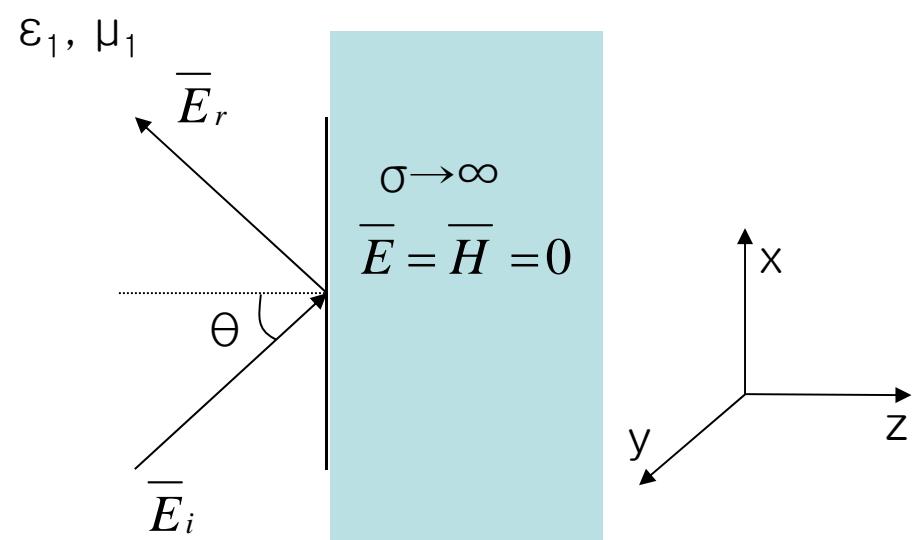


# Lect. 11: Oblique Incidence at Conductor

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(Cheng 8-7.1)



$$\bar{E}_i = \bar{y} E_0 e^{-j\beta_x x} e^{-j\beta_z z}$$

(Perpendicular polarization)

$$\beta_x = k \sin \theta, \quad \beta_z = k \cos \theta$$

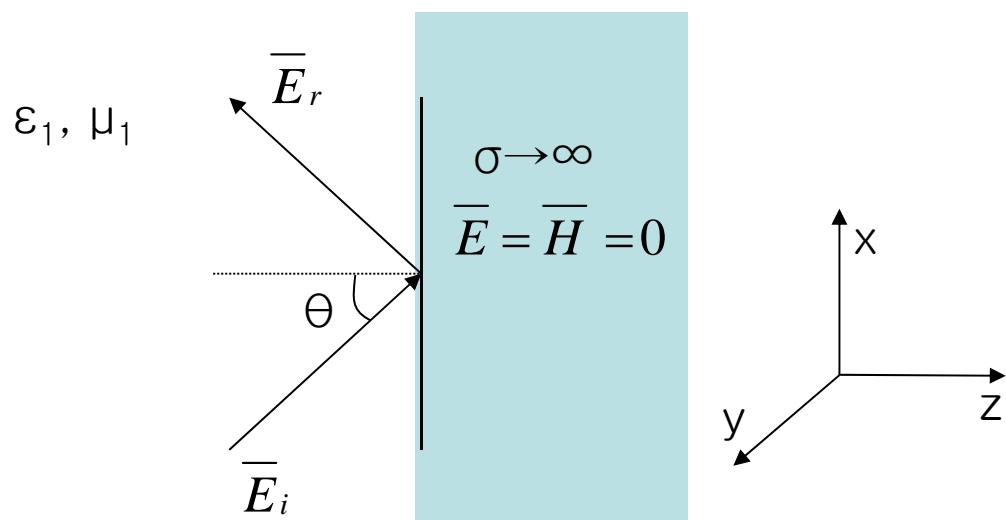
$$\bar{E}_r, \bar{H}_i, \bar{H}_r = ?$$

# Lect. 11: Oblique Incidence at Conductor

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$$\bar{E}_r = \bar{y} E_r e^{-j\beta_{rx}x} e^{j\beta_{rz}z}$$

