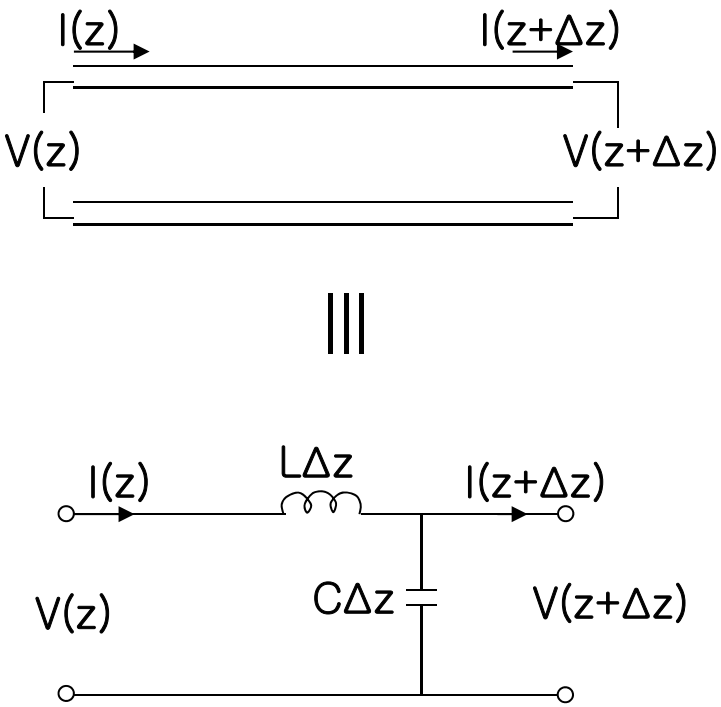
Lect. 16: Circuit Models for Transmission Lines

Consider a small segment of transmission line as a circuit



$$V(z) - L\Delta z \frac{dI(z)}{dt} = V(z+\Delta z)$$

$$\frac{V(z) - V(z+\Delta z)}{\Delta z} = L\frac{dI(z)}{dt}$$

$$\Delta z \to 0 \qquad -\frac{dV}{dz} = L\frac{dI}{dt}$$

$$-\frac{dV}{dz} = j\omega LI$$

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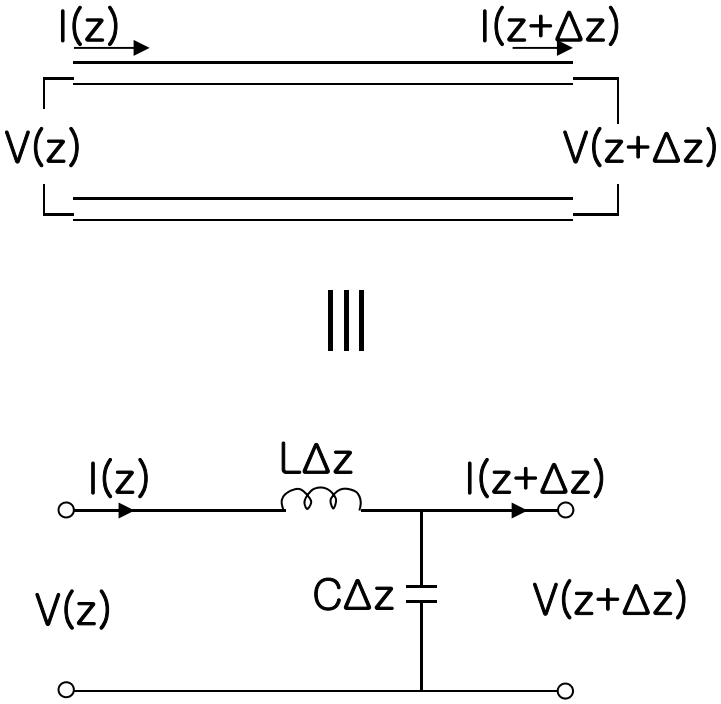
Lect. 16: Circuit Models for Transmission Lines



$$I(z) - C\Delta z \frac{dV(z+\Delta z)}{dt} = I(z+\Delta z)$$

$$\Delta z \to 0 \qquad -\frac{dI}{dz} = C\frac{dV}{dt}$$

$$\frac{dI}{dz} = -j\omega CV$$

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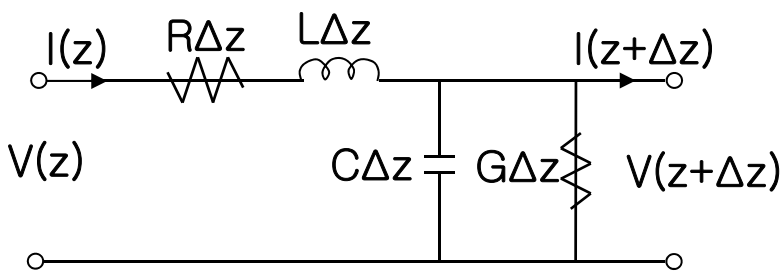
Lect. 16: Circuit Models for Transmission Lines

