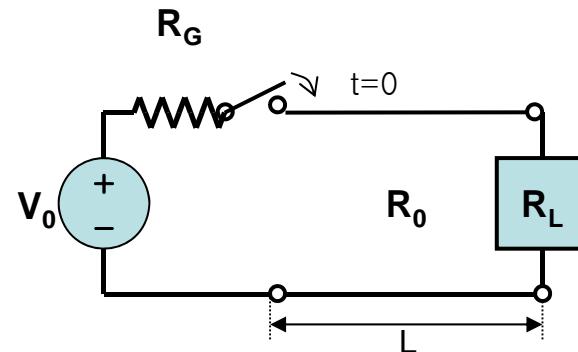


Lect. 19: Transients in Transmission Lines

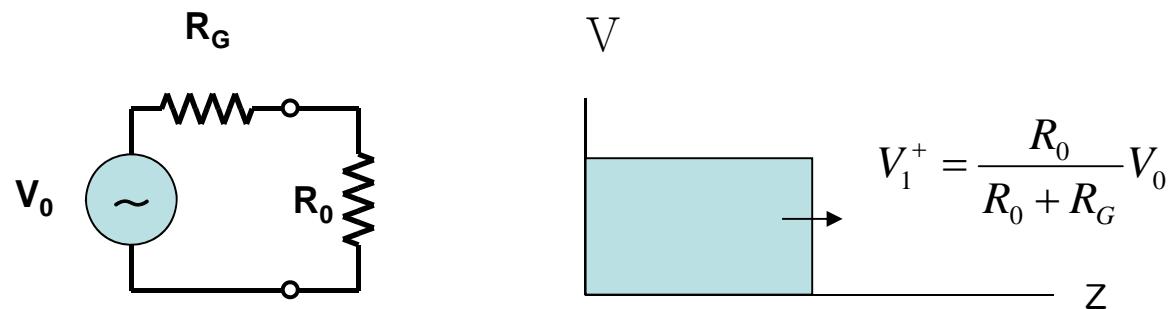
(Cheng 9–5.2, 5.3, 5.4) =

So far we have considered only sinusoidal steady-state responses for TLs

How about transient responses? Step responses

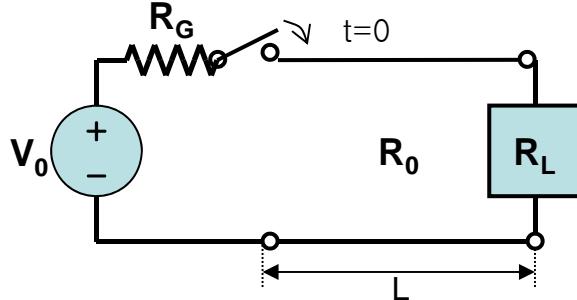


i) At $t = 0^+$



Initially, the voltage wave “sees” only R_0 on TL \Rightarrow Voltage divider

Lect. 19: Transients in Transmission Lines



ii) At $t = \frac{L}{u}$ $V_1^- = \Gamma_L V_1^+$, $\Gamma_L = \frac{R_L - R_0}{R_L + R_0}$

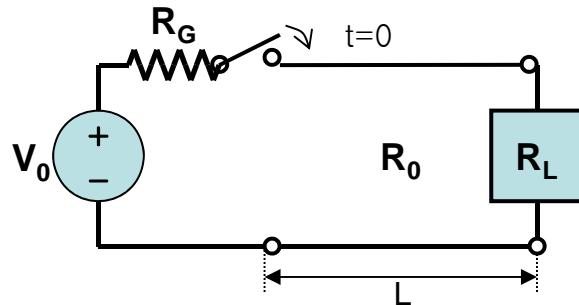
iii) At $t = \frac{2L}{u}$ $V_2^+ = \Gamma_G V_1^-$, $\Gamma_G = \frac{R_G - R_0}{R_G + R_0}$

iv) At $t = \frac{3L}{u}$ $V_2^- = \Gamma_L V_2^+$,

$$\begin{aligned}
 V_{total(t=\infty)} &= V_1^+ + V_1^- + V_2^+ + V_2^- + \dots \\
 &= V_1^+ \left(1 + \Gamma_L + \Gamma_L \Gamma_G + \Gamma_L^2 \Gamma_G + \Gamma_L^2 \Gamma_G^2 + \Gamma_L^3 \Gamma_G^2 + \dots \right) \\
 &= V_1^+ [(1 + \Gamma_L \Gamma_G + \Gamma_L^2 \Gamma_G^2 + \dots) + \Gamma_L (1 + \Gamma_L \Gamma_G + \Gamma_L^2 \Gamma_G^2 + \dots)] \\
 &= V_1^+ \left(\frac{1 + \Gamma_L}{1 - \Gamma_L \Gamma_G} \right) = \left(\frac{R_0}{R_0 + R_G} \right) V_0 \left(\frac{1 + \Gamma_L}{1 - \Gamma_L \Gamma_G} \right) \\
 &= \left(\frac{R_L}{R_L + R_G} \right) V_0
 \end{aligned}$$

