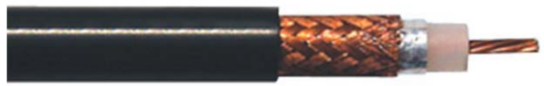
Lect. 15: Voltage, Current Waves in Transmission Lines

(Cheng 9-2,3)



Coaxial cables

Why use coaxial cables?

- Deliver high-frequency electric signals without loss and distortion
- Signals are well guided within coax cables
- E, H fields within transmission lines?
- Relationship between E, H fields and V, I ?

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Transmission Lines: support voltage and current waves

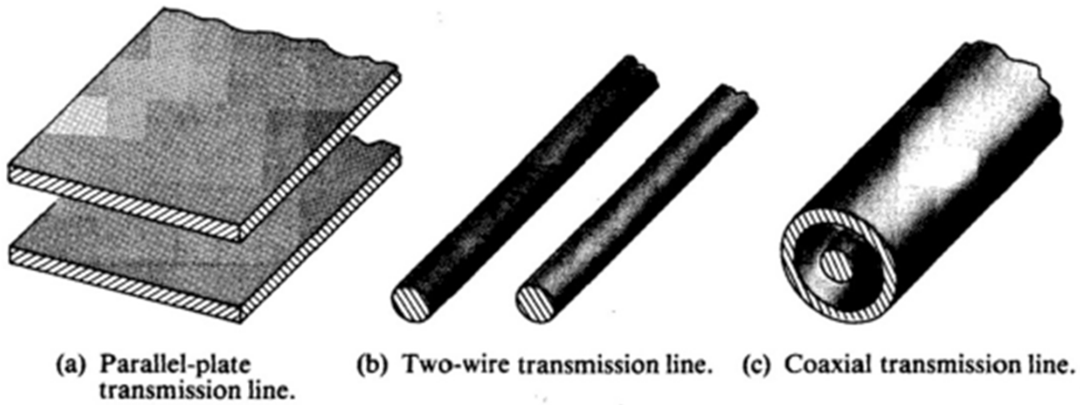

FIGURE 9–1
Common types of transmission lines.

Focus on parallel-plate transmission lines for easy analysis

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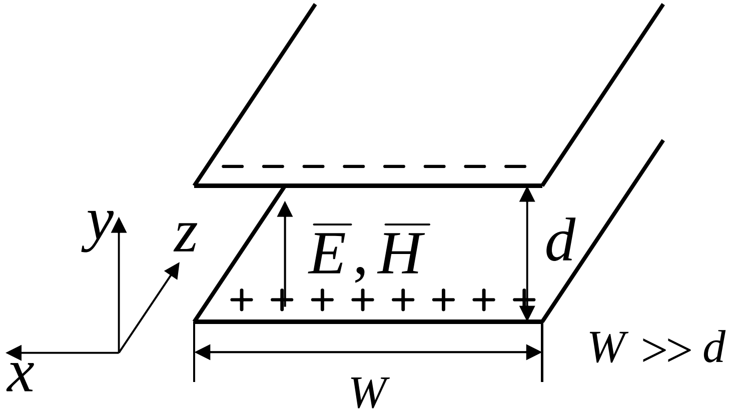
Determine E and H when they are confined within two parallel plates

by surface charges and currents



$$\overline{E} = \overline{y}\, E_0 e^{-j\beta z} \quad (0 < y < d)$$

$$\overline{H} = -\overline{x}\frac{E_0}{\eta} e^{-j\beta z} \quad (0 < y < d)$$

Charge and current densities

$$\rho_s(y=0) = \varepsilon E_0 e^{-j\beta z}$$

$$\rho_s(y=d) = -\varepsilon E_0 e^{-j\beta z}$$

$$\overline{J_s}\,(y=0) = \overline{z}\frac{E_0}{\eta} e^{-j\beta z}$$

$$\overline{J_s}\,(y=d) = -\overline{z}\frac{E_0}{\eta} e^{-j\beta z}$$

Voltages and currents?

