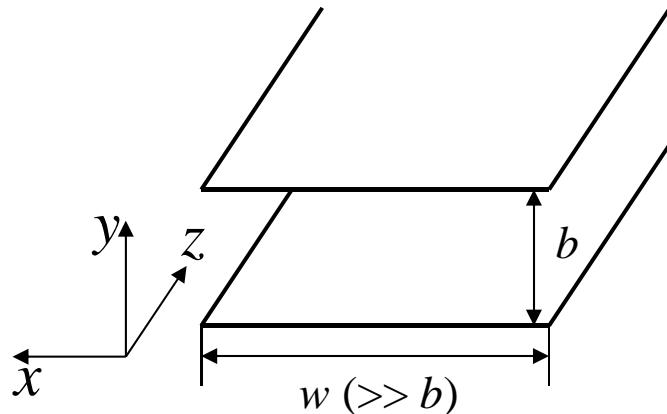


Lect. 24 TE waves

(Cheng 10-3.2)



$$\nabla^2 \bar{E}(x, y, z, t) - \mu\epsilon \frac{\partial^2}{\partial t^2} \bar{E}(x, y, z, t) = 0$$

$$\bar{E}(x, y, z, t) = \bar{E}(x, y) \cdot e^{-\gamma z} \cdot e^{j\omega t} \quad (\gamma = \alpha + j\beta)$$

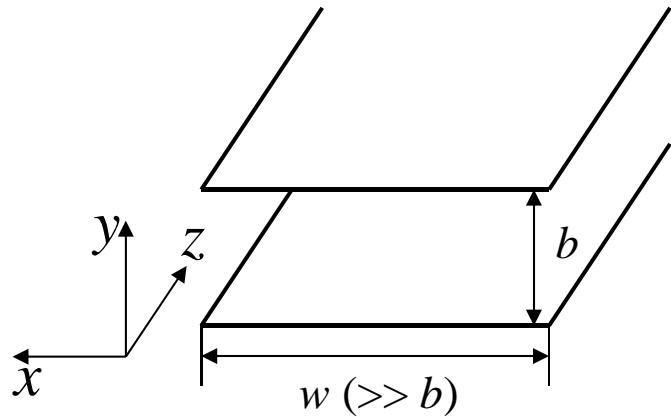
$$\nabla_{xy}^2 \bar{E}(x, y) + h^2 \bar{E}(x, y) = 0$$

$$h^2 = \gamma^2 + \omega^2 \mu\epsilon$$

- 1) Transverse Electro-Magnetic (TEM) waves: $E_z = H_z = 0$ and $h = 0$
→ Plane wave, Voltage/Current waves on Transmission line
 - 2) Transverse Magnetic (TM) waves:
 $H_z = 0$ but E_z nonzero
 - 3) Transverse Electric (TE) waves:
 $E_z = 0$ but H_z nonzero
-

Lect. 24 TE waves

(Cheng 10-3.2)



$$\nabla^2 \bar{E}(x, y, z, t) - \mu\epsilon \frac{\partial^2}{\partial t^2} \bar{E}(x, y, z, t) = 0$$

$$\bar{E}(x, y, z, t) = \bar{E}(x, y) \cdot e^{-\gamma z} \cdot e^{j\omega t} \quad (\gamma = \alpha + j\beta)$$

$$\nabla_{xy}^2 \bar{E}(x, y) + h^2 \bar{E}(x, y) = 0$$

$$h^2 = \gamma^2 + \omega^2 \mu\epsilon$$

$$H_x = -\frac{1}{h^2} \left(\gamma \frac{\partial H_z}{\partial x} - j\omega\epsilon \frac{\partial E_z}{\partial y} \right)$$

$$H_y = -\frac{1}{h^2} \left(\gamma \frac{\partial H_z}{\partial y} + j\omega\epsilon \frac{\partial E_z}{\partial x} \right)$$

$$E_x = -\frac{1}{h^2} \left(\gamma \frac{\partial E_z}{\partial x} + j\omega\mu \frac{\partial H_z}{\partial y} \right)$$

$$E_y = -\frac{1}{h^2} \left(\gamma \frac{\partial E_z}{\partial y} - j\omega\mu \frac{\partial H_z}{\partial x} \right)$$

