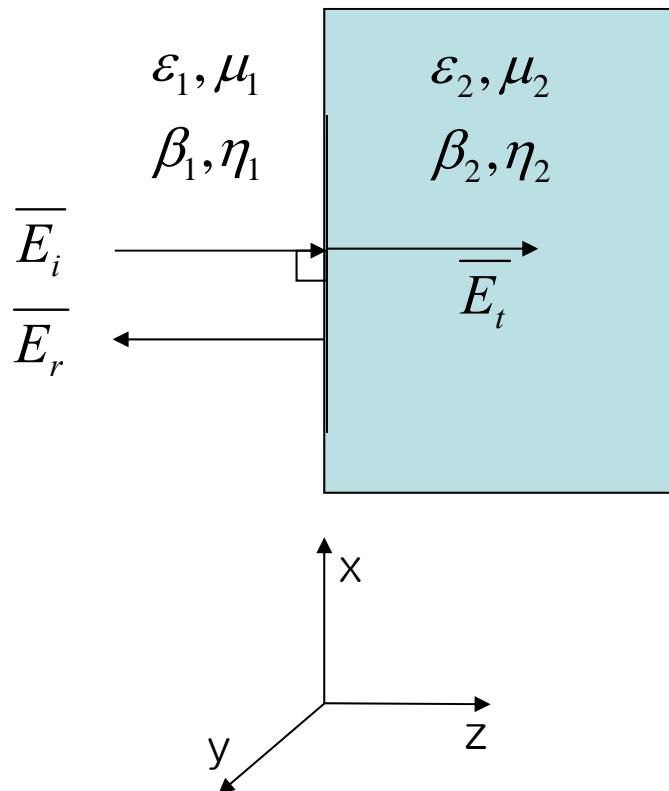


# Lect. 12: Normal Incidence at Dielectric Interface

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$$\sigma = 0$$



(Cheng 8-8)

$$\bar{E}_i = \bar{y} E_0 \exp(-j\beta_1 z), \quad \bar{H}_i = -\bar{x} \frac{E_0}{\eta_1} \exp(-j\beta_1 z)$$

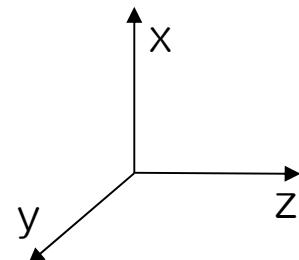
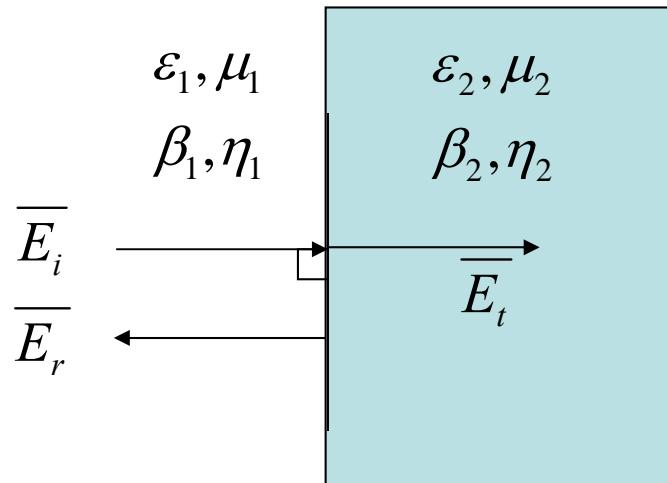
$$\bar{E}_r = \bar{y} E_r \exp(j\beta_1 z), \quad \bar{H}_r = \bar{x} \frac{E_r}{\eta_1} \exp(j\beta_1 z)$$

$$\bar{E}_t = \bar{y} E_t \exp(-j\beta_2 z), \quad \bar{H}_t = -\bar{x} \frac{E_t}{\eta_2} \exp(-j\beta_2 z)$$

Determine  $E_r, E_t$

# Lect. 12: Normal Incidence at Dielectric Interface

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$$\bar{E}_i = \bar{y}E_0 \exp(-j\beta_1 z), \quad \bar{H}_i = -\bar{x} \frac{E_0}{\eta_1} \exp(-j\beta_1 z)$$

$$\bar{E}_r = \bar{y}E_r \exp(j\beta_1 z), \quad \bar{H}_r = \bar{x} \frac{E_r}{\eta_1} \exp(j\beta_1 z)$$

$$\bar{E}_t = \bar{y}E_t \exp(-j\beta_2 z), \quad \bar{H}_t = -\bar{x} \frac{E_t}{\eta_2} \exp(-j\beta_2 z)$$