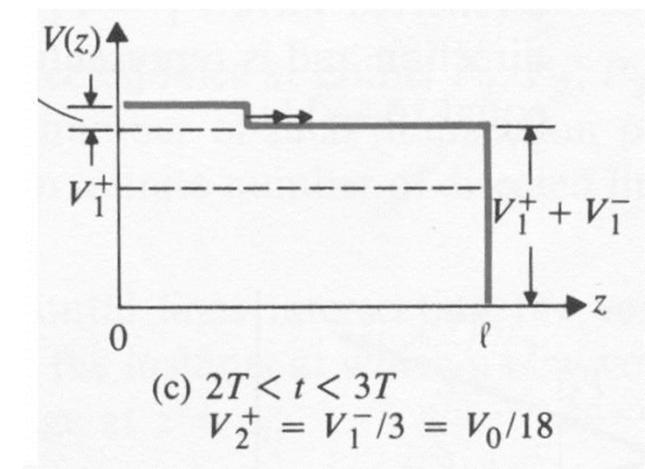
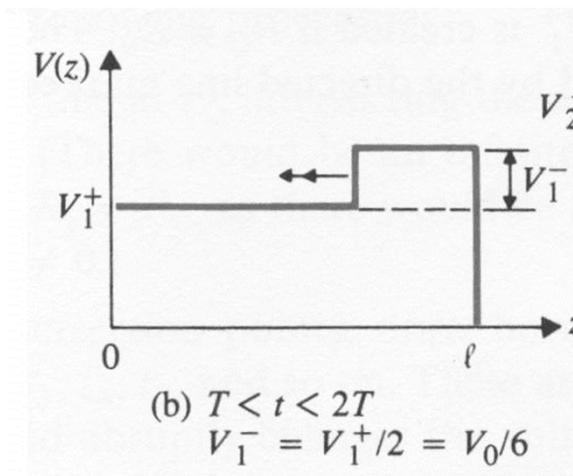
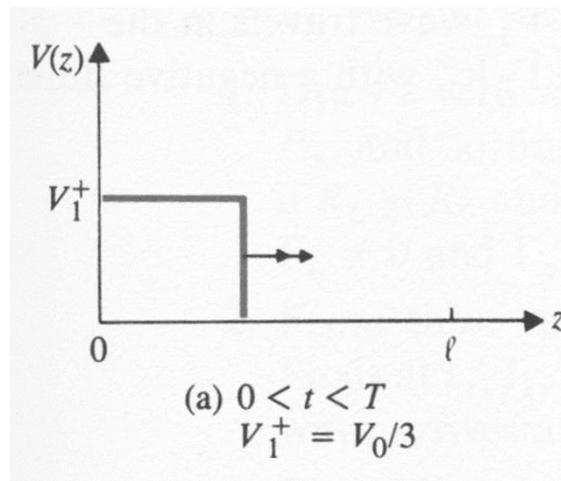
No TL effect:
 All the wave characteristics have died out!

Lect. 19: Transients in Transmission Lines



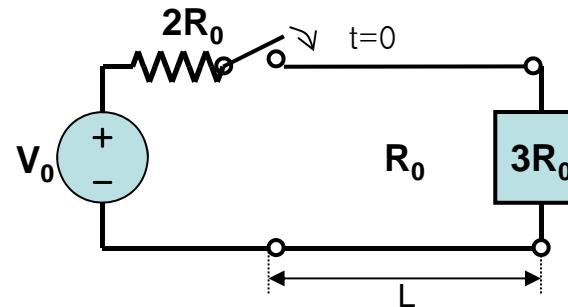
Example) $R_L = 3R_0, R_G = 2R_0$

$$\Gamma_L = \frac{R_L - R_0}{R_L + R_0} = \frac{2R_0}{4R_0} = \frac{1}{2} \quad \Gamma_G = \frac{R_G - R_0}{R_G + R_0} = \frac{R_0}{3R_0} = \frac{1}{3}$$