TM waves: $H_z=0$ but E_z nonzero

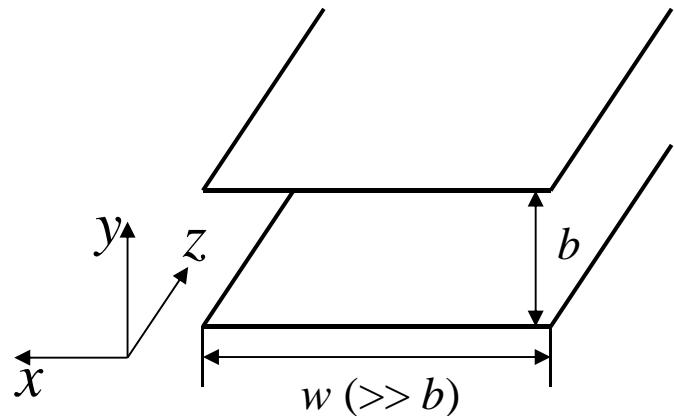
$$\nabla_{x,y}^2 E_z(x, y) + h^2 E_z(x, y) = 0$$

$$\frac{d^2}{dy^2} E_z(y) + h^2 E_z(y) = 0$$

$$E_z(y) = A_n \sin(hy), \quad h = \frac{n\pi}{b}$$

Lect. 24 TE waves

(Cheng 10-3.2)



TM waves

$$E_x(y) = 0$$

$$E_y(y) = -\frac{\gamma}{h} A_n \cos(hy)$$

$$E_z(y) = A_n \sin(hy)$$

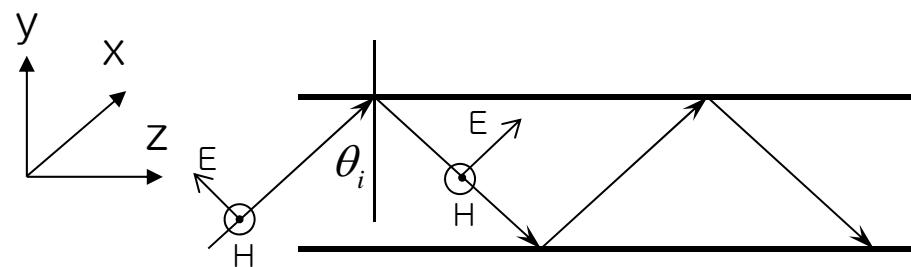
$$h = \frac{n\pi}{b}$$

$$H_x(y) = \frac{j\omega\epsilon}{h} A_n \cos(hy)$$

$$H_y(y) = 0$$

$$H_z(y) = 0$$

→ Mode: Quantization of h, β



$$k = \omega \sqrt{\mu \epsilon}$$

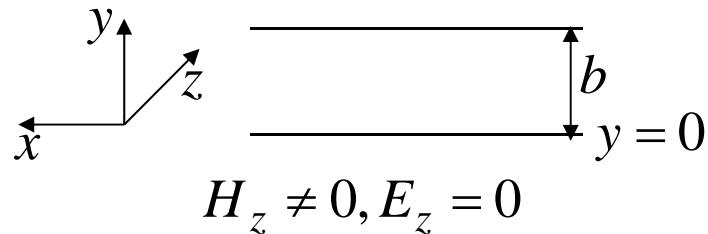
$$h = k \cos \theta_i$$

$$\beta = k \sin \theta_i$$

Lect. 24 TE waves

(Cheng 10-3.2)

TE waves:



$$\begin{aligned}\nabla_{x,y}^2 H_z(x,y) + h^2 H_z(x,y) &= 0 \\ \Rightarrow \frac{d^2}{dy^2} H_z(y) + h^2 H_z(y) &= 0 \\ \Rightarrow H_z(y) &= A \sin(hy) + B \cos(hy)\end{aligned}$$

At the surface of electrodes

$E_{tan} = 0$ and $H_{nor} = 0$

→ BC cannot be directly applied to H_z

At the boundary

$$E_x = 0 \quad \therefore \frac{\partial H_z}{\partial y} = 0$$

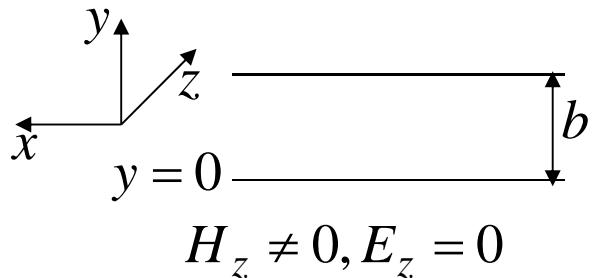
$$H_x = -\frac{1}{h^2} \left(\gamma \frac{\partial H_z}{\partial x} - j\omega \epsilon \frac{\partial E_z}{\partial y} \right)$$

$$H_y = -\frac{1}{h^2} \left(\gamma \frac{\partial H_z}{\partial y} + j\omega \epsilon \frac{\partial E_z}{\partial x} \right)$$

$$E_x = -\frac{1}{h^2} \left(\gamma \frac{\partial E_z}{\partial x} + j\omega \mu \frac{\partial H_z}{\partial y} \right)$$

$$E_y = -\frac{1}{h^2} \left(\gamma \frac{\partial E_z}{\partial y} - j\omega \mu \frac{\partial H_z}{\partial x} \right)$$