More general case



R : resistance per unit length in the conductor

G: conductance per unit length in the dielectric

$$V(z) - I(z)R\Delta z - L\Delta z \frac{dI(z)}{dt} = V(z+\Delta z)$$

$$\frac{V(z) - V(z+\Delta z)}{\Delta z} = I(z)R + L\frac{dI(z)}{dt}$$

$$-\frac{dV(z)}{dz} = I(z)R + L\frac{dI(z)}{dt}$$

$$-\frac{dV(z)}{dz} = I(z)R + j\omega L I(z) = (R + j\omega L)I(z)$$

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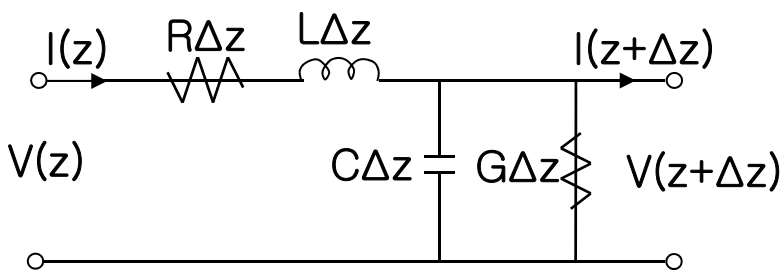

R : resistance per unit length in the conductor

G: conductance per unit length in the dielectric

$$I(z) - \frac{C\Delta z \cdot dV(z+\Delta z)}{dt} - G\Delta z \cdot V(z+\Delta z) = I(z+\Delta z)$$

$$\frac{I(z) - I(z+\Delta z)}{\Delta z} = \frac{C \cdot dV(z+\Delta z)}{dt} + G \cdot V(z+\Delta z)$$

$$-\frac{dI(z))}{dz} = \frac{CdV(z)}{dt} + GV(z)$$

$$-\frac{dI(z)}{dz} = j\omega CV(z) + GV(z) = (G + j\omega C)V(z)$$

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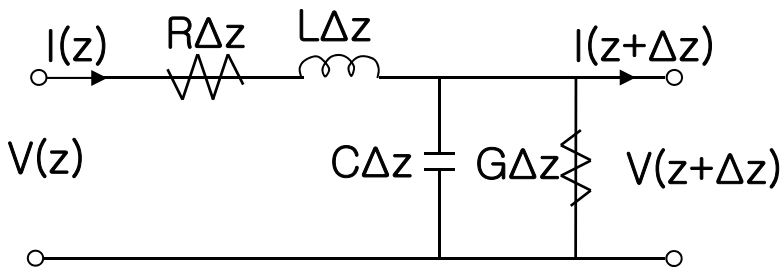

$$-\frac{dV(z)}{dz} = (R + j\omega L)I(z) \qquad -\frac{dI(z)}{dz} = (G + j\omega C)V(z)$$

$$\frac{d^2V}{dz^2} = (R + j\omega L)(G + j\omega C)V = \gamma^2 V$$

$$\frac{d^2I}{dz^2} = (R + j\omega L)(G + j\omega C)I = \gamma^2 I$$

R : resistance per unit length in the conductor

G: conductance per unit length in the dielectric

Generalized transmission line wave equations!

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Solution for voltage wave equation: $\frac{d^2V}{dz^2} = (R + j\omega L)(G + j\omega C)V = \gamma^2 V$

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$\frac{dV}{dz} = -\gamma V_0^+ e^{-\gamma z} + \gamma V_0^- e^{\gamma z}$$

$$\frac{d^2V}{dz^2} = \gamma^2 V(z)$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = j\beta + \alpha = j(\beta - j\alpha)$$

$$= jk$$

$R = G = 0$ 일 때

$$\gamma = \sqrt{j\omega L \cdot j\omega C} = j\omega\sqrt{LC} = j\beta$$

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Impedance Z=V/I

For $V(z) = V_0^+ e^{-\gamma z}$

From $\dfrac{dV}{dz} = -(R + j\omega L)I$

$$-\gamma V = -(R + j\omega L)I$$

$$\therefore Z = \frac{V}{I} = \frac{(R + j\omega L)}{\gamma} = \frac{(R + j\omega L)}{\sqrt{(R + j\omega L)(G + j\omega C)}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Complex impedance!

