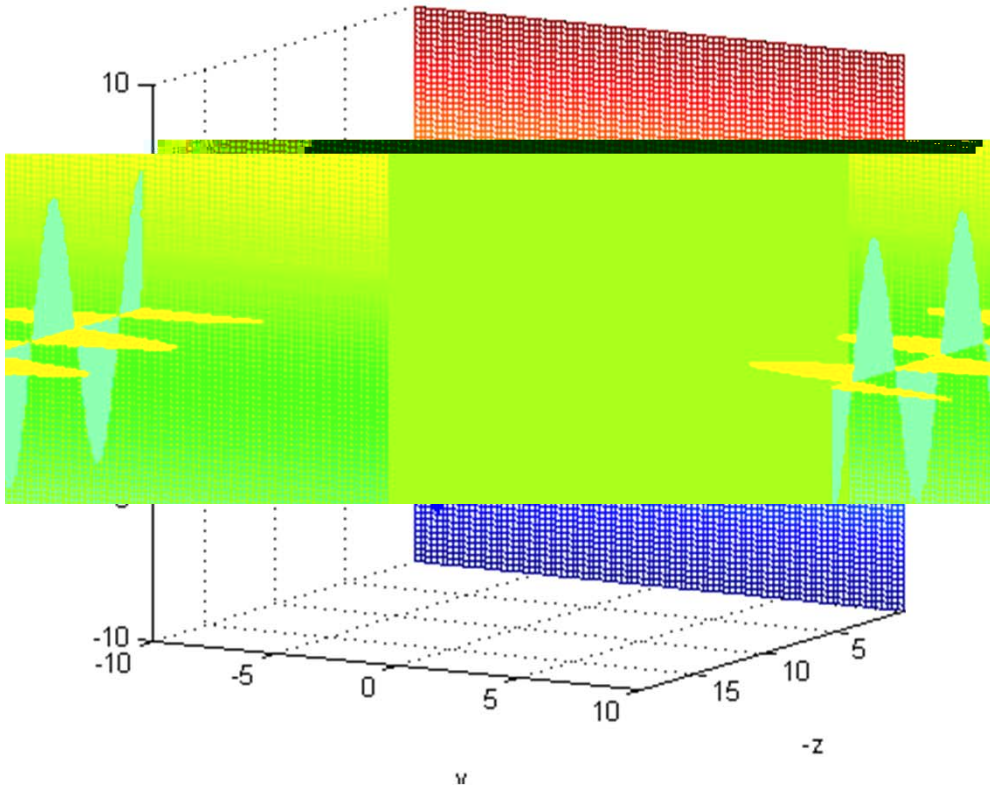
$$\bar{H}_{total}(z,t) = \bar{y}2\frac{E_0}{\eta_1}\cos(\beta z)\exp(j\omega t)$$

$$\mathrm{Re}\left[\bar{H}_{total}(z,t)\right] = \bar{y}2\frac{E_0}{\eta_1}\cos(\beta z)\cos(\omega t)$$

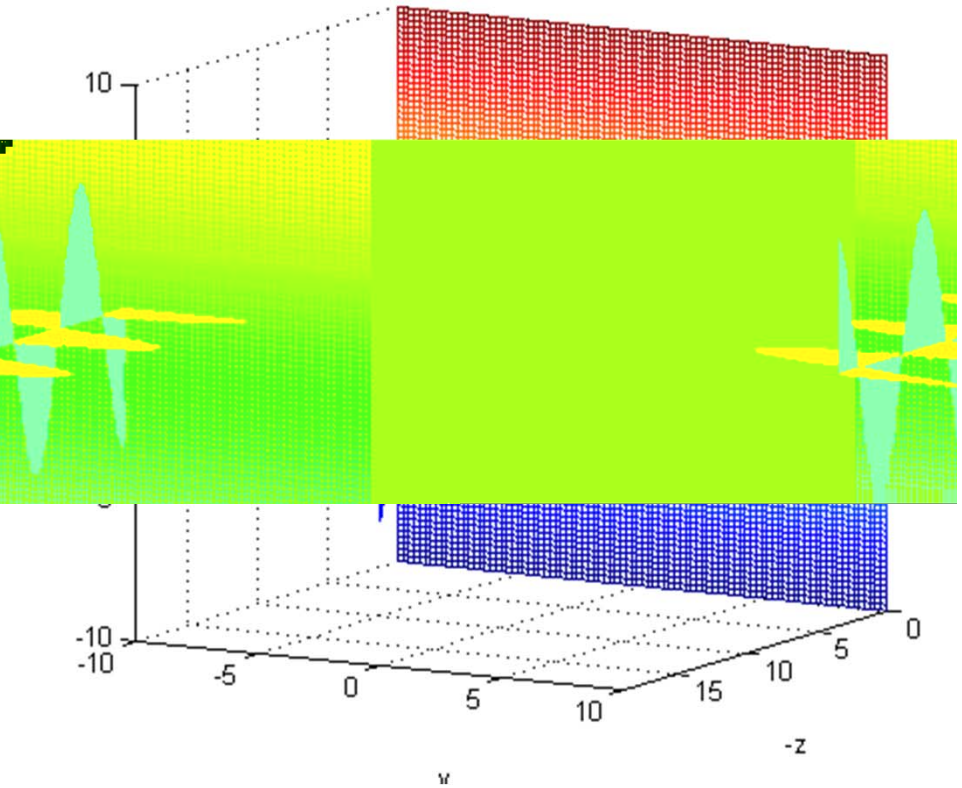
➔ Standing Wave (out of phase with E-field)

Lect. 10: EM Waves Incident at Conductor

Incident Wave



Reflected Wave



Lect. 10: EM Waves Incident at Conductor

Total Wave

Lect. 10: EM Waves Incident at Conductor

EM Power propagation

$\overline{P} = \overline{E} \times \overline{H}$ Poynting vector

Average power density? $\overline{P}_{av} = \frac{1}{2}\mathrm{Re}\left[\overline{E} \times \overline{H}^*\right]$

$$\overline{E} = \overline{E}_i + \overline{E}_r = \overline{x}E_0(-2j)\sin(\beta z)$$

$$\overline{H} = H_i + H_r = \overline{y}\frac{2E_0}{\eta}\cos(\beta z)$$

$$\overline{E} \times \overline{H}^* = \overline{z}\frac{-4j\left|E_0\right|^2}{\eta}\sin(\beta z)\cos(\beta z)$$

$$\therefore \overline{P}_{av} = 0$$