1)  $E_{tan}$  should be continuous at  $z=0$

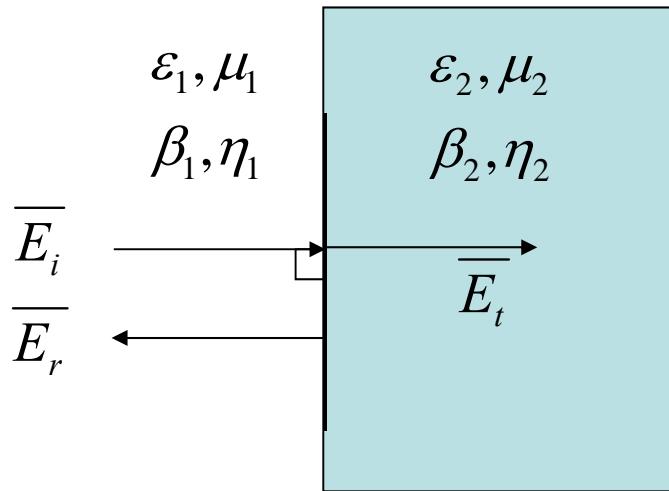
$$E_0 + E_r = E_t$$

Define  $E_r = \Gamma E_0$ , and  $E_t = \tau E_0$

$\Gamma$ : reflection coef.	$\Rightarrow$	$1 + \Gamma = \tau$
$\tau$ : transmission coef.		

# Lect. 12: Normal Incidence at Dielectric Interface

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2)  $H_{tan}$  should be continuous at  $z=0$

$$\bar{a}_n \times (\bar{H}_2 - \bar{H}_1) = \bar{J}_s = 0$$

$$H_i + H_r = H_t$$

$$-\frac{E_0}{\eta_1} + \frac{E_r}{\eta_1} = -\frac{E_t}{\eta_2}$$

$$\therefore -\frac{1}{\eta_1} + \frac{\Gamma}{\eta_1} = -\frac{\tau}{\eta_2}, \quad 1 + \Gamma = \tau$$

$$\bar{H}_i = -\bar{x} \frac{E_0}{\eta_1} \exp(-j\beta_1 z)$$

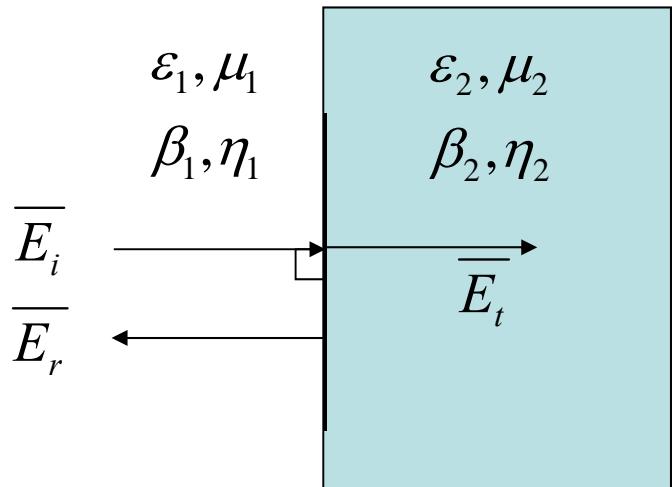
$$\bar{H}_r = \bar{x} \frac{E_r}{\eta_1} \exp(j\beta_1 z)$$

$$\bar{H}_t = -\bar{x} \frac{E_t}{\eta_2} \exp(-j\beta_2 z)$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}, \quad \tau = \frac{2\eta_2}{\eta_2 + \eta_1}$$

# Lect. 12: Normal Incidence at Dielectric Interface

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$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}, \quad \tau = \frac{2\eta_2}{\eta_2 + \eta_1}$$

Max.  $\Gamma$ :  $0, \quad \eta_2 \gg \eta_1$   
 $\varepsilon_2 \ll \varepsilon_1$  (Assuming  $\mu_1 = \mu_2$ )