- Boundary condition at  $z=0$



$$\bar{E}_i = \bar{y} E_0 e^{-j\beta_x x} e^{-j\beta_z z}$$

$$\bar{E}_i + \bar{E}_r = 0$$

$$E_0 e^{-j\beta_x x} + E_r e^{-j\beta_{rx}x} = 0$$

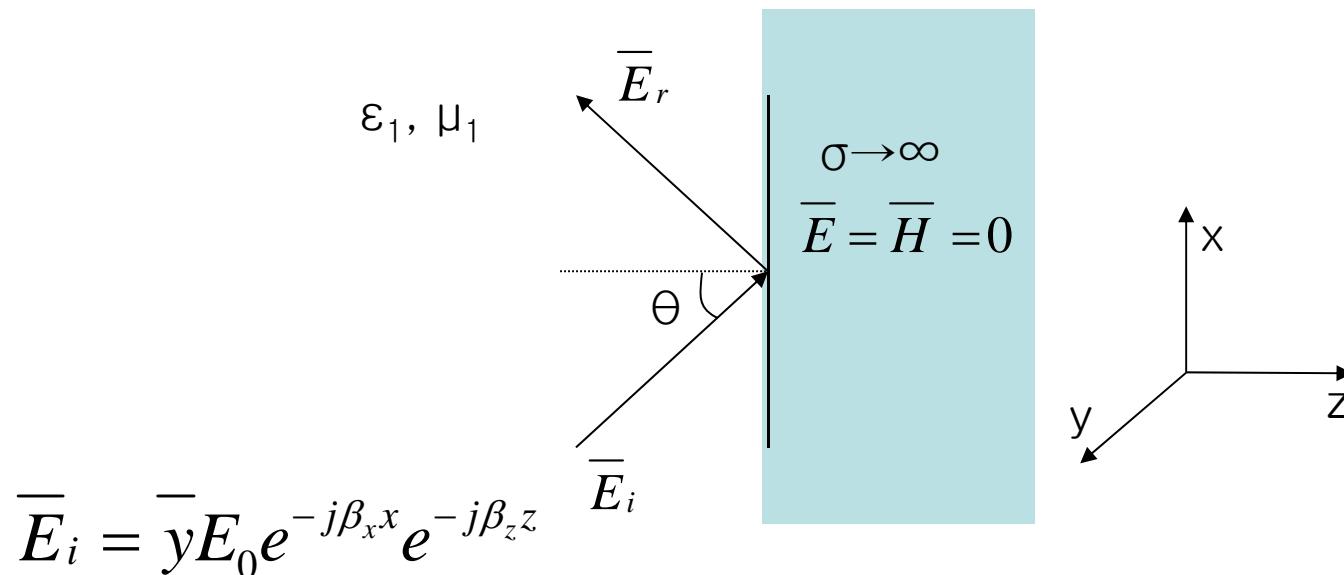
$$\beta_x = \beta_{rx} \quad \text{and} \quad E_0 = -E_r$$

$$\theta_i = \theta_r$$

$$\beta_z = \beta_{rz}$$

# Lect. 11: Oblique Incidence at Conductor

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$$\begin{aligned}\bar{E}_{total} &= \bar{E}_i + \bar{E}_r = \bar{y} E_0 e^{-j\beta_x x} e^{-j\beta_z z} - \bar{y} E_0 e^{-j\beta_x x} e^{j\beta_z z} \\ &= \bar{y} E_0 e^{-j\beta_x x} (-2j) \sin(\beta_z z)\end{aligned}$$

X direction: Plane wave propagation

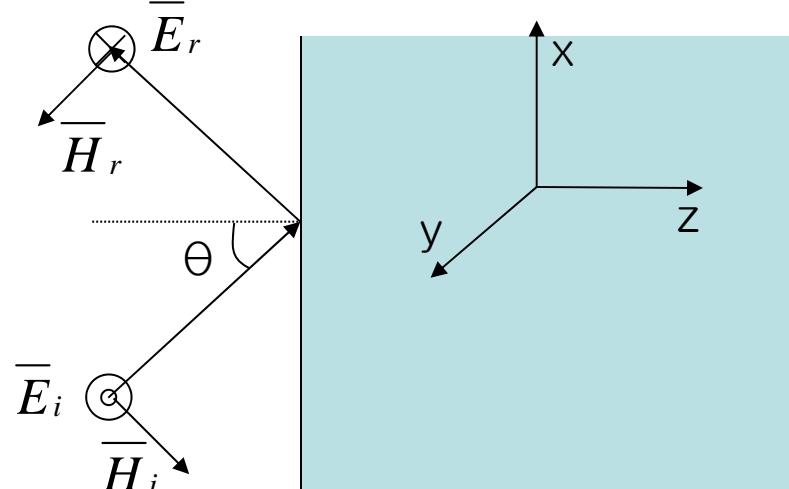
Z direction: Standing wave

# Lect. 11: Oblique Incidence at Conductor

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$$\bar{E}_r = -\bar{y}E_0 e^{-j\beta_x x} e^{j\beta_z z}$$