Lect. 24 TE waves



$$\frac{d^2}{dy^2} H_z(y) + h^2 H_z(y) = 0$$
$$H_z(y) = A \sin(hy) + B \cos(hy) \quad \text{BC: } \frac{\partial H_z}{\partial y}(y=b) = 0$$

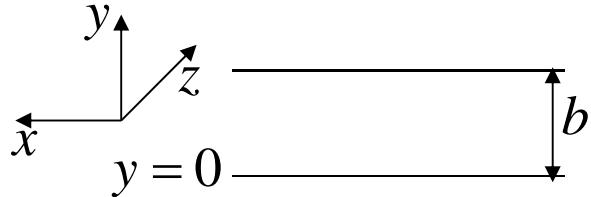
$$\frac{\partial H_z}{\partial y}(y=0) = hA \cos(0) - hB \sin(0) = 0 \quad \text{at } y=0 \text{ (1)}$$
$$\frac{\partial H_z}{\partial y}(y=b) = hA \cos(hb) - hB \sin(hb) = 0 \quad \text{at } y=b \text{ (2)}$$

$$(1) \quad A = 0, \quad H_z(y) = B \cos(hy)$$

$$(2) \quad \frac{\partial H_z}{\partial y}(y=b) = -hB \sin(hb) = 0 \Rightarrow hb = n\pi, \quad h = \frac{n\pi}{b}$$

$$\therefore H_z(y) = B \cos(hy), \quad h = \frac{n\pi}{b} \quad \text{TE}_n \text{ Mode}$$

Lect. 24 TE waves



$$H_z \neq 0, E_z = 0$$

$$H_x = -\frac{1}{h^2} \left(\gamma \frac{\partial H_z}{\partial x} - j\omega\varepsilon \frac{\partial E_z}{\partial y} \right)$$

$$H_y = -\frac{1}{h^2} \left(\gamma \frac{\partial H_z}{\partial y} + j\omega\varepsilon \frac{\partial E_z}{\partial x} \right)$$

$$E_x = -\frac{1}{h^2} \left(\gamma \frac{\partial E_z}{\partial x} + j\omega\mu \frac{\partial H_z}{\partial y} \right)$$

$$E_y = -\frac{1}{h^2} \left(\gamma \frac{\partial E_z}{\partial y} - j\omega\mu \frac{\partial H_z}{\partial x} \right)$$

$$H_z(y) = B \cos(hy), \quad h = \frac{n\pi}{b}$$

$$H_x(y) = 0$$

$$H_y(y) = \frac{\gamma}{h} B \sin(hy)$$

$$H_z(y) = B \cos(hy)$$

$$E_x(y) = \frac{j\omega\mu}{h} B \sin(hy)$$

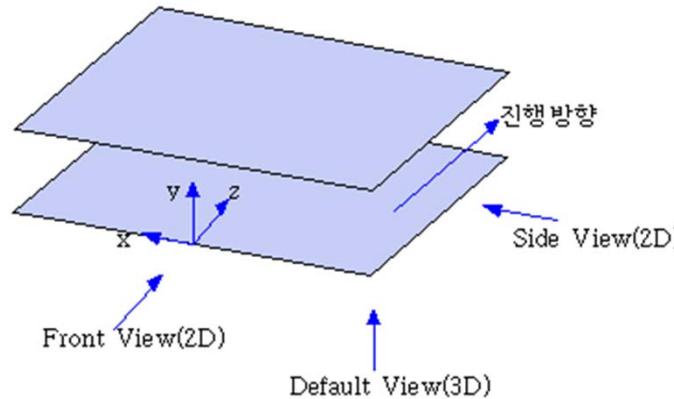
$$E_y(y) = 0$$

$$E_z(y) = 0$$

TE₀ Mode?

Lect. 24 TE waves

Wave propagation animation for TE_1 and TE_2



$$E_x(y) = \frac{j\omega\mu}{h} B \sin(hy)$$

$$E_y(y) = 0$$

$$E_z(y) = 0$$

$$H_x(y) = 0$$

$$H_y(y) = \frac{\gamma}{h} B \sin(hy)$$

$$H_z(y) = B \cos(hy)$$

For following animations, $\bar{H}(x, y, z, t) = \text{Re} \left[\bar{H}(x, y) e^{-j\beta z} e^{j\omega t} \right]$

TE₁

$$H_x(y) = 0$$

$$H_y(y) = -\frac{\beta}{\pi/b} B_1 \sin\left(\frac{\pi}{b} y\right) \sin(\omega t - \beta z)$$

$$H_z(y) = B_1 \cos\left(\frac{\pi}{b} y\right) \cos(\omega t - \beta z)$$

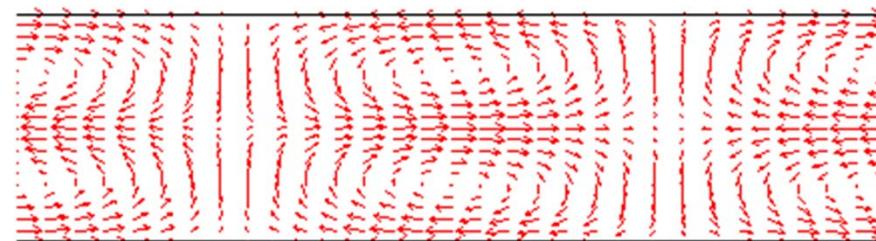
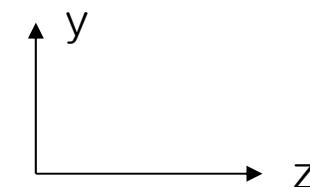
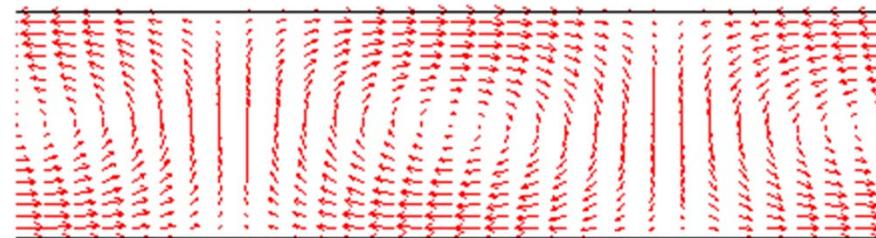
TE₂

$$H_x(y) = 0$$

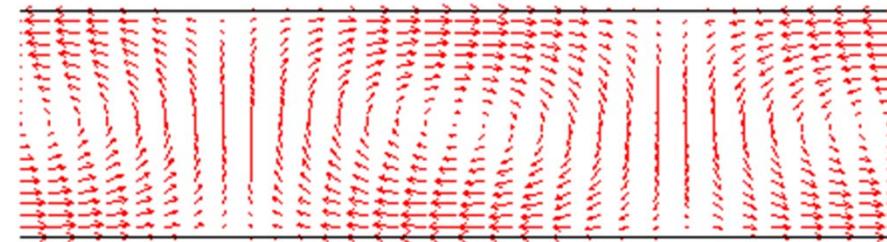
$$H_y(y) = -\frac{\beta}{2\pi/b} B_2 \sin\left(\frac{2\pi}{b} y\right) \sin(\omega t - \beta z)$$

$$H_z(y) = B_2 \cos\left(\frac{2\pi}{b} y\right) \cos(\omega t - \beta z)$$

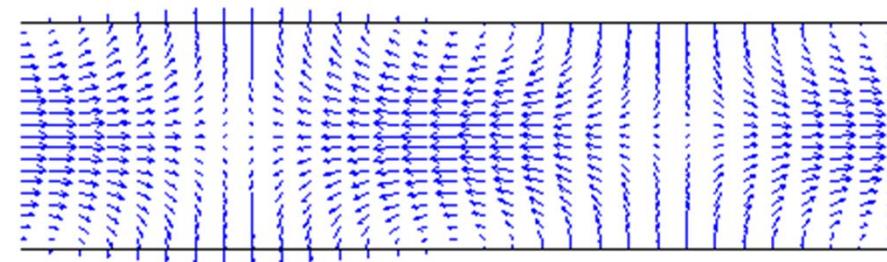
Side View



Side View

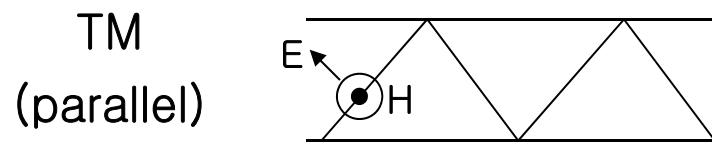