$\tau$ : 2

Min.  $\Gamma$ :  $-1, \quad \eta_2 \ll \eta_1$   
 $\varepsilon_2 \gg \varepsilon_1$  (Assuming  $\mu_1 = \mu_2$ )

$\tau$ : 0

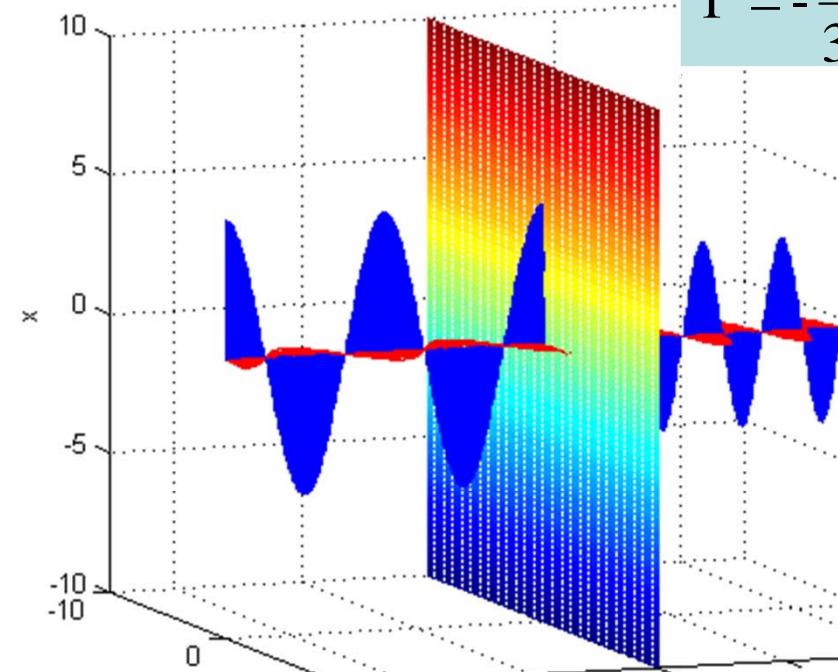
→ Same as the case for EM wave incident  
on a perfect conductor

$$\eta = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\pi f \mu}{\sigma}} (1+j)$$

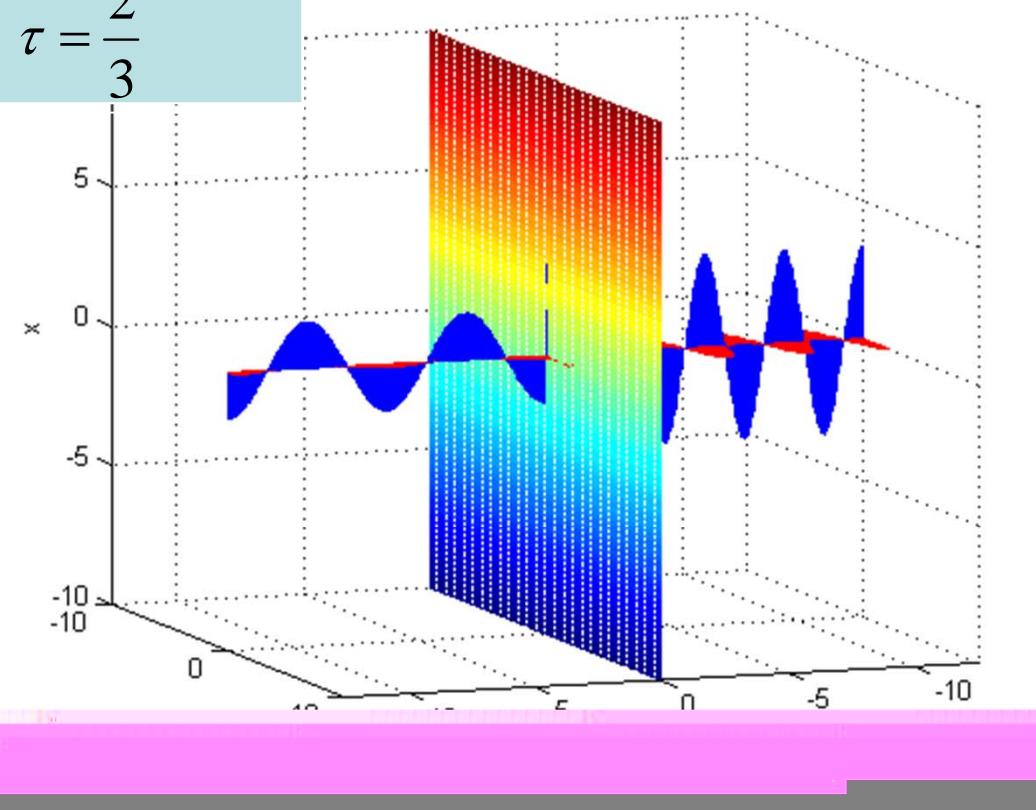
# Lect. 12: Normal Incidence at Dielectric Interface