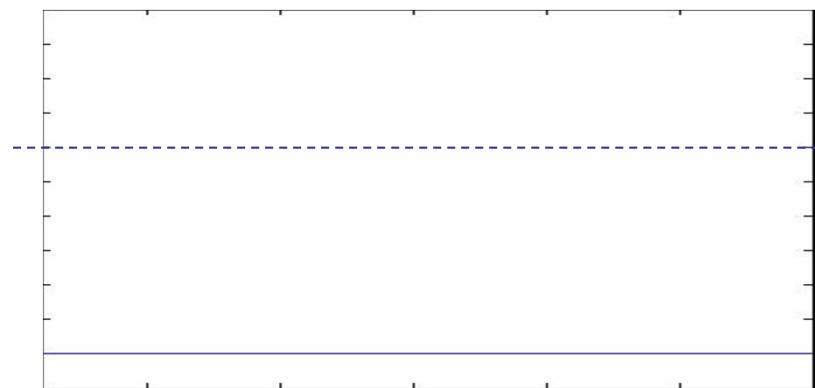


$$V_{tot(t=\infty)} = \frac{3}{5}V_0$$

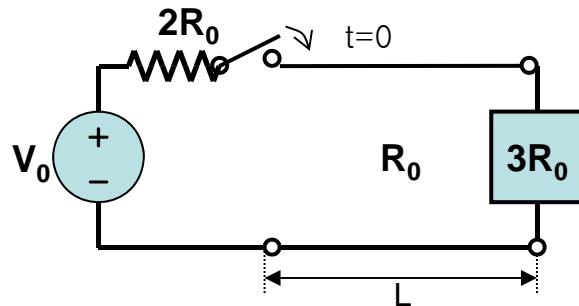
Lect. 19: Transients in Transmission Lines



voltage

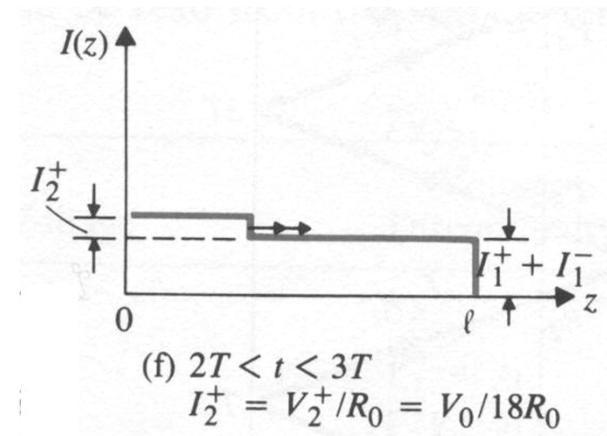
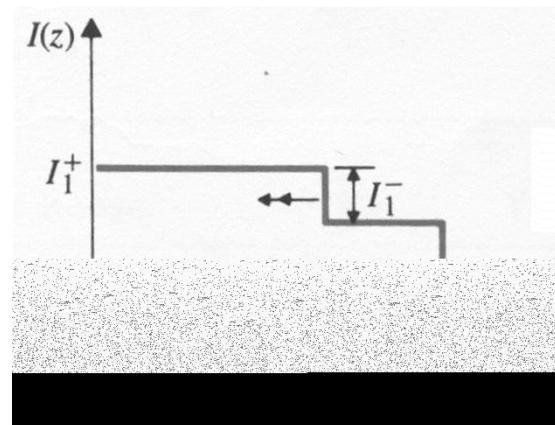
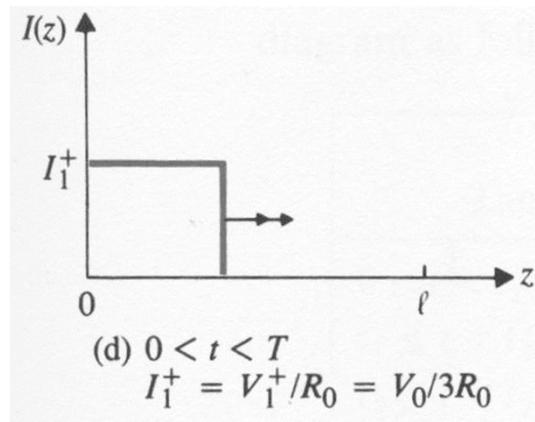


