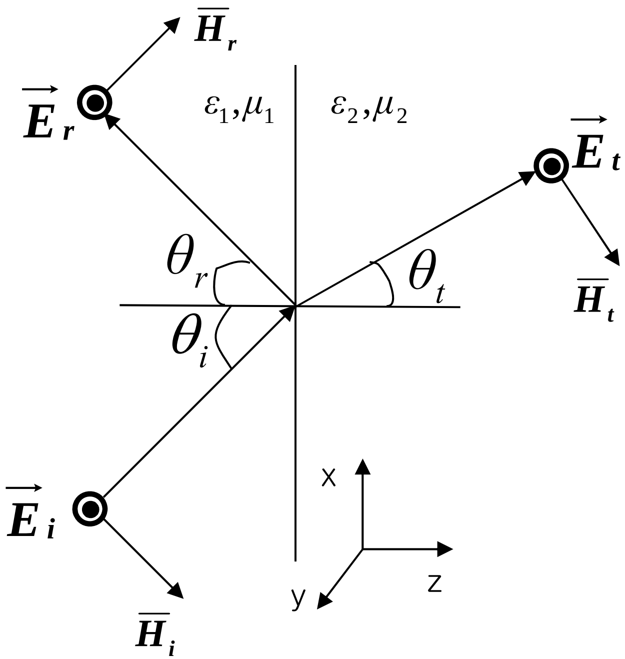
Lect. 13: Oblique Incidence at Dielectric Interface

(Cheng 8-10)



Perpendicular Polarization

$$\overline{E_i} = \overline{y} E_i \exp(-j\beta_{1x} x) \exp(-j\beta_{1z} z)$$

$$\overline{E_r} = \overline{y}\, \Gamma E_i \exp(-j\beta_{rx} x) \exp(j\beta_{rz} z)$$

$$\overline{E_t} = \overline{y}\, \tau E_i \exp(-j\beta_{2x} x) \exp(-j\beta_{2z} z)$$

$$\overline{H_i} = (-\overline{x}\cos\theta_i + \overline{z}\sin\theta_i) \frac{E_i}{\eta_1} \exp(-j\beta_{1x} x) \exp(-j\beta_{1z} z)$$

$$\overline{H_r} = (\overline{x}\cos\theta_r + \overline{z}\sin\theta_r) \frac{\Gamma E_i}{\eta_1} \exp(-j\beta_{rx} x) \exp(j\beta_{rz} z)$$

$$\overline{H_t} = (-\overline{x}\cos\theta_t + \overline{z}\sin\theta_t) \frac{\tau E_i}{\eta_2} \exp(-j\beta_{2x} x) \exp(-j\beta_{2z} z)$$

Unknowns: $\Gamma, \tau, \theta_r, \theta_t$

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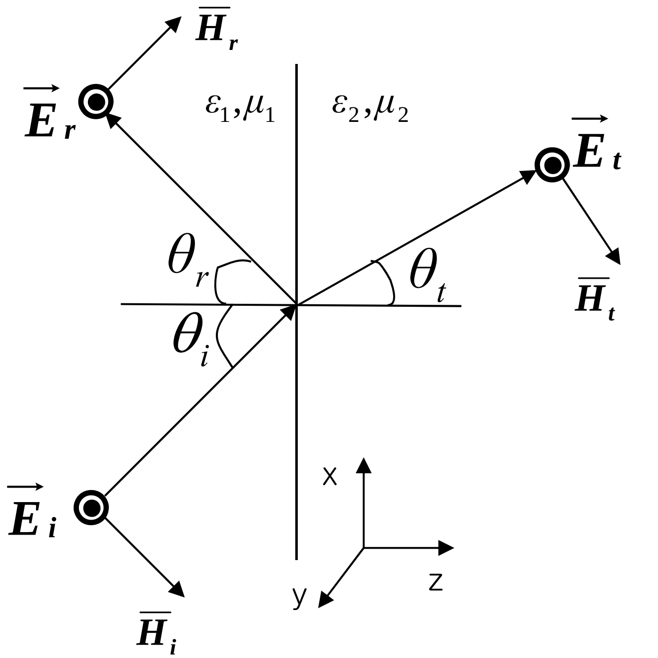

$$\overline{E_i} = \overline{y} E_i \exp(-j\beta_{1x}x)\exp(-j\beta_{1z}z)$$

$$\overline{E_r} = \overline{y}\,\Gamma E_i \exp(-j\beta_{rx}x)\exp(j\beta_{rz}z)$$

$$\overline{E_t} = \overline{y}\,\tau E_i \exp(-j\beta_{2x}x)\exp(-j\beta_{2z}z)$$

B.C.'s at z=0

1) $\overline{\mathrm{E}}_{\tan}$ continuous $(\overline{\mathrm{E_i}} + \overline{\mathrm{E_r}} = \overline{\mathrm{E_t}})$

$$\exp(-j\beta_{1x}x) + \Gamma\exp(-j\beta_{rx}x) = \tau\exp(-j\beta_{2x}x)$$

$$\therefore\ (a)\ \beta_{1x} = \beta_{rx} = \beta_{2x} \quad \text{and} \quad (b)\ 1+\Gamma=\tau$$

From $\beta_{1x} = \beta_{rx} = \beta_{2x}$: $\beta_{\tan}$ all identical

$$\sin\theta_i = \sin\theta_r \qquad \omega\sqrt{\mu_1\varepsilon_1}\sin\theta_i = \omega\sqrt{\mu_2\varepsilon_2}\sin\theta_t$$

With $\mu_1 = \mu_2$ and $n = \sqrt{\varepsilon_r}$

$n_1\sin\theta_i = n_2\sin\theta_t$: Snell's Law

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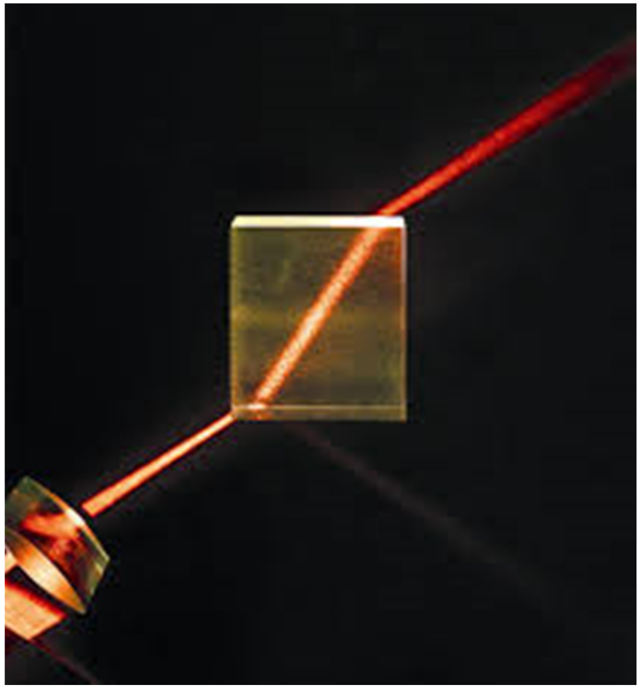
Snell's Law: $n_1 \sin\theta_i = n_2 \sin\theta_t$