For $V(z) = V_0^- e^{\gamma z}$?

Phase velocity?

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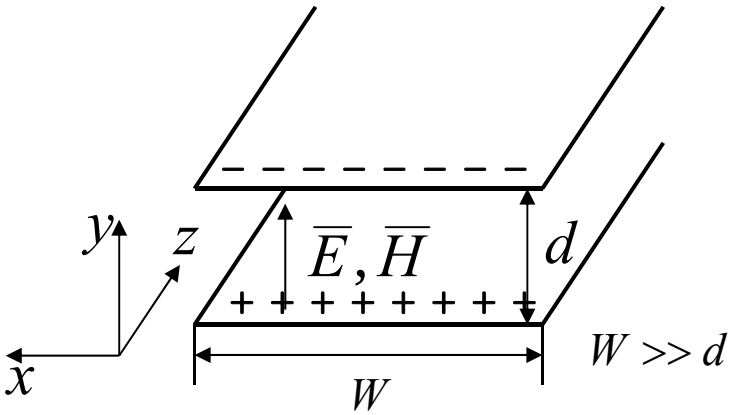

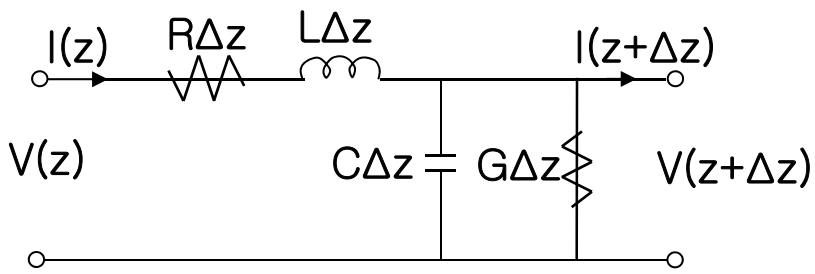

$$\frac{d^2V}{dz^2} = (R + j\omega L)(G + j\omega C)V = \gamma^2 V \qquad \frac{d^2I}{dz^2} = (R + j\omega L)(G + j\omega C)I = \gamma^2 I$$

G: conductance per unit length in the dielectric

Time constant for a capacitor: $\dfrac{C}{G} = \dfrac{\varepsilon}{\sigma_d}$

$$\therefore G = \frac{C\sigma_d}{\varepsilon} = \frac{\varepsilon W}{d}\frac{\sigma_d}{\varepsilon} = \frac{\sigma_d W}{d}$$

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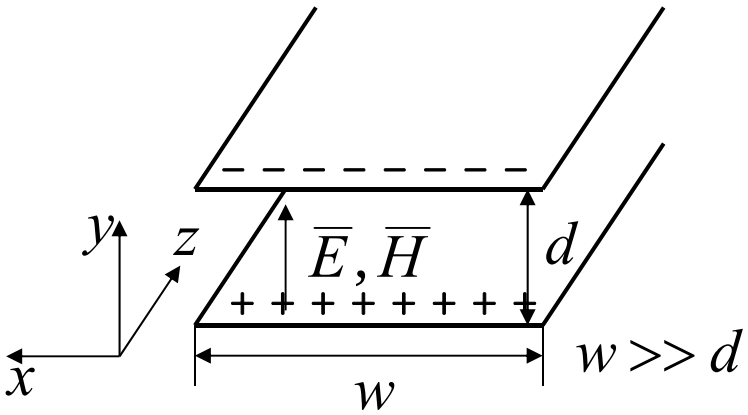

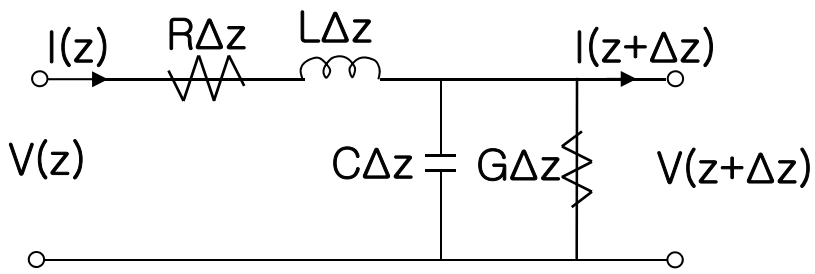

$$\frac{d^2V}{dz^2} = (R + j\omega L)(G + j\omega C)V = \gamma^2 V \qquad \frac{d^2I}{dz^2} = (R + j\omega L)(G + j\omega C)I = \gamma^2 I$$

R : resistance per unit length in the conductor

It can be shown for parallel plate TL $R = \frac{2}{w}\sqrt{\frac{\pi f \mu_c}{\sigma_c}}$

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