How about H-fields?



$$\bar{E}_i = \bar{y}E_0 e^{-j\beta_x x} e^{-j\beta_z z}$$

$$\bar{H}_i = \frac{1}{\eta} (\bar{a}_k \times \bar{E}_i) \quad \bar{a}_k = \bar{x} \sin \theta + \bar{z} \cos \theta$$

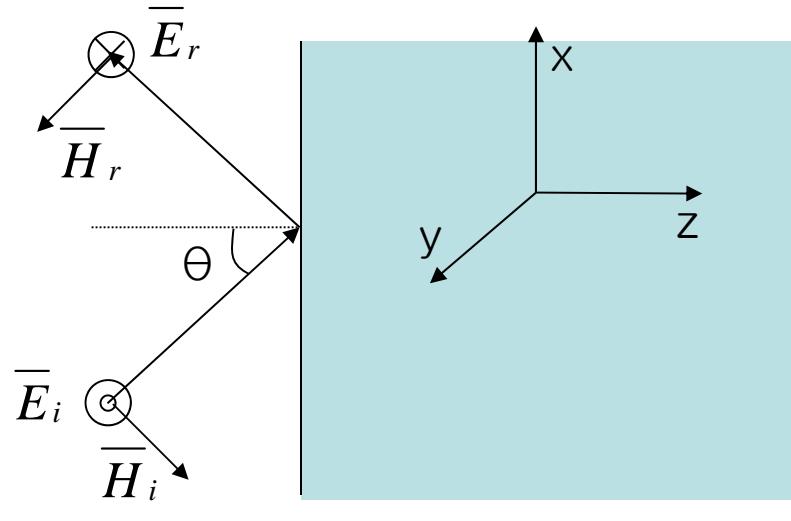
$$= \frac{E_0}{\eta} (\bar{z} \sin \theta - \bar{x} \cos \theta) e^{-j\beta_x x} e^{-j\beta_z z}$$

$$\bar{H}_r = \frac{1}{\eta} (\bar{a}_k \times \bar{E}_r) \quad \bar{a}_k = \bar{x} \sin \theta - \bar{z} \cos \theta$$

$$\bar{H}_r = -\frac{E_0}{\eta} (\bar{z} \sin \theta + \bar{x} \cos \theta) e^{-j\beta_x x} e^{j\beta_z z}$$

# Lect. 11: Oblique Incidence at Conductor

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$$\bar{E}_i = \bar{y}E_0 e^{-j\beta_x x} e^{-j\beta_z z}$$

$$\bar{E}_r = -\bar{y}E_0 e^{-j\beta_x x} e^{j\beta_z z}$$

$$\bar{H}_i = \frac{E_0}{\eta} (\bar{z} \sin \theta - \bar{x} \cos \theta) e^{-j\beta_x x} e^{-j\beta_z z}$$

$$\bar{H}_r = -\frac{E_0}{\eta} (\bar{z} \sin \theta + \bar{x} \cos \theta) e^{-j\beta_x x} e^{j\beta_z z}$$