Lect. 19: Transients in Transmission Lines



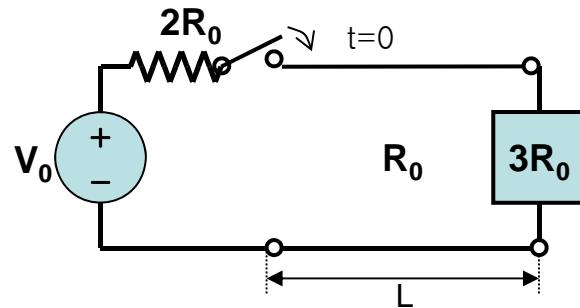
$$R_L = 3R_0, R_G = 2R_0, \Gamma_L = \frac{1}{2}, \Gamma_G = \frac{1}{3}$$

How about currents?

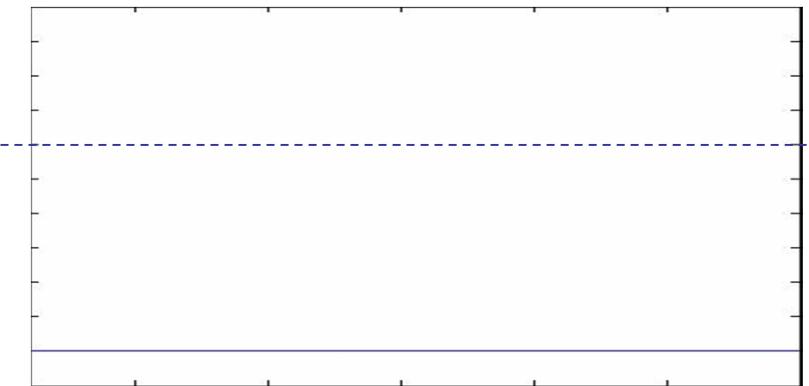


$$I_{total(t=\infty)} = \frac{1}{5} \frac{V_0}{R_0}$$

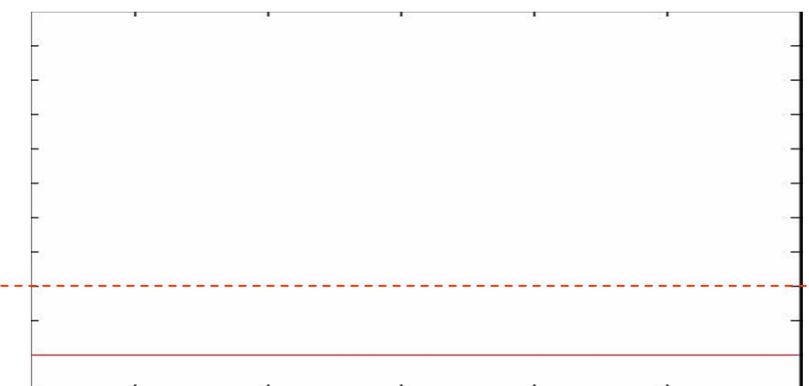
Lect. 19: Transients in Transmission Lines