➔ Refraction

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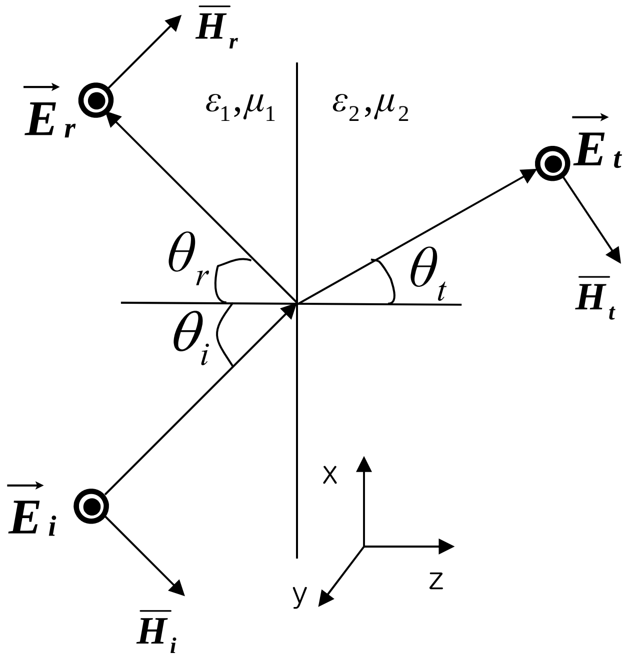

B.C.'s at z=0

2) $\overline{H}_{tan}$ continuous

$$-\frac{\cos\theta_i}{\eta_1}+\frac{\cos\theta_i}{\eta_1}\Gamma=-\frac{\cos\theta_t}{\eta_2}\tau$$

With $1+\Gamma=\tau$,

$$\Gamma_\perp=\frac{\eta_2/\cos\theta_t-\eta_1/\cos\theta_i}{\eta_2/\cos\theta_t+\eta_1/\cos\theta_i}$$

$$\tau_\perp=\frac{2\eta_2/\cos\theta_t}{\eta_2/\cos\theta_t+\eta_1/\cos\theta_i}$$

$$\overline{H_i}=(-\bar{x}\cos\theta_i+\bar{z}\sin\theta_i)\frac{E_i}{\eta_1}\exp(-j\beta_{1x}x)\exp(-j\beta_{1z}z)$$

$$\overline{H_r}=(\bar{x}\cos\theta_i+\bar{z}\sin\theta_i)\frac{\Gamma E_i}{\eta_1}\exp(-j\beta_{1x}x)\exp(j\beta_{1z}z)$$

$$\overline{H_t}=(-\bar{x}\cos\theta_t+\bar{z}\sin\theta_t)\frac{\tau E_i}{\eta_2}\exp(-j\beta_{2x}x)\exp(-j\beta_{2z}z)$$

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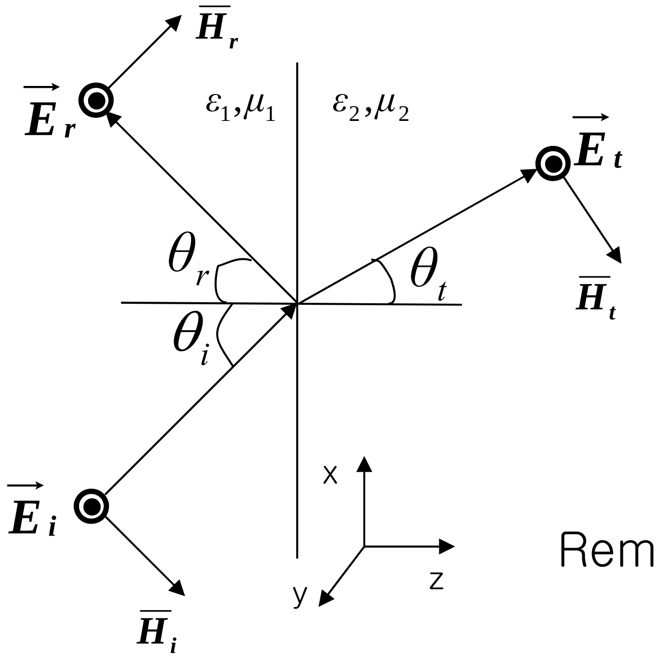

$$\Gamma_\perp = \frac{\eta_2/\cos\theta_t - \eta_1/\cos\theta_i}{\eta_2/\cos\theta_t + \eta_1/\cos\theta_i}$$

$$\tau_\perp = \frac{2\eta_2/\cos\theta_t}{\eta_2/\cos\theta_t + \eta_1/\cos\theta_i}$$

Remember for normal incident

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}, \quad \tau = \frac{2\eta_2}{\eta_2 + \eta_1}$$

For oblique incidence of perpendicular polarization, effectively

$$\eta_1 \Rightarrow \frac{\eta_1}{\cos\theta_i}, \quad \eta_2 \Rightarrow \frac{\eta_2}{\cos\theta_t}$$

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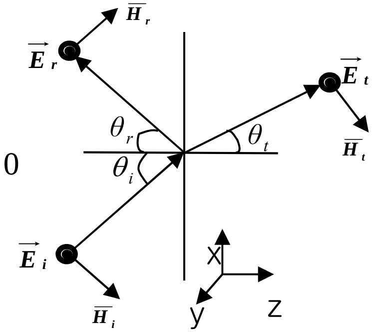
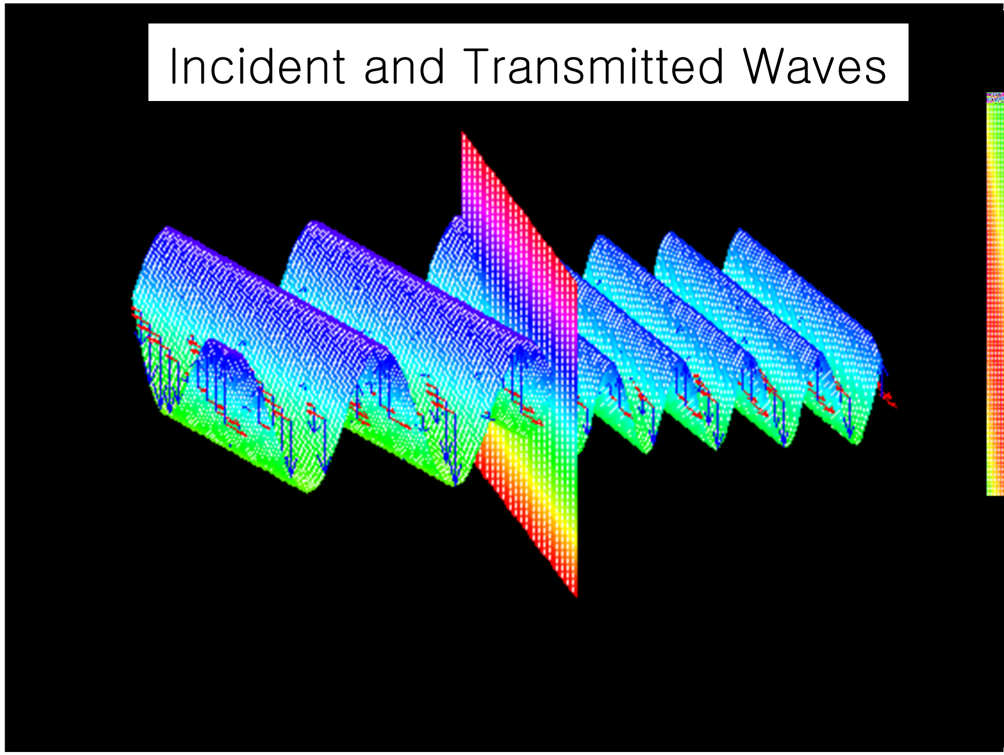
ε_0, μ_0

