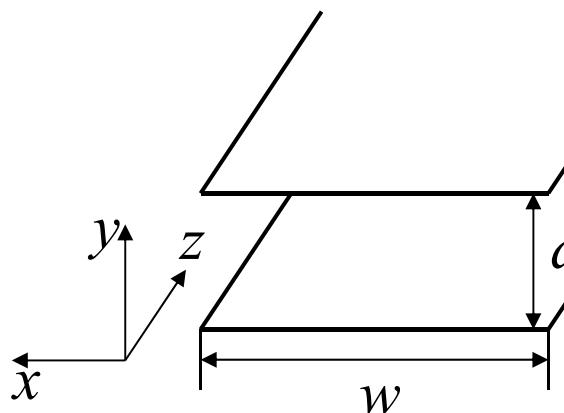


Lect. 22 EM Waves in a Waveguide (TEM)

(Cheng 10-2) =

EM waves inside a waveguide propagating in z-direction



$$\bar{E} = \bar{y} E_0 e^{-jkz}$$

$$\bar{H} = -\bar{x} \frac{E_0}{\eta} e^{-jkz}$$

→ Transmission Lines
with voltage and current waves

Other solutions?

$$\text{Wave Eq.: } \nabla^2 \bar{E} - \mu \epsilon \frac{\partial^2}{\partial t^2} \bar{E} = 0$$

$$\text{Assume } \bar{E}(x, y, z, t) = \bar{E}(x, y) \cdot e^{-\gamma z} \cdot e^{j\omega t} \quad (\gamma = \alpha + j\beta)$$

(Propagating into z-direction and time-harmonic)

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$$\nabla^2 \bar{E} - \mu\epsilon \frac{\partial^2}{\partial t^2} \bar{E} = 0 \quad \text{with} \quad \bar{E}(x, y, z, t) = \bar{E}(x, y) \cdot e^{-\gamma z} \cdot e^{j\omega t}$$

$$\nabla^2 \bar{E} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \bar{E} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \gamma^2 \right) \bar{E} = (\nabla_{xy}^2 + \gamma^2) \bar{E}$$

$$\frac{\partial^2}{\partial t^2} \bar{E} = -\omega^2 \bar{E}$$

$$\nabla_{xy}^2 \bar{E}(x, y) + (\gamma^2 + \omega^2 \mu\epsilon) \bar{E}(x, y) = 0$$

Let $h^2 = \gamma^2 + \omega^2 \mu\epsilon$

$$\nabla_{xy}^2 \bar{E}(x, y) + h^2 \bar{E}(x, y) = 0 \quad \text{2-D Wave Eq.}$$

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For $\nabla \times \bar{E} = -j\omega\mu\bar{H}$

$$\bar{E}(x, y, z, t) = \bar{E}(x, y) \cdot e^{-\gamma z} \cdot e^{j\omega t}$$

$$\nabla \times \bar{E} = \begin{vmatrix} \bar{x} & \bar{y} & \bar{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \bar{x} \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) + \bar{y} \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) + \bar{z} \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) = -j\omega\mu\bar{H}$$

$$\frac{\partial}{\partial z} \rightarrow -\gamma$$

Likewise from $\nabla \times H = j\omega\varepsilon E$

$$(Eq.1) \quad \frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega\mu H_x$$

$$(Eq.2) \quad -\gamma E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$(Eq.3) \quad \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$$

$$(Eq.4) \quad \frac{\partial H_z}{\partial y} + \gamma H_y = j\omega\varepsilon E_x$$

$$(Eq.5) \quad -\gamma H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$$

$$(Eq.6) \quad \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z$$

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- From above six equations, H_x, H_y, E_x, E_y can be expressed in terms of E_z and H_z .

Solve for E_z or H_z first.

Then, all other components can be determined.