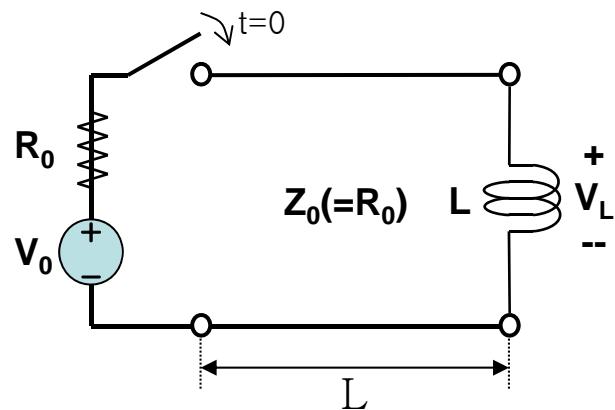
voltage



current



Lect. 19: Transients in Transmission Lines



$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \quad Z_L = ?$$

$Z_L = j\omega L \rightarrow$ only for sinusoidal signals

Remember $V(t) = L \frac{dI(t)}{dt}$

Initially Open
Finally Short

i) At $t = 0^+$ $V_1^+ = \frac{R_0}{R_0 + R_0} V_0 = \frac{1}{2} V_0$

ii) At $t = \frac{L}{u} = T$ $V_L = V_1^+ + V_1^-$

$$i_L = \frac{V_1^+ - V_1^-}{R_0}$$

$$V_L = L \frac{di_L}{dt}$$

$$V_L + i_L R_0 = 2V_1^+ = V_0$$

$$\therefore \frac{di_L}{dt} + \frac{R_0}{L} i_L = \frac{V_0}{L}$$

Lect. 19: Transients in Transmission Lines

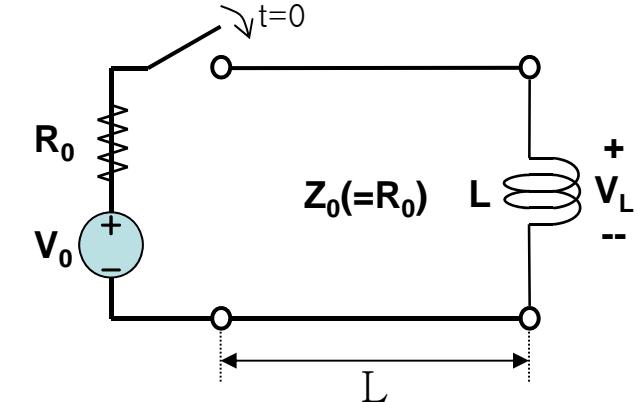
$$\frac{di_L}{dt} + \frac{R_0}{L} i_L = \frac{V_0}{L} \quad (\text{for } t \geq T, \text{ otherwise } i_L = 0)$$

We can solve above differential equation and show

$$i_L(t) = \frac{V_0}{R_0} \left(1 - e^{-(t-T)\frac{R_0}{L}} \right)$$

$$V_L(t) = L \frac{di_L}{dt} = V_0 e^{-(t-T)\frac{R_0}{L}}$$

$$V_1^-(t) = V_L - V_1^+ = V_0 \left(e^{-(t-T)\frac{R_0}{L}} - \frac{1}{2} \right)$$



No further reflection in this problem since impedance is matched at the source side

Lect. 19: Transients in Transmission Lines

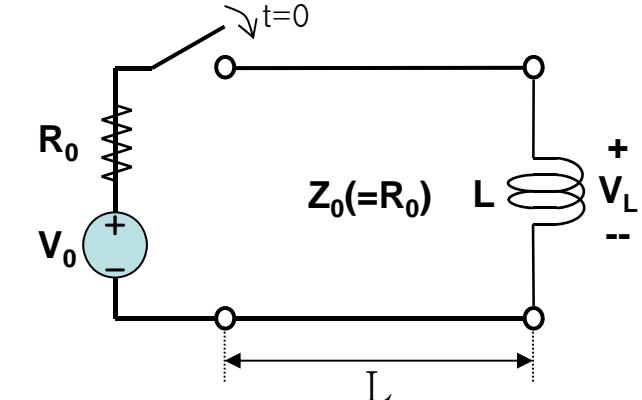
$$\frac{di_L}{dt} + \frac{R_0}{L} i_L = \frac{V_0}{L} \quad (\text{for } t \geq T, \text{ otherwise } i_L = 0)$$

$$i_{L,h}(t) = C e^{-\frac{R_0}{L}t} \quad i_{L,p}(t) = \frac{V_0}{R_0}$$

$$\text{Using } i_L(T) = 0, \quad i_L(t) = \frac{V_0}{R_0} \left(1 - e^{-(t-T)\frac{R_0}{L}} \right)$$

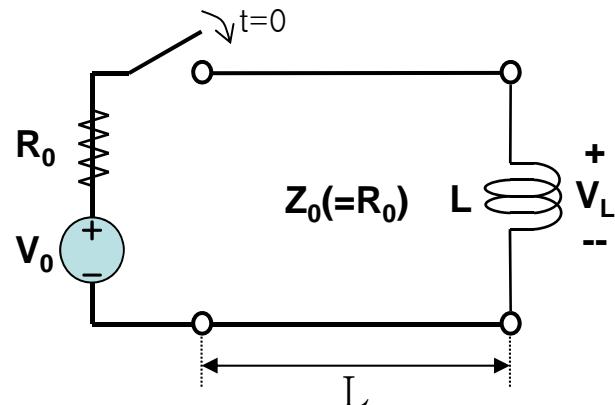
$$V_L(t) = L \frac{di_L}{dt} = V_0 e^{-(t-T)\frac{R_0}{L}}$$

$$V_1^-(t) = V_L - V_1^+ = V_0 \left(e^{-(t-T)\frac{R_0}{L}} - \frac{1}{2} \right)$$