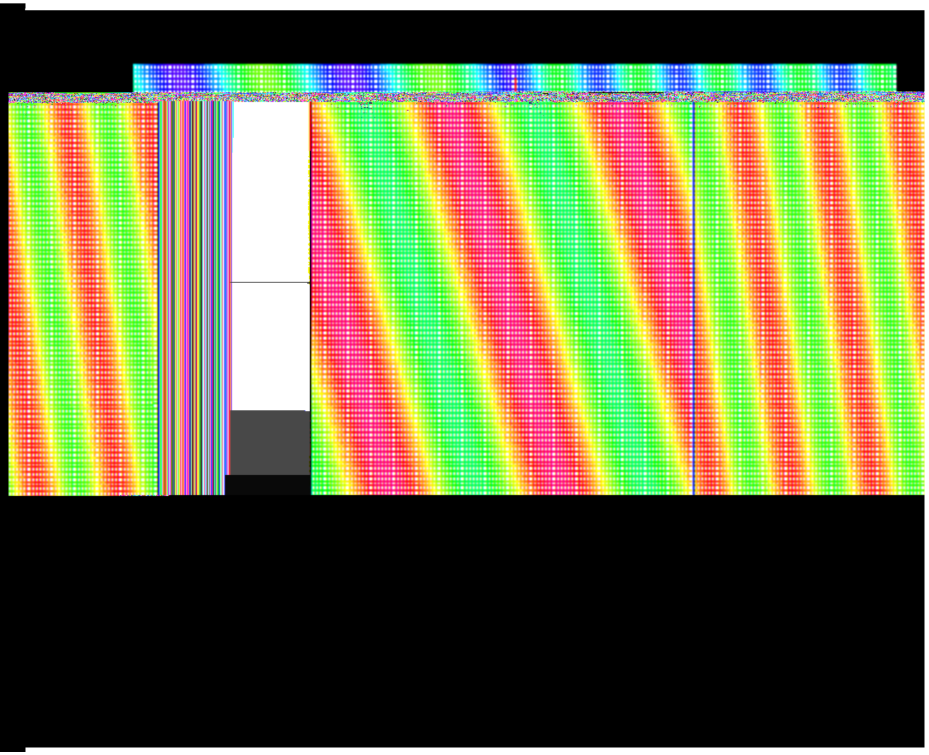


$\varepsilon = 4\varepsilon_0, \mu_0$

For $\theta_i = 20^\circ \quad \theta_t = \sin^{-1}\left[\frac{1}{2}\sin(20^\circ)\right] \simeq 9.85^\circ$

Γ = -0.3542, τ = 0.6458





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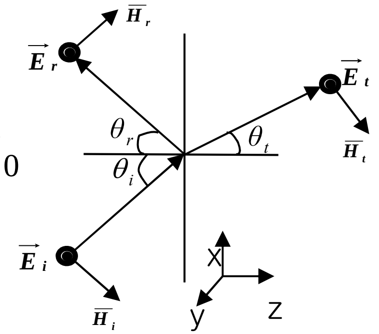
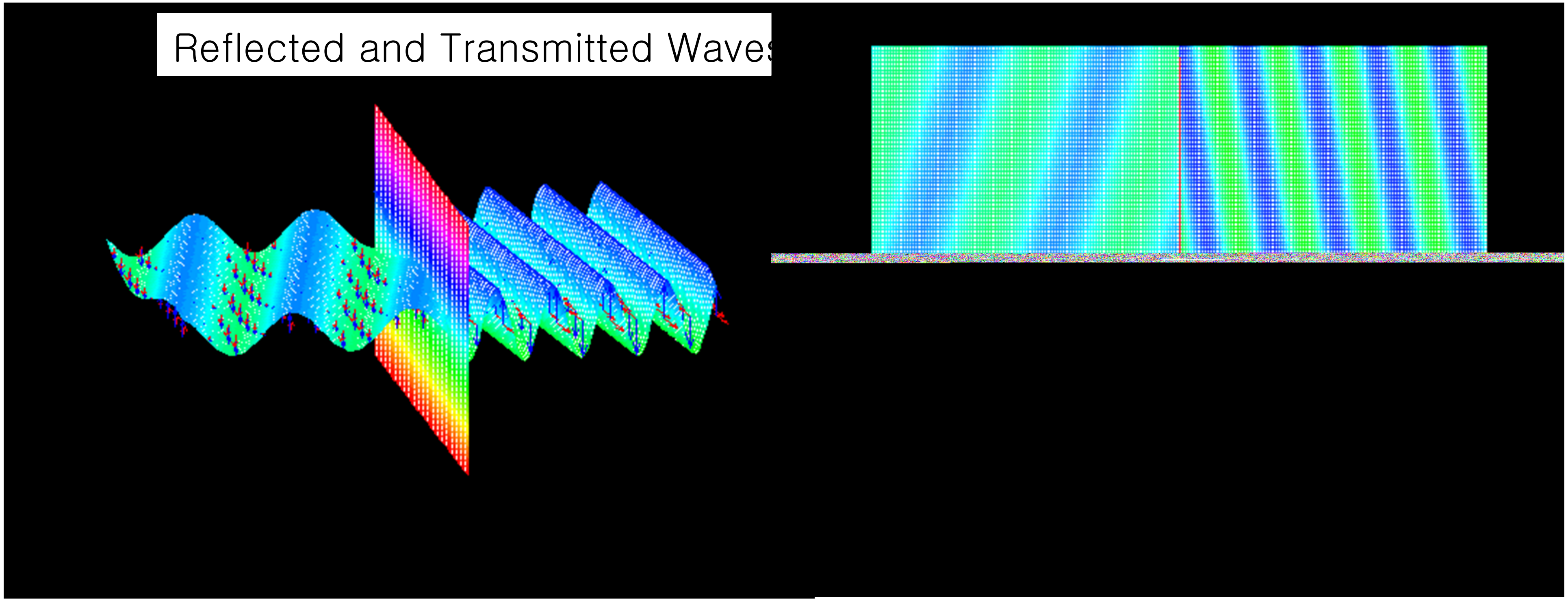
ε_0, μ_0