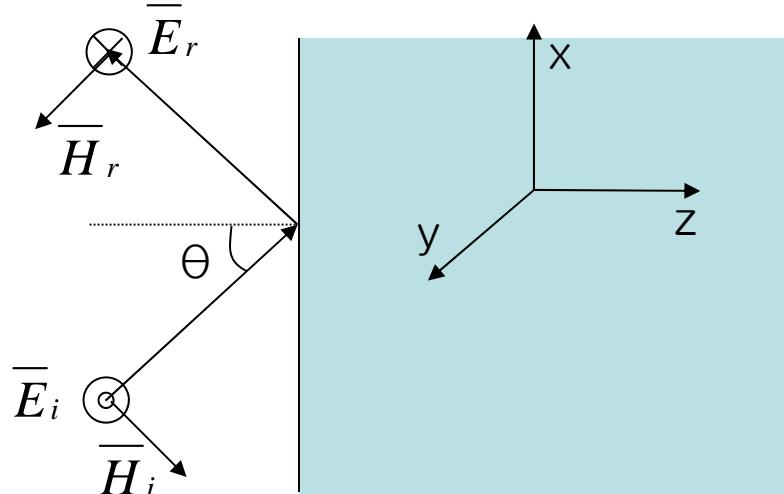
$$\rho_s = ? \quad \bar{J}_s = ?$$

$$\text{At } z=0, \quad \bar{a}_n \times (\bar{H}_2 - \bar{H}_1) = \bar{J}_s$$

$$\bar{a}_n = \bar{z} \quad \bar{H}_2 = 0 \quad \bar{J}_s = \bar{y} \frac{2E_0}{\eta} \cos \theta e^{-j\beta_x x}$$

Propagation of current densities into  $x$ -direction!

# Lect. 11: Oblique Incidence at Conductor



$$\bar{E}_i = \bar{y}E_0 e^{-j\beta_x x} e^{-j\beta_z z}$$

$$\bar{E}_r = -\bar{y}E_0 e^{-j\beta_x x} e^{j\beta_z z}$$

$$\bar{H}_i = \frac{E_0}{\eta} (\bar{z} \sin \theta - \bar{x} \cos \theta) e^{-j\beta_x x} e^{-j\beta_z z}$$

$$\bar{H}_r = -\frac{E_0}{\eta} (\bar{z} \sin \theta + \bar{x} \cos \theta) e^{-j\beta_x x} e^{j\beta_z z}$$

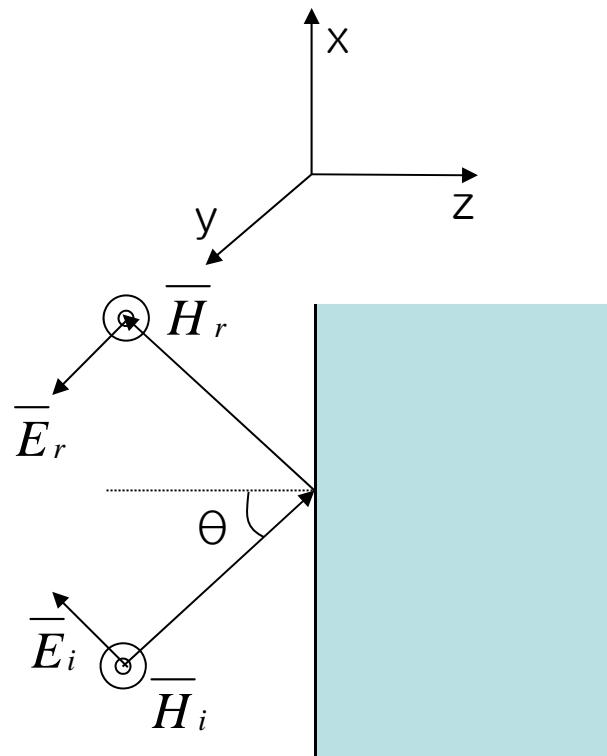
$$\bar{E}_{total} = \bar{E}_i + \bar{E}_r = \bar{y}E_0 e^{-j\beta_x x} (-2j \sin \beta_z z)$$

$$\begin{aligned}\bar{H}_{total} &= \bar{H}_i + \bar{H}_r = \frac{E_0}{\eta} e^{-j\beta_x x} \left[ \bar{z} \sin \theta (e^{-j\beta_z z} - e^{j\beta_z z}) - \bar{x} \cos \theta (e^{-j\beta_z z} + e^{j\beta_z z}) \right] \\ &= \frac{E_0}{\eta} e^{-j\beta_x x} \left[ \bar{z} \sin \theta \cdot (-2j) \sin(\beta_z z) - \bar{x} \cos \theta \cdot 2 \cos(\beta_z z) \right]\end{aligned}$$

$$\bar{P}_{av} = \frac{1}{2} \operatorname{Re} [\bar{E} \times \bar{H}^*] = \bar{x} \frac{2E_0^2}{\eta} \sin \theta \sin^2(\beta_z z)$$

# Lect. 11: Oblique Incidence at Conductor

Parallel polarization



$$\overline{H}_i = \overline{y} \frac{E_0}{\eta} e^{-j\beta_x x} e^{-j\beta_z z}$$