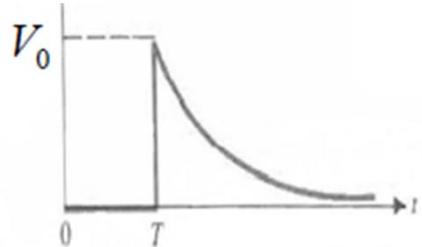


No further reflection in this problem since impedance is matched at the source side

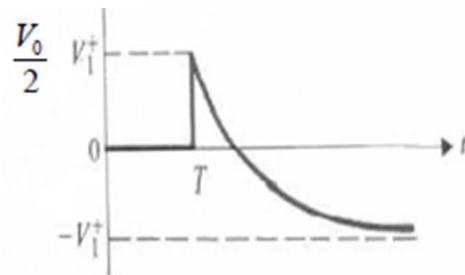
Lect. 19: Transients in Transmission Lines



$$V_L(t) = V_0 e^{-(t-T)\frac{R_0}{L}}$$



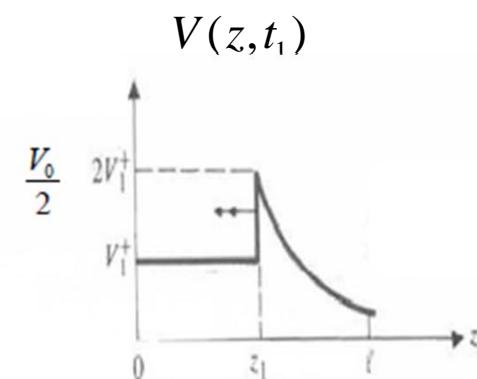
$$V_1^- = V_L - V_1^+ = V_0 \left(e^{-(t-T)\frac{R_0}{L}} - \frac{1}{2} \right)$$



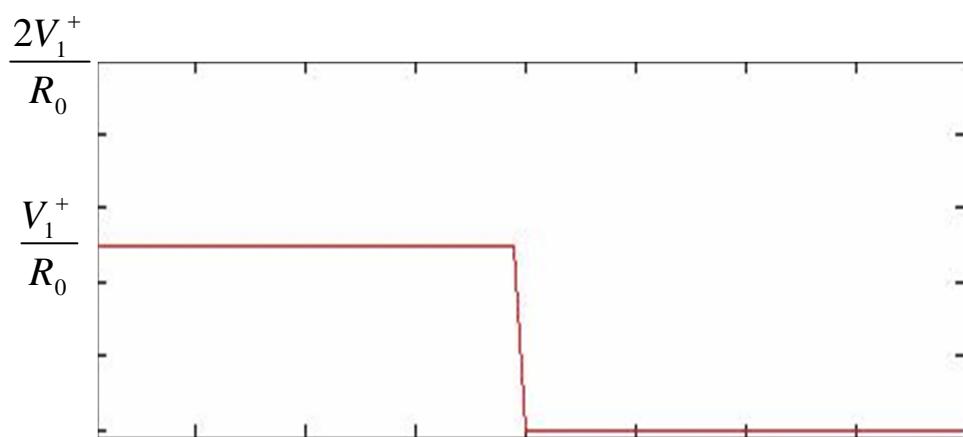
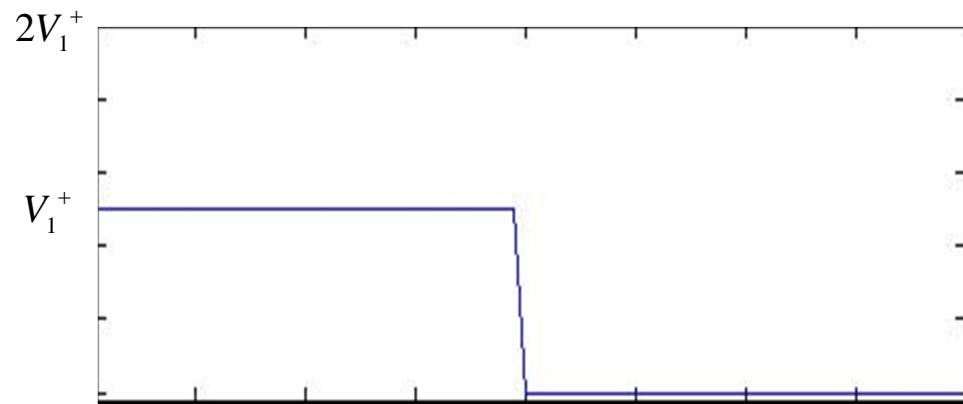
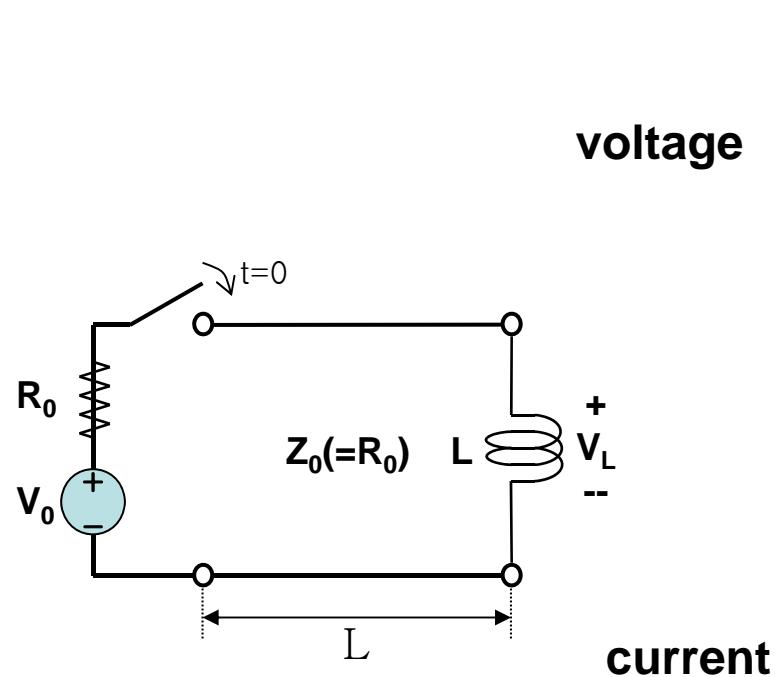
Initially, the inductor is open $V_L(t=T) = V_0$