$$4\epsilon_1 = \epsilon_2 \quad \text{and} \quad \mu_1 = \mu_2$$

$$\Gamma = -\frac{1}{3}, \quad \tau = \frac{2}{3}$$



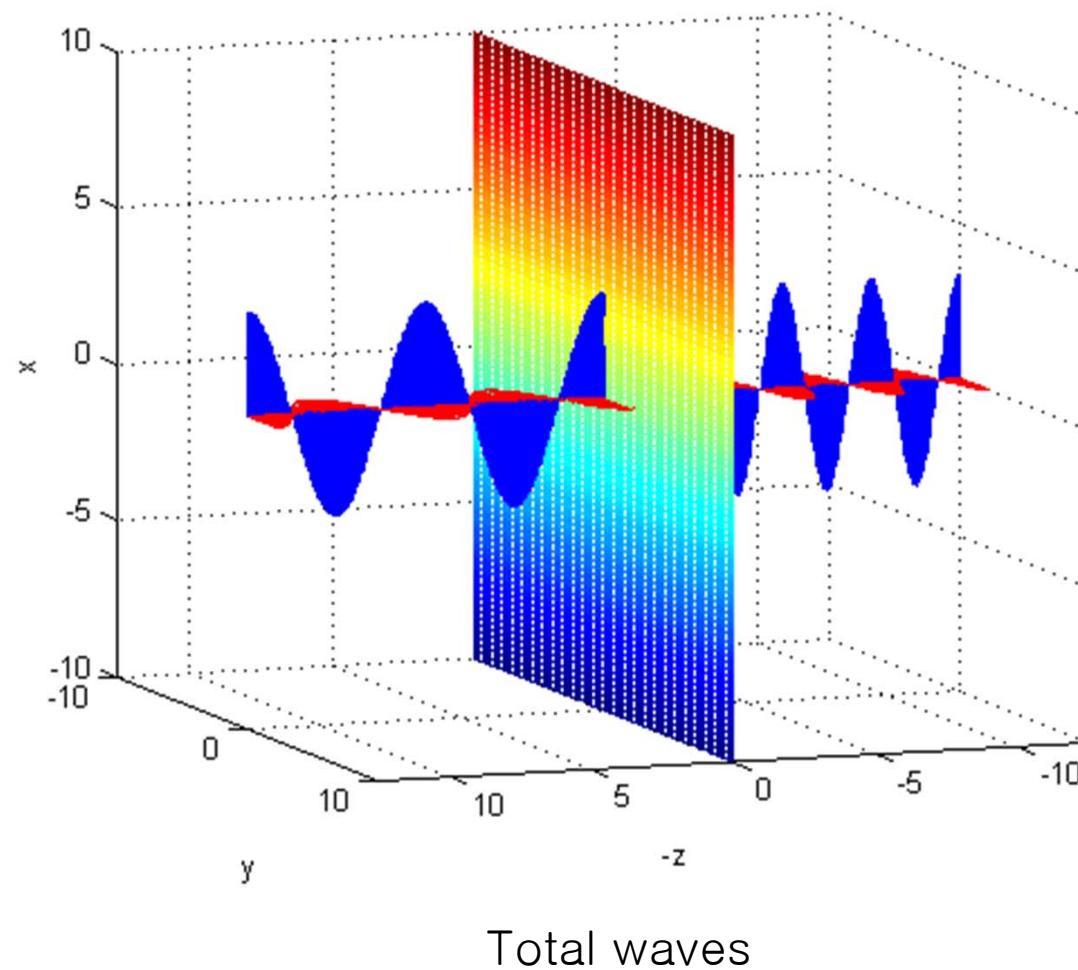
Incident and Transmitted waves



Reflected and Transmitted waves