$\varepsilon = 4\varepsilon_0, \mu_0$

For $\theta_i = 20^\circ$ $\quad \theta_t \simeq 9.85^\circ$

Γ = -0.3542, τ = 0.6458



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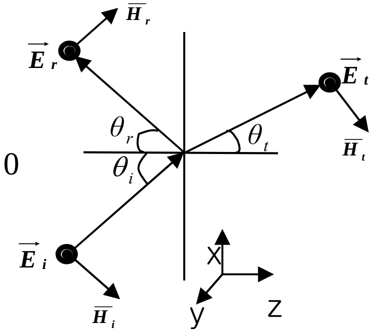
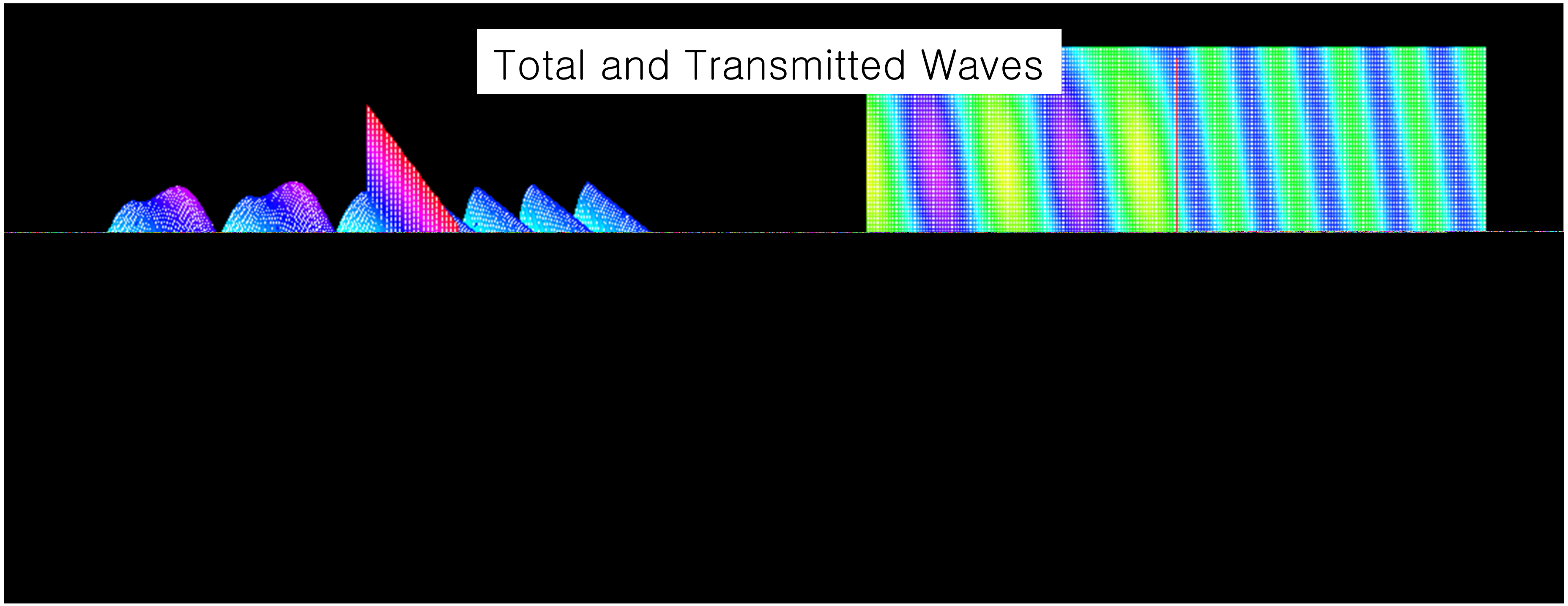
ε_0, μ_0



$\varepsilon = 4\varepsilon_0, \mu_0$

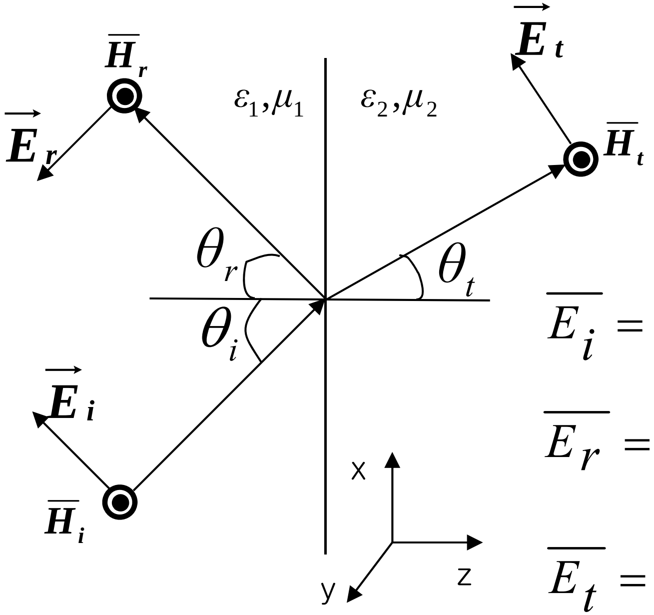
For $\theta_i = 20^\circ$ $\theta_t \simeq 9.85^\circ$

Γ = -0.3542, τ = 0.6458



Lect. 13: Oblique Incidence at Dielectric Interface

✓ Parallel Polarization



$$\overline{H_i} = \overline{y} H_i \exp(-j\beta_{1x} x)\exp(-j\beta_{1z} z)$$

$$\overline{H_r} = \overline{y} H_r \exp(-j\beta_{rx} x)\exp(j\beta_{rz} z)$$

$$\overline{H_t} = \overline{y} H_t \exp(-j\beta_{2x} x)\exp(-j\beta_{2z} z)$$

$$\overline{E_i} = (\overline{x}\cos\theta_i - \overline{z}\sin\theta_i) H_i \eta_1 \exp(-j\beta_{1x} x)\exp(-j\beta_{1z} z)$$

$$\overline{E_r} = (-\overline{x}\cos\theta_r - \overline{z}\sin\theta_r) H_r \eta_1 \exp(-j\beta_{rx} x)\exp(j\beta_{rz} z)$$

$$\overline{E_t} = (\overline{x}\cos\theta_t - \overline{z}\sin\theta_t) H_t \eta_2 \exp(-j\beta_{2x} x)\exp(-j\beta_{2z} z)$$

$$\frac{E_r}{E_i} = \Gamma \qquad \frac{H_r}{H_i} = -\Gamma \qquad \frac{H_t}{H_i} = \frac{E_t / \eta_2}{E_i / \eta_1} = \tau \frac{\eta_1}{\eta_2}$$