Finally, the inductor is short $V_L(t=\infty) = 0$

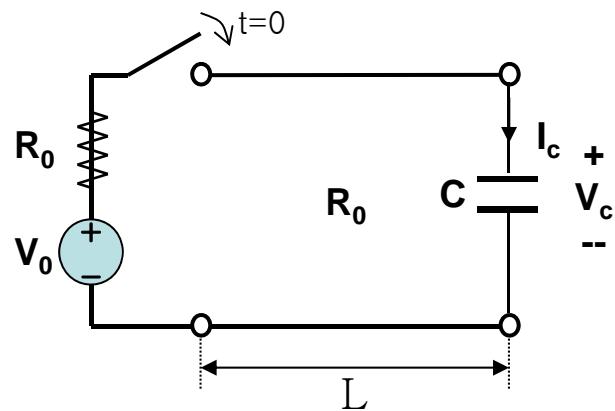
In-between, inductor voltage changes exponentially with time constant R_0/L



Lect. 19: Transients in Transmission Lines



Lect. 19: Transients in Transmission Lines



$$V_C(t) = V_0 \left(1 - e^{-\frac{(t-T)}{R_0 C}} \right)$$

$$i_C(t) = \frac{V_0}{R_0} e^{-\frac{(t-T)}{R_0 C}}$$

$$V_1^- = V_C - V_1^+ = V_C - \frac{V_0}{2}$$

$$I(t) = C \frac{dV(t)}{dt}$$

initially short
finally open

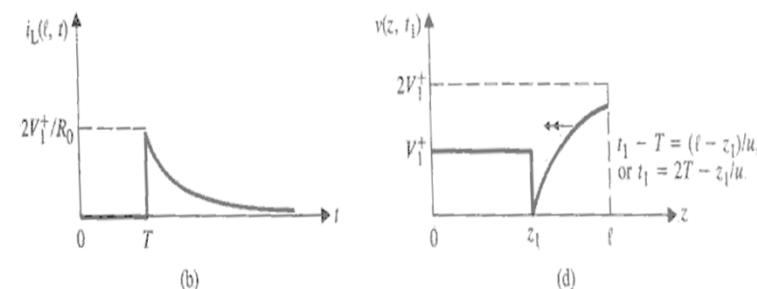
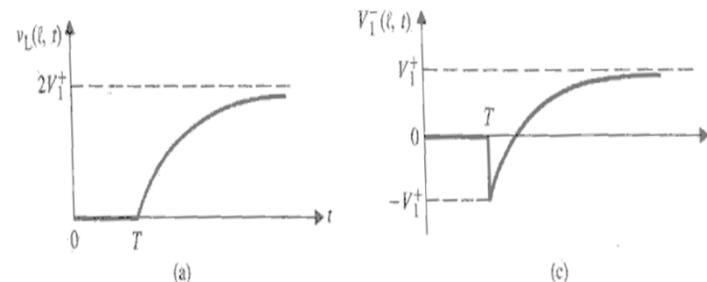