- Solutions can be classified into three types:

$$H_x = -\frac{1}{h^2} \left(\gamma \frac{\partial H_z}{\partial x} - j\omega\epsilon \frac{\partial E_z}{\partial y} \right)$$

$$H_y = -\frac{1}{h^2} \left(\gamma \frac{\partial H_z}{\partial y} + j\omega\epsilon \frac{\partial E_z}{\partial x} \right)$$

$$E_x = -\frac{1}{h^2} \left(\gamma \frac{\partial E_z}{\partial x} + j\omega\mu \frac{\partial H_z}{\partial y} \right)$$

$$E_y = -\frac{1}{h^2} \left(\gamma \frac{\partial E_z}{\partial y} - j\omega\mu \frac{\partial H_z}{\partial x} \right)$$

$$h^2 = \gamma^2 + \omega^2 \mu \epsilon$$

1) Transverse Electro-Magnetic (TEM)
waves: $E_z = H_z = 0$

2) Transverse Magnetic (TM) waves:
 $H_z = 0$ but E_z nonzero

3) Transverse Electric (TE) waves:
 $E_z = 0$ but H_z nonzero

Lect. 22 EM Waves in a Waveguide (TEM)

$$H_x = -\frac{1}{h^2} \left(\gamma \frac{\partial H_z}{\partial x} - j\omega\epsilon \frac{\partial E_z}{\partial y} \right)$$

$$H_y = -\frac{1}{h^2} \left(\gamma \frac{\partial H_z}{\partial y} + j\omega\epsilon \frac{\partial E_z}{\partial x} \right)$$

$$E_x = -\frac{1}{h^2} \left(\gamma \frac{\partial E_z}{\partial x} + j\omega\mu \frac{\partial H_z}{\partial y} \right)$$

$$E_y = -\frac{1}{h^2} \left(\gamma \frac{\partial E_z}{\partial y} - j\omega\mu \frac{\partial H_z}{\partial x} \right)$$

$$h^2 = \gamma^2 + \omega^2 \mu \epsilon$$

1) Transverse Electro-Magnetic (TEM) waves:
 $E_z = H_z = 0$

$\rightarrow E_z = H_z = 0$ is meaningful only if $h=0$

$$\gamma^2 = -\omega^2 \mu \epsilon$$

Remember $\gamma = \alpha + j\beta$

If ϵ, μ are real, or lossless, $\beta^2 = \omega^2 \mu \epsilon$

$$\bar{E}(x, y, z, t) = \bar{E}(x, y) \cdot e^{-\gamma z} \cdot e^{j\omega t} = \bar{E}(x, y) e^{-j\beta z} e^{j\omega t}$$

If $E(x, y)$ is constant \rightarrow plane waves

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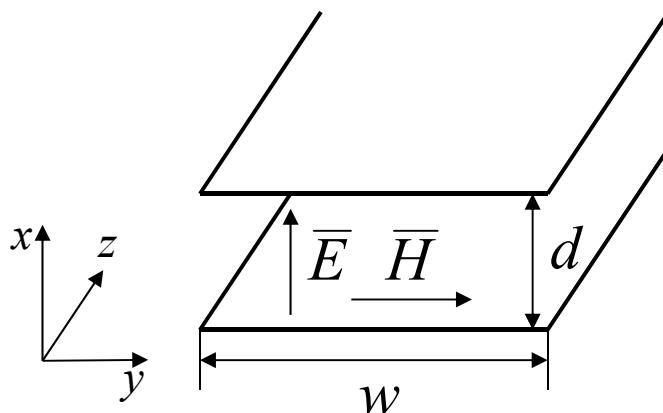
- TEM waves (Transverse Electro-Magnetic Waves):

$$E_z = H_z = 0 \text{ and } h = 0$$

$$\bar{E}(x, y, z, t) = \bar{E}(x, y) e^{-j\beta z} e^{j\omega t}$$

$$\beta = \omega \sqrt{\mu \epsilon} \quad (\omega \sqrt{LC})$$

If $E(x, y)$ is confined in a certain region



$$\text{For } 0 < x < d, \quad \bar{E} = \bar{x} E_0 e^{-jkz}$$

$$\bar{H} = \bar{y} H_0 e^{-jkz}$$

$$\begin{aligned} Z \text{ (impedance)} &= \frac{E_x}{H_y} = \frac{j\omega\mu}{\gamma} \quad (\text{Eq. 2}) \\ &= \frac{j\omega\mu}{j\beta} \\ &= \sqrt{\frac{\mu}{\epsilon}} \left(\sqrt{\frac{L}{C}} \right) \end{aligned}$$

→ Transmission line

(Voltage, current waves)