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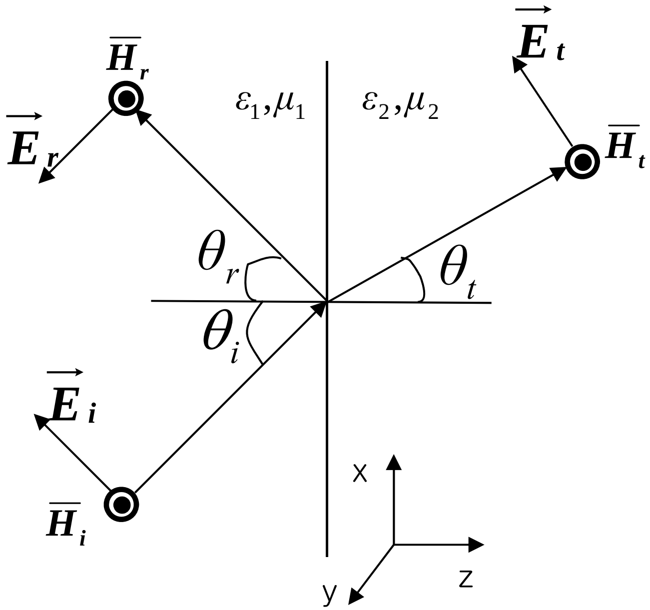
✓ Parallel Polarization



$$\frac{H_r}{H_i} = -\Gamma \qquad \frac{H_t}{H_i} = \frac{E_t/\eta_2}{E_i/\eta_1} = \tau\frac{\eta_1}{\eta_2}$$

$$\overline{H_i} = \overline{y} H_i \exp(-j\beta_{1x}x)\exp(-j\beta_{1z}z)$$

$$\overline{H_r} = \overline{y}(-\Gamma)H_i \exp(-j\beta_{rx}x)\exp(j\beta_{rz}z)$$

$$\overline{H_t} = \overline{y}\frac{\eta_1}{\eta_2}\tau H_i \exp(-j\beta_{2x}x)\exp(-j\beta_{2z}z)$$

$$\overline{E_i} = (\overline{x}\cos\theta_i - \overline{z}\sin\theta_i)H_i\eta_1 \exp(-j\beta_{1x}x)\exp(-j\beta_{1z}z)$$

$$\overline{E_r} = (-\overline{x}\cos\theta_r - \overline{z}\sin\theta_r)H_i\eta_1(-\Gamma)\exp(-j\beta_{rx}x)\exp(j\beta_{rz}z)$$

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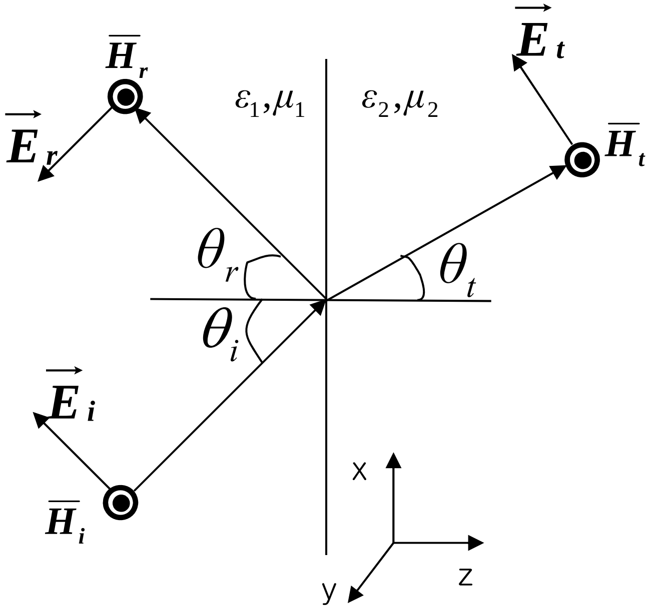
Lect. 13: Oblique Incidence at Dielectric Interface



$$\overline{H_i} = \overline{y} H_i \exp(-j\beta_{1x}x)\exp(-j\beta_{1z}z)$$

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$$\overline{H_t} = \overline{y}\frac{\eta_1}{\eta_2}\tau H_i \exp(-j\beta_{2x}x)\exp(-j\beta_{2z}z)$$

B.C.'s at z=0

1) $\overline{H}_{\tan}$ coutinuous $(\overline{H}_i + \overline{H}_r = \overline{H}_t)$

$$\exp(-j\beta_{1x}x) - \Gamma\exp(-j\beta_{rx}x) = \frac{\eta_1}{\eta_2}\tau\exp(-j\beta_{2x}x)$$

$\therefore$ (a) $\beta_{1x} = \beta_{rx} = \beta_{2x}$ (b) $1 - \Gamma = \frac{\eta_1}{\eta_2}\tau$

From $\beta_{1x} = \beta_{rx} = \beta_{2x}$: $\beta_{\tan}$ all identical

$\therefore \sin\theta_i = \sin\theta_r$ and $\omega\sqrt{\mu_1\varepsilon_1}\sin\theta_i = \omega\sqrt{\mu_2\varepsilon_2}\sin\theta_t$

With $\mu_1 = \mu_2$ and $n = \sqrt{\varepsilon_r}$

$n_1\sin\theta_i = n_2\sin\theta_t$: Snell's Law

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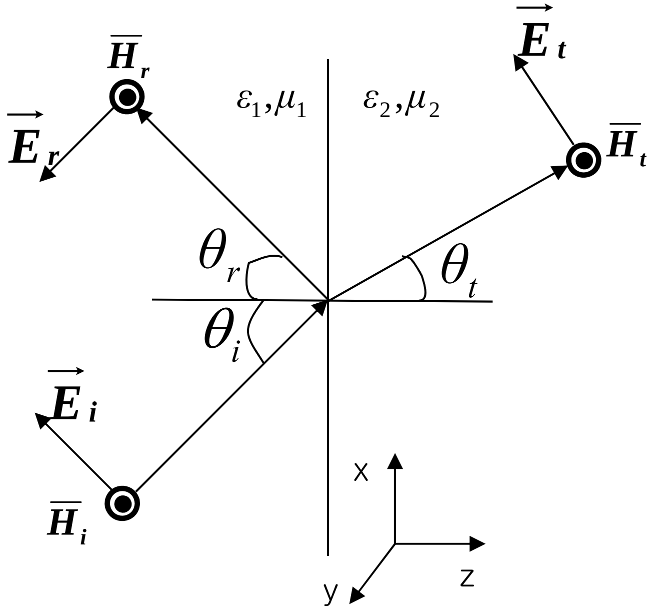