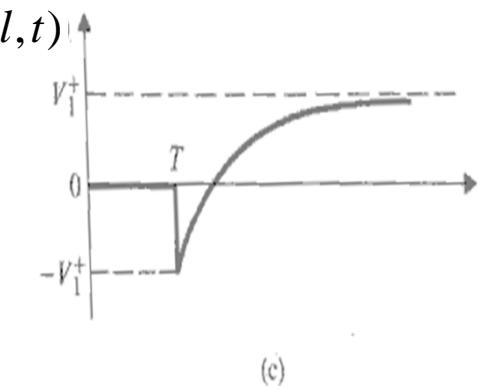
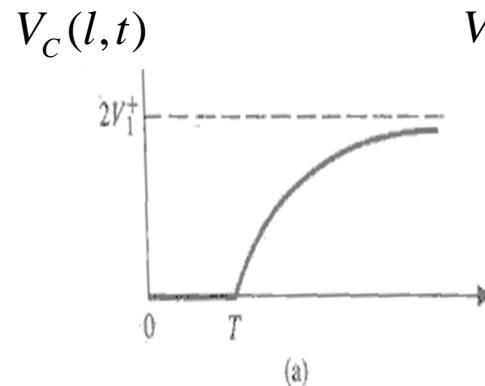
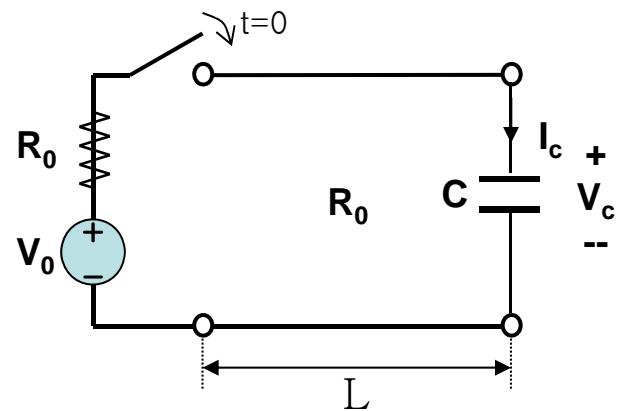


FIGURE 9-29
Transient responses of a lossless line with a capacitive termination.

Lect. 19: Transients in Transmission Lines



$$V_C(t) = V_0 \left(1 - e^{-\frac{(t-T)}{R_0 C}} \right)$$

$$i_C(t) = \frac{V_0}{R_0} e^{-\frac{(t-T)}{R_0 C}}$$

$$V_1^- = V_C - V_1^+ = V_C - \frac{V_0}{2}$$

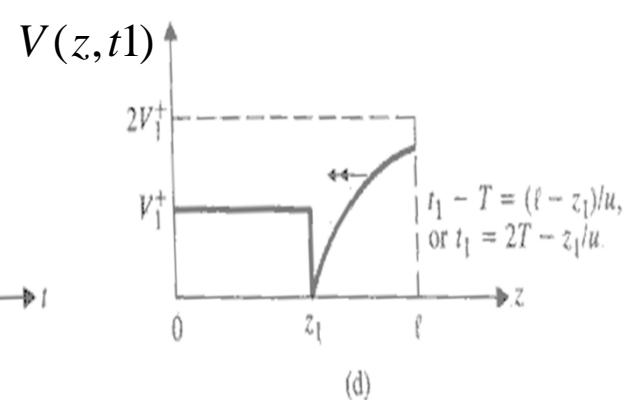
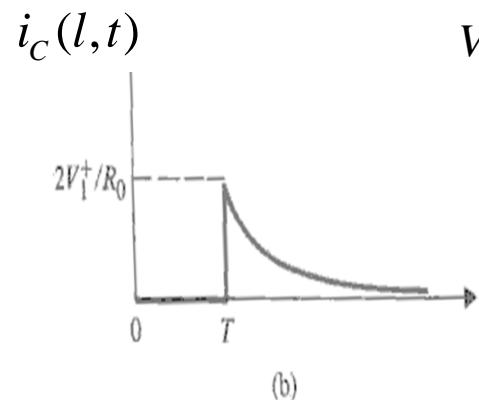


FIGURE 9-29
Transient responses of a lossless line with a capacitive termination.