# Lect. 12: Normal Incidence at Dielectric Interface

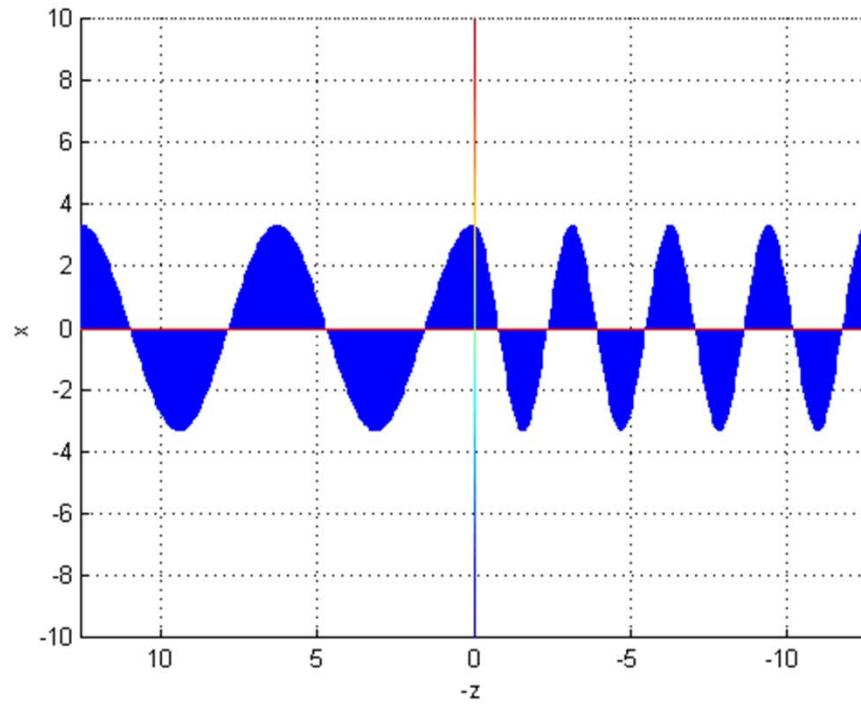
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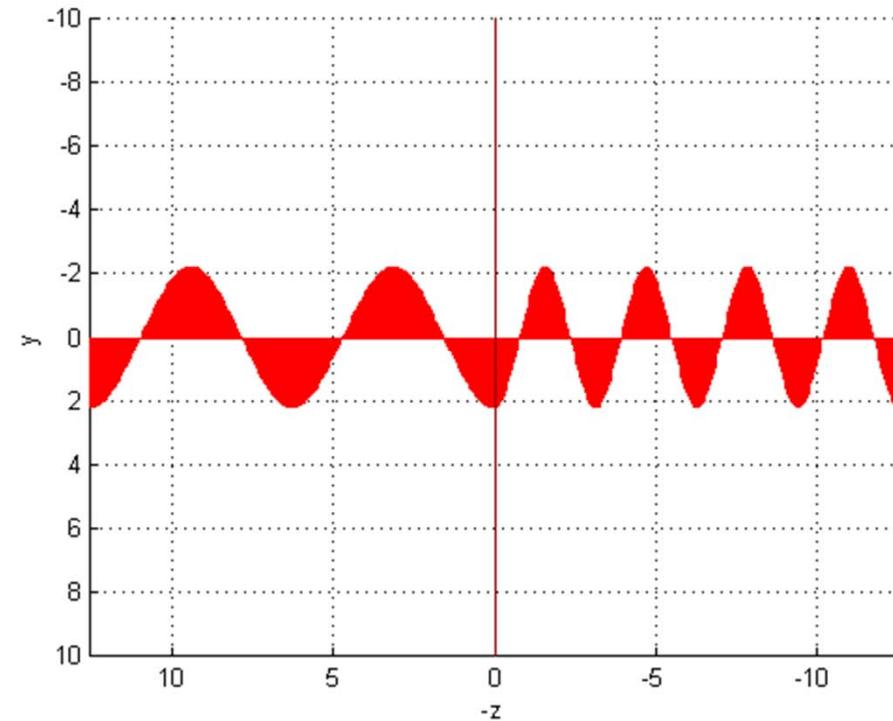
# Lect. 12: Normal Incidence at Dielectric Interface