B.C.'s at z=0

2) $\overline{E}_{\tan}$ continuous

$$\cos\theta_i + \Gamma\cos\theta_i = \tau\cos\theta_t$$

With $1-\ \Gamma = \dfrac{\eta_1}{\eta_2}\tau,$

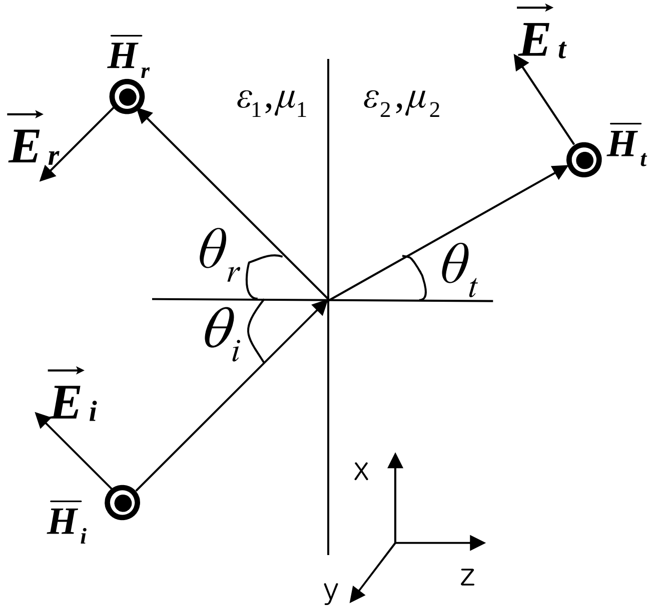
$$\boxed{\Gamma_{ll} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}\ ,\ \ \tau_{ll} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}}$$

$$\overline{E_i} = (\overline{x}\cos\theta_i - \overline{z}\sin\theta_i)H_i\eta_1\exp(-j\beta_{1x}x)\exp(-j\beta_{1z}z)$$

$$\overline{E_r} = (-\overline{x}\cos\theta_r - \overline{z}\sin\theta_r)H_i\eta_1(-\Gamma)\exp(-j\beta_{rx}x)\exp(j\beta_{rz}z)$$

$$\overline{E_t} = (\overline{x}\cos\theta_t - \overline{z}\sin\theta_t)H_i\eta_1\tau\exp(-j\beta_{2x}x)\exp(-j\beta_{2z}z)$$

Lect. 13: Oblique Incidence at Dielectric Interface



$$\Gamma_{ll} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}\ ,\ \tau_{ll} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

Remember for normal incident

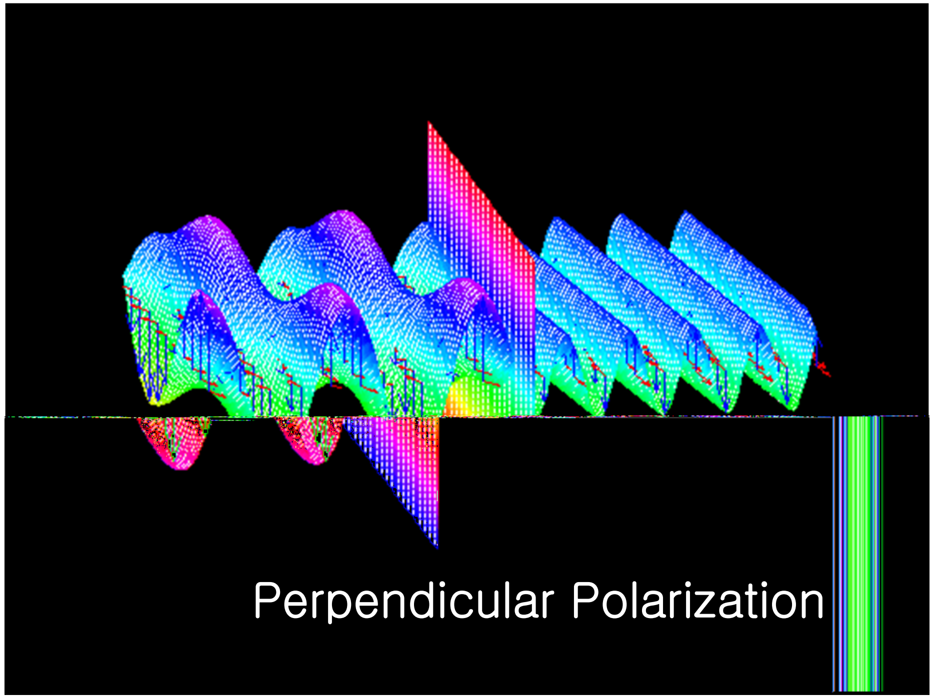
$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1},\ \tau = \frac{2\eta_2}{\eta_2 + \eta_1}$$

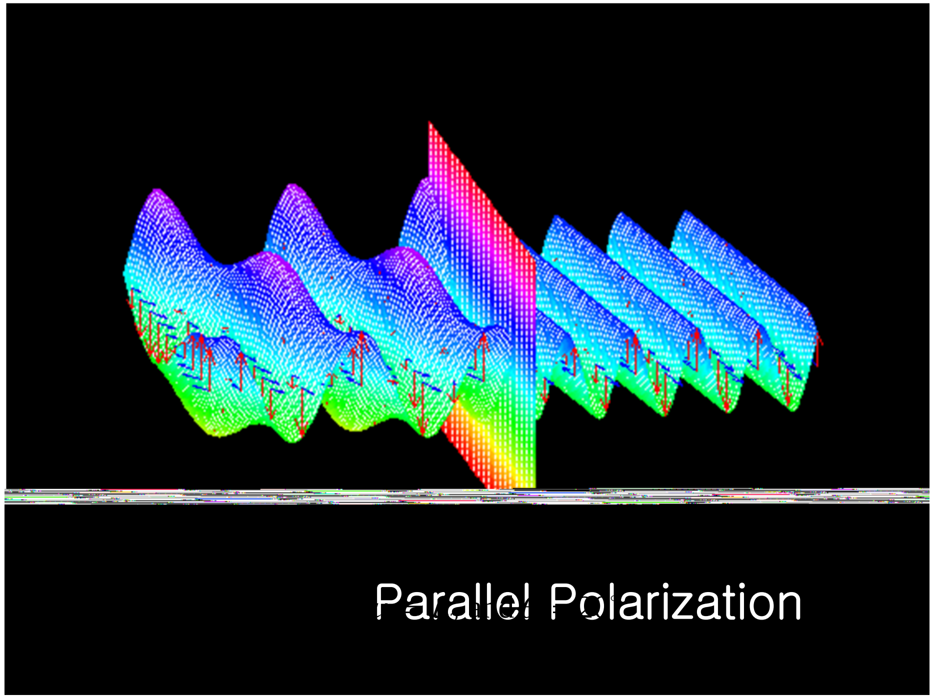
For oblique incidence of parallel polarization, effectively

$$\eta_1 \Rightarrow \eta_1\cos\theta_i,\ \eta_2 \Rightarrow \eta_2\cos\theta_t$$

Lect. 13: Oblique Incidence at Dielectric Interface

$$4\varepsilon_1 = \varepsilon_2,\ \ \mu_1 = \mu_2 \text{ and } \theta_i = 20^\circ$$