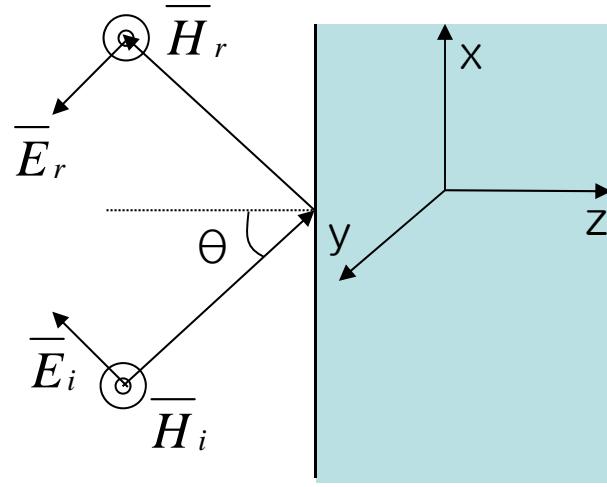
$$\overline{E}_i = E_0 (\overline{x} \cos \theta - \overline{z} \sin \theta) e^{-j\beta_x x} e^{-j\beta_z z}$$

$$\text{Let } \overline{H}_r = \overline{y} \frac{E_r}{\eta} e^{-j\beta_x x} e^{+j\beta_z z}$$

$$\overline{E}_r = -E_r (\overline{x} \cos \theta + \overline{z} \sin \theta) e^{-j\beta_x x} e^{+j\beta_z z}$$

# Lect. 11: Oblique Incidence at Conductor

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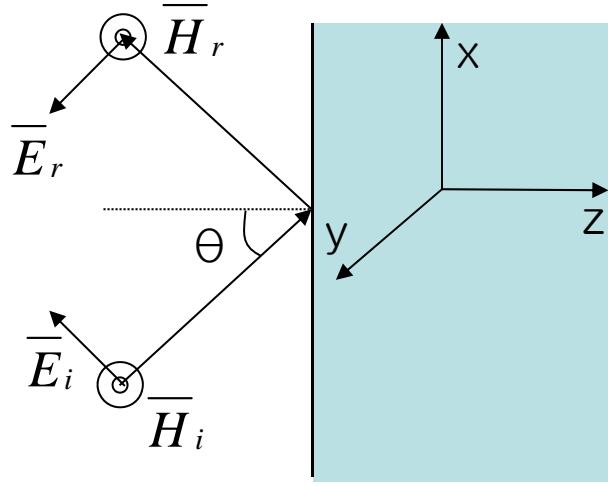
$$\begin{aligned}\bar{H}_i &= \bar{y} \frac{\bar{E}_0}{\eta} e^{-j\beta_x x} e^{-j\beta_z z} \\ \bar{E}_i &= \bar{E}_0 (\bar{x} \cos \theta - \bar{z} \sin \theta) e^{-j\beta_x x} e^{-j\beta_z z} \\ \bar{H}_r &= \bar{y} \frac{\bar{E}_r}{\eta} e^{-j\beta_x x} e^{+j\beta_z z} \\ \bar{E}_r &= -\bar{E}_r (\bar{x} \cos \theta + \bar{z} \sin \theta) e^{-j\beta_x x} e^{+j\beta_z z}\end{aligned}$$

Using B.C. at  $z = 0$

$$E_0 = E_r$$

# Lect. 11: Oblique Incidence at Conductor

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$$\begin{aligned}\overline{H}_i &= \overline{y} \frac{E_0}{\eta} e^{-j\beta_x x} e^{-j\beta_z z} \\ \overline{E}_i &= E_0 (\overline{x} \cos \theta - \overline{z} \sin \theta) e^{-j\beta_x x} e^{-j\beta_z z} \\ \overline{H}_r &= \overline{y} \frac{E_r}{\eta} e^{-j\beta_x x} e^{+j\beta_z z} \\ \overline{E}_r &= -E_0 (\overline{x} \cos \theta + \overline{z} \sin \theta) e^{-j\beta_x x} e^{+j\beta_z z}\end{aligned}$$

at  $z = 0$

$$\rho_s = 2\epsilon_1 E_0 \sin \theta E e^{-j\beta_x x}$$

$$\overline{J}_s = \overline{x} \frac{2E_0}{\eta} e^{-j\beta_x x}$$

$$\overline{P}_a = \overline{x} 2 \frac{E_0^2}{\eta} \sin \theta \cos^2(\beta_z z)$$