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E-field



H-field



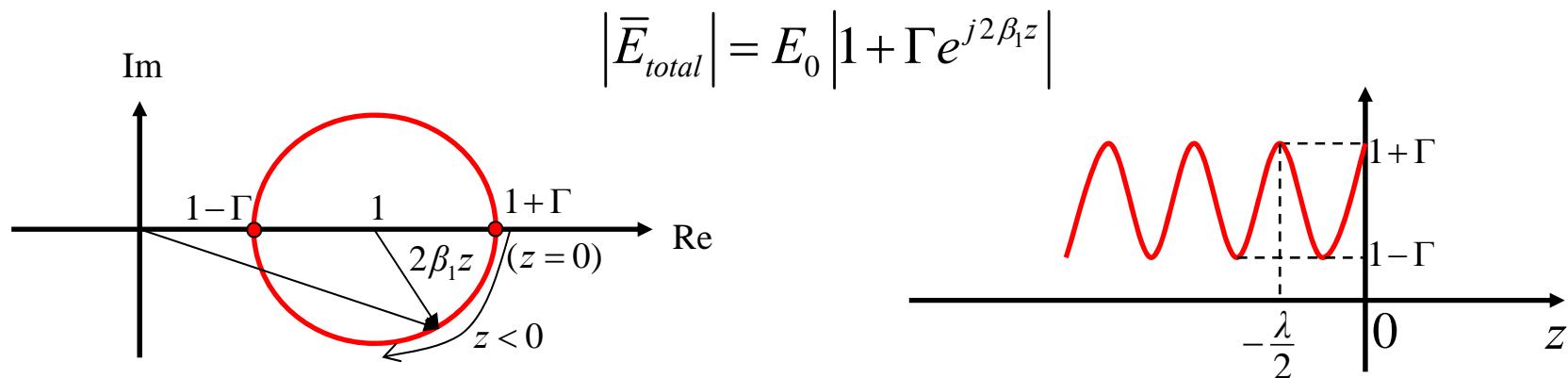
# Lect. 12: Normal Incidence at Dielectric Interface

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Total E-field for  $z < 0$

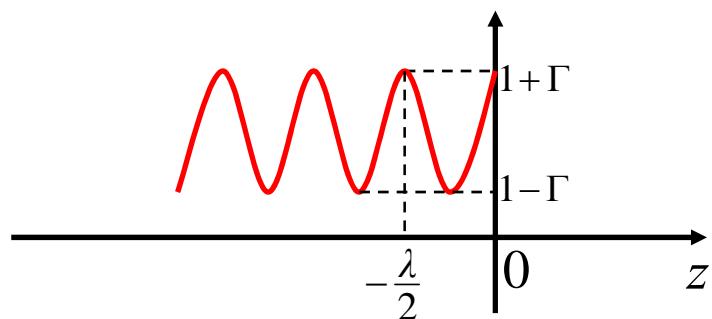
$$\begin{aligned}\bar{E}_{total}(z < 0) &= \bar{E}_i + \bar{E}_r = \bar{y}E_0(e^{-j\beta_l z} + \Gamma e^{j\beta_l z}) \\ &= \bar{y}E_0 \left\{ \underbrace{(1 + \Gamma)e^{-j\beta_l z}}_{\text{transmitting wave}} + \underbrace{\Gamma(e^{j\beta_l z} - e^{-j\beta_l z})}_{\text{standing wave}} \right\}\end{aligned}$$

$$\bar{E}_{total}(z < 0) = \bar{E}_i + \bar{E}_r = \bar{y}E_0(e^{-j\beta_l z} + \Gamma e^{j\beta_l z}) = \bar{y}E_0 e^{-j\beta_l z} (1 + \Gamma e^{j2\beta_l z})$$



# Lect. 12: Normal Incidence at Dielectric Interface

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✓ Define standing wave ratio