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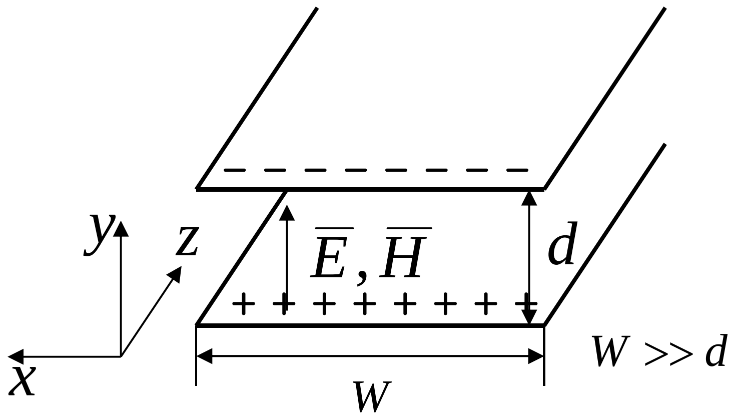

$$V(y=d,z)-V(y=0,z)= -\int_{y=0}^{d} E_y(y)dy$$

$$= -E_O e^{-j\beta z} d$$

$$I(y=d,z) = J_s W = -\frac{E_0}{\eta} e^{-j\beta z} W$$

$$\overline{E} = \overline{y}\, E_0 e^{-j\beta z} \quad (0 < y < d)$$

$$\overline{H} = -\overline{x}\frac{E_0}{\eta} e^{-j\beta z} \quad (0 < y < d)$$

Voltage and Current Waves!

$$V(z) = -E_y(z)d \qquad I(y=d,z) = H_x(x)W$$

Wave equations for V(z) and I(z) ?

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$$V(z) = -E_y(z)d \quad \text{and} \quad I(z) = H_x(z)W$$

From $\nabla \times \overline{\mathrm{E}} = -\mathrm{j}\omega\mu\overline{\mathrm{H}}$, $\quad \nabla \times \overline{\mathrm{E}} = \begin{vmatrix} \overline{\mathrm{x}} & \overline{\mathrm{y}} & \overline{\mathrm{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix} = \overline{x}\left(-\frac{\partial E_y}{\partial z}\right) \qquad \therefore \frac{dE_y}{dz} = \mathrm{j}\omega\mu\mathrm{H}_\mathrm{x}$

$$\frac{dV}{dz} = -\frac{dE_y}{dz}d \quad = -\mathrm{j}\omega\mu\mathrm{H}_\mathrm{x}d \quad = -\mathrm{j}\omega\mu\frac{I(z)}{W}d$$

Let $L = \dfrac{\mu d}{W}$, inductance per unit length

$$\therefore \frac{dV}{dz} = -j\omega LI$$

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$$V(z) = -E_y(z)d \text{ and } I(z) = H_x(z)W$$

From $\nabla \times \overline{\mathrm{H}} = \mathrm{j}\omega\varepsilon\overline{\mathrm{E}}$ $\quad \nabla \times \overline{\mathrm{H}} = \begin{vmatrix} \overline{\mathrm{x}} & \overline{\mathrm{y}} & \overline{\mathrm{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & 0 & 0 \end{vmatrix} = \overline{y}(\frac{\partial H_x}{\partial z})$ $\quad \therefore \frac{dH_x}{dz} = \mathrm{j}\omega\varepsilon E_y$

$$\frac{dI}{dz} = \frac{dH_x}{dz}W = j\omega\varepsilon E_y W = -j\omega\varepsilon\frac{V(z)}{d}W$$

Let $\mathrm{C} = \dfrac{\varepsilon W}{d}$, capacitance per unit length

$$\frac{dI}{dz} = -j\omega CV$$

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$$\frac{dV}{dz} = -j\omega LI \qquad \frac{dI}{dz} = -j\omega CV \qquad (L = \frac{\mu d}{W},\ C = \frac{\varepsilon W}{d})$$

With additional differentiation,

$$\frac{d^2V}{dz^2} = -j\omega L\frac{dI}{dz} = -\omega^2 LCV \quad : \text{Wave Eq. for } V(z)$$

$$\frac{d^2I}{dz^2} = -j\omega C\frac{dV}{dz} = -\omega^2 LCI \quad : \text{Wave Eq. for } I(z)$$

Remember

$$\nabla^2 \overline{E} = \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2} \qquad \nabla^2 \overline{H} = \mu\varepsilon \frac{\partial^2 \overline{H}}{\partial t^2}$$

$$\nabla^2 \overline{E} = -\omega^2 \mu\varepsilon \overline{E} \qquad \nabla^2 \overline{H} = -\omega^2 \mu\varepsilon \overline{H}$$