$\theta_t = 9.85^\circ, \Gamma = -0.354, \tau = 0.646$

$\theta_t = 9.85^\circ, \Gamma = -0.312, \tau = 0.656$

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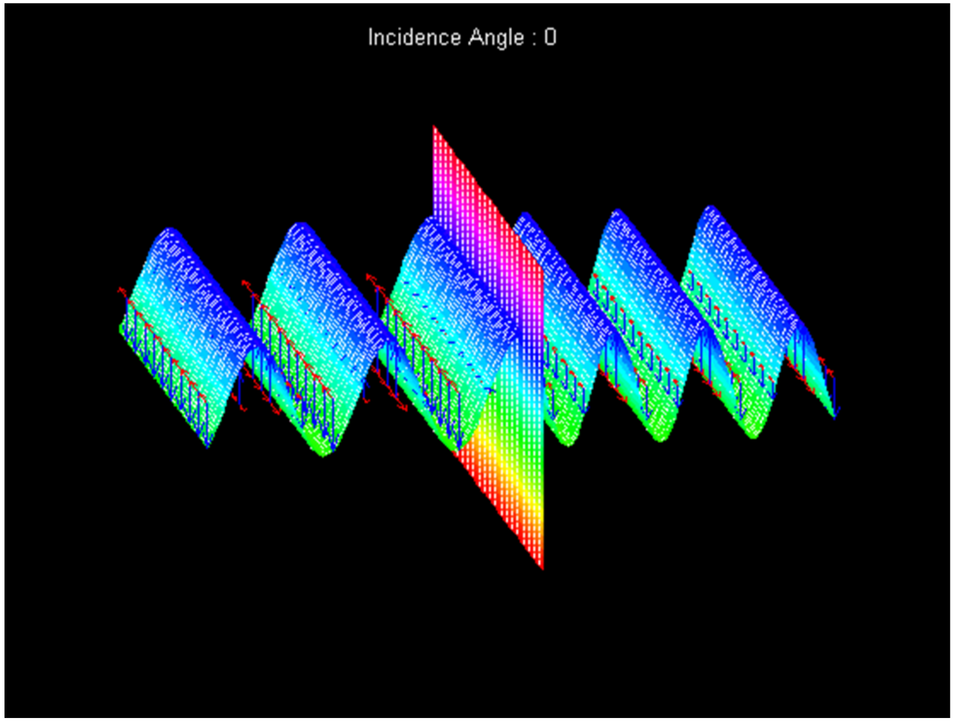
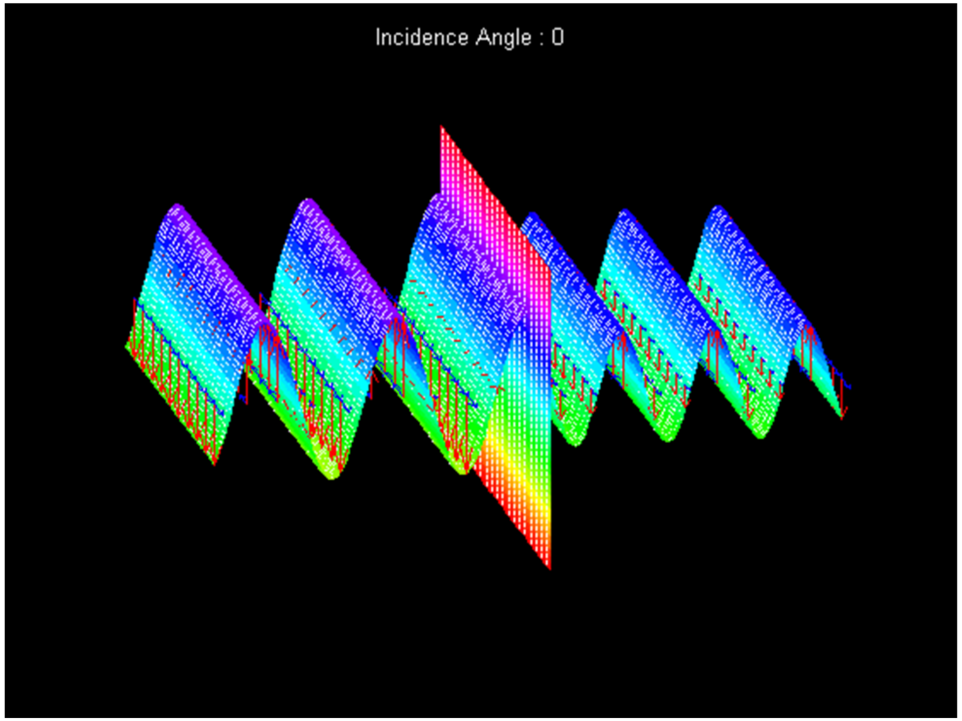
$$2\varepsilon_1 = \varepsilon_2,\ \ \mu_1 = \mu_2 \text{ and } \theta_i : \text{from } 0^\circ \text{ to } 90^\circ$$



Perpendicular



Parallel

Do you see any difference?

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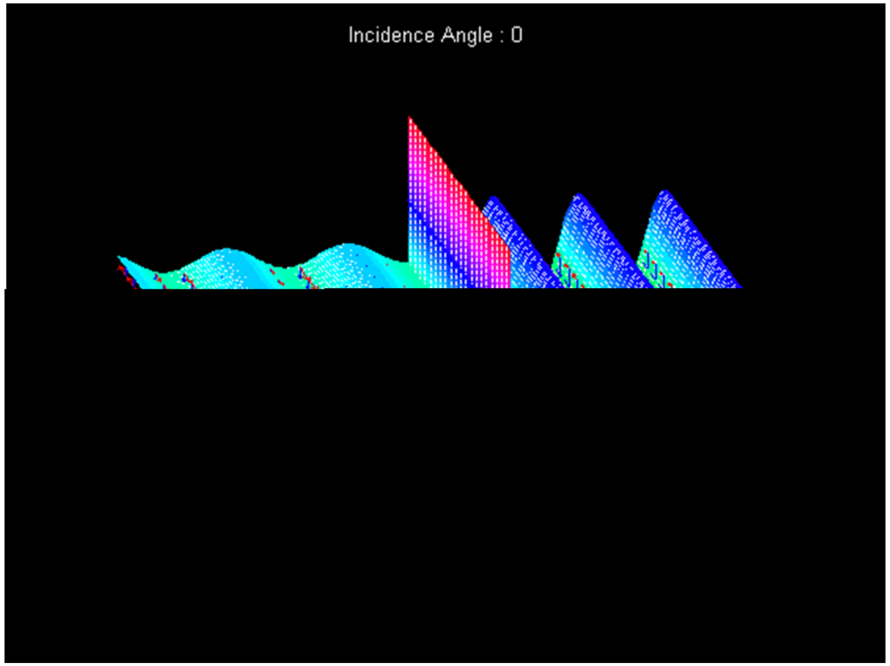
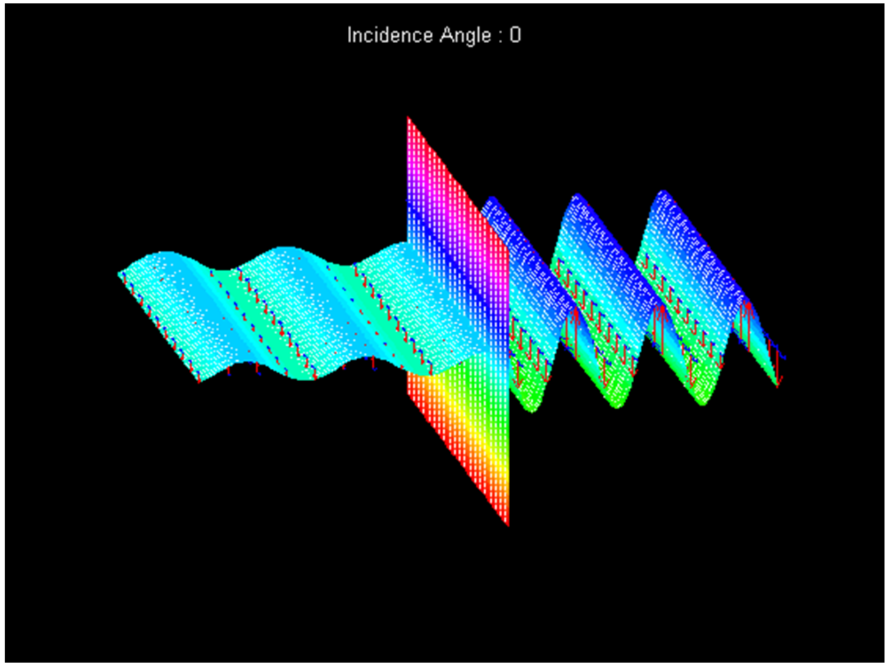
$$2\varepsilon_1 = \varepsilon_2,\ \ \mu_1 = \mu_2 \text{ and } \theta_i : \text{from } 0^\circ \text{ to } 90^\circ$$

Reflected and Transmitted Waves only





Perpendicular

Parallel

Do you see any difference?

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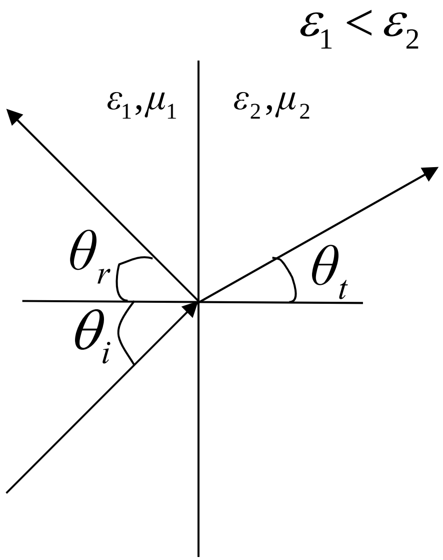

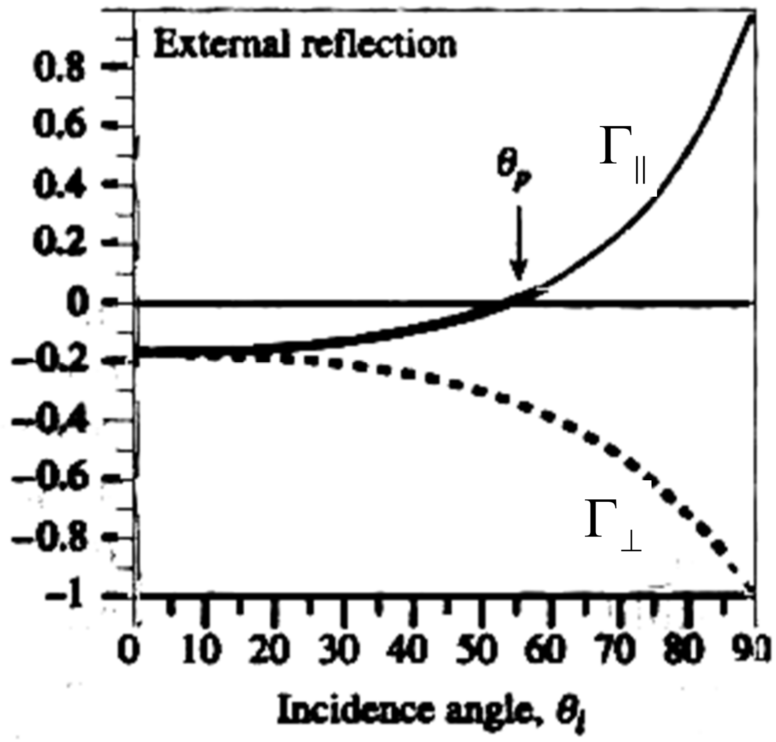

For a certain incident angle, there is no reflection for parallel polarization

➔ Brewster angle

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$$\text{For } \Gamma_{\perp} = \frac{\eta_2/\cos\theta_t - \eta_1/\cos\theta_i}{\eta_2/\cos\theta_t + \eta_1/\cos\theta_i} = 0$$

$$\sqrt{\frac{\mu}{\varepsilon_2}}\frac{1}{\cos\theta_t} - \sqrt{\frac{\mu}{\varepsilon_1}}\frac{1}{\cos\theta_i} = 0$$

$$\therefore n_1\cos\theta_i = n_2\cos\theta_t,$$

$$\text{But } \; n_1\sin\theta_i = n_2\sin\theta_t \quad \text{(Snell's law)}$$

$$\therefore \text{No } \theta_i \text{ exists.}$$

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$$\text{For } \Gamma_{ll} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = 0$$

$$\sqrt{\frac{\mu}{\varepsilon_2}} \cos\theta_t - \sqrt{\frac{\mu}{\varepsilon_1}} \cos\theta_i = 0$$

$$\therefore n_2 \cos\theta_i = n_1 \cos\theta_t \quad \text{and} \quad n_1 \sin\theta_i = n_2 \sin\theta_t$$

$$\sin^2\theta_i = \frac{n_2^{\,2}}{n_1^{\,2} + n_2^{\,2}} \quad \text{and} \quad \cos^2\theta_i = \frac{n_1^{\,2}}{n_1^{\,2} + n_2^{\,2}} \qquad \therefore \theta_i = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

$$\text{For the previous demo, } \theta_i = \tan^{-1}\left(\sqrt{2}\right) = 54.7^\circ$$

For dielectric materials, Brewster angle exists only for parallel polarization

(Assuming $\mu_1 = \mu_2$)

Lect. 13: Oblique Incidence at Dielectric Interface

- Incident sun light has both perpendicular and parallel polarization
- At Brewster angle, only light with perpendicular polarization gets reflected
- A polarizer can block light with certain polarization
- Then, reflection of sun light can be entirely blocked

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