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

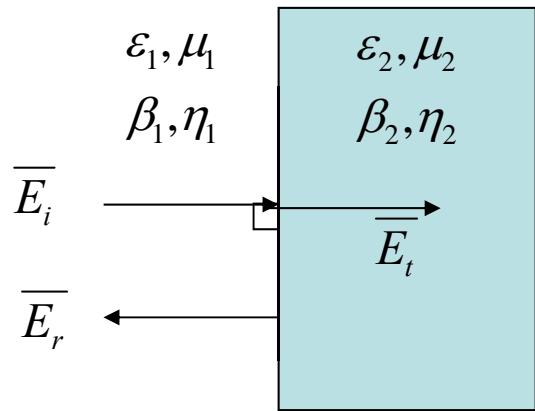
$$|\Gamma| = 1 \quad \Rightarrow \quad S \rightarrow \infty$$

$$\Gamma = 0 \quad \Rightarrow \quad S = 1$$

$$1 \leq S < \infty$$

# Lect. 12: Normal Incidence at Dielectric Interface

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$$\begin{aligned}\bar{E}_i &= \bar{y} E_0 \exp(-j\beta_1 z), & \bar{H}_i &= -\bar{x} \frac{E_0}{\eta_1} \exp(-j\beta_1 z) \\ \bar{E}_r &= \bar{y} \Gamma E_0 \exp(j\beta_1 z), & \bar{H}_r &= \bar{x} \frac{\Gamma E_0}{\eta_1} \exp(j\beta_1 z) \\ \bar{E}_t &= \bar{y} \tau E_0 \exp(-j\beta_2 z), & \bar{H}_t &= -\bar{x} \frac{\tau E_0}{\eta_2} \exp(-j\beta_2 z)\end{aligned}$$

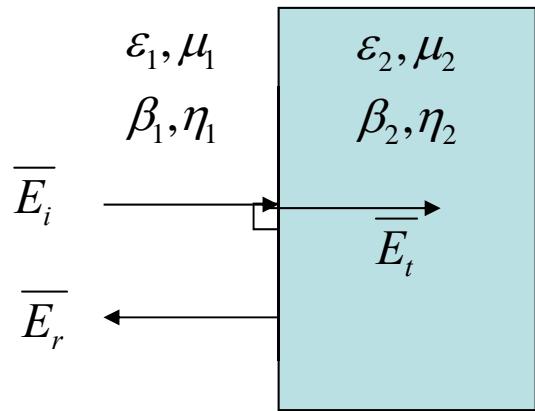
Average power density propagation  $(\bar{P}_{av} = \frac{1}{2} \text{Re} [\bar{E} \times \bar{H}^*])$

In medium 2,

$$\bar{P}_{av} = \frac{1}{2} \text{Re} [\bar{E} \times \bar{H}^*] = \bar{z} \frac{E_0^2}{2\eta_2} \tau^2$$

# Lect. 12: Normal Incidence at Dielectric Interface

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$$\begin{aligned}\bar{E}_i &= \bar{y}E_0 \exp(-j\beta_1 z), & \bar{H}_i &= -\bar{x} \frac{E_0}{\eta_1} \exp(-j\beta_1 z) \\ \bar{E}_r &= \bar{y}\Gamma E_0 \exp(j\beta_1 z), & \bar{H}_r &= \bar{x} \frac{\Gamma E_0}{\eta_1} \exp(j\beta_1 z) \\ \bar{E}_t &= \bar{y}\tau E_0 \exp(-j\beta_2 z), & \bar{H}_t &= -\bar{x} \frac{\tau E_0}{\eta_2} \exp(-j\beta_2 z)\end{aligned}$$

In medium 1,

$$\begin{aligned}\bar{E} \times \bar{H}^* &= \bar{y}E_0 [\exp(-j\beta_1 z) + \Gamma \exp(j\beta_1 z)] \times \left( -\bar{x} \frac{E_0}{\eta_1} \right) [\exp(j\beta_1 z) - \Gamma \exp(-j\beta_1 z)] \\ &= \bar{z} \frac{E_0^2}{\eta_1} [1 - \Gamma^2 + \Gamma(\exp^{j2\beta_1 z} - \exp^{-j2\beta_1 z})] = \bar{z} \frac{E_0^2}{\eta_1} [1 - \Gamma^2 + \Gamma 2j \sin(2\beta_1 z)]\end{aligned}$$

$$\bar{P}_{av} = \frac{1}{2} \operatorname{Re} [\bar{E} \times \bar{H}^*] = \bar{z} \frac{E_0^2}{2\eta_1} (1 - \Gamma^2) \quad \therefore \frac{\tau^2}{\eta_2} = \frac{1 - \Gamma^2}{\eta_1}$$