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$$\frac{d^2V}{dz^2} = -\omega^2 LCV \qquad \frac{d^2I}{dz^2} = -\omega^2 LCI \qquad (L = \frac{\mu d}{W},\ \mathrm{C} = \frac{\varepsilon W}{d})$$

Solutions:

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z} \qquad I(z) = I_0^+ e^{-j\beta z} + I_0^- e^{+j\beta z}$$

$$-\beta^2 V(z) = -\omega^2 LCV(z) \qquad -\beta^2 I(z) = -\omega^2 LCI(z)$$

$$\therefore\ \beta = \omega\sqrt{LC} = \omega\sqrt{\mu\varepsilon} \qquad \therefore\ \beta = \omega\sqrt{LC} = \omega\sqrt{\mu\varepsilon}$$

$$u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{LC}}$$

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How about impedance?

$$V(z) = V_0^+ e^{-j\beta z} \qquad I(z) = I_0^+ e^{-j\beta z}$$

$$\frac{dI(z)}{dz} = -j\beta I_0^+ e^{-j\beta z}$$

Remember $\dfrac{dI(z)}{dz} = -j\omega C V(z) = -j\omega C V_0^+ e^{-j\beta z}$

$$\therefore \frac{V_0^+}{I_0^+} = \frac{\beta}{\omega C} = \frac{\omega\sqrt{LC}}{\omega C} = \sqrt{\frac{L}{C}} = \sqrt{\frac{\mu d/W}{\varepsilon W/d}} = \eta\frac{d}{W}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$ (TL Characteristic Impedance)

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How about impedance?

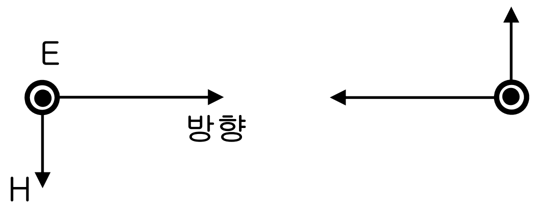
$$V(z) = V_0^- e^{j\beta z} \qquad I(z) = I_0^- e^{j\beta z}$$

$$\frac{dI(z)}{dz} = j\beta I_0^- e^{j\beta z}$$

Remember $\displaystyle \frac{dI(z)}{dz} = -j\omega C V(z) = -j\omega C V_0^- e^{j\beta z}$

$$\therefore \frac{V_0^-}{I_0^-} = -\frac{\beta}{\omega C} = -\frac{\omega\sqrt{LC}}{\omega C} = -\sqrt{\frac{L}{C}} = -Z_0$$

Why negative impedance?



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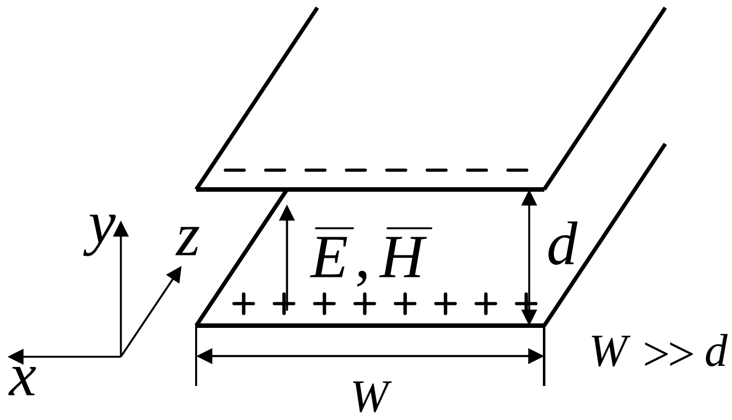

From $\nabla\times\overline{E} = -j\omega\mu\overline{H}$, $\quad \dfrac{dV}{dz} = -j\omega LI \quad (L = \dfrac{\mu d}{W})$

From $\nabla\times\overline{H} = j\omega\varepsilon\overline{E}$ $\quad \dfrac{dI}{dz} = -j\omega CV \quad (C = \dfrac{\varepsilon W}{d})$

$$\frac{d^2V}{dz^2} = -\omega^2 LCV \qquad \frac{d^2I}{dz^2} = -\omega^2 LCI$$

$$V(z) = V_0^+e^{-j\beta z} + V_0^-e^{+j\beta z} \qquad I(z) = I_0^+e^{-j\beta z} + I_0^-e^{+j\beta z}$$

$$\beta = \omega\sqrt{LC} \qquad Z_0 = \sqrt{\frac{L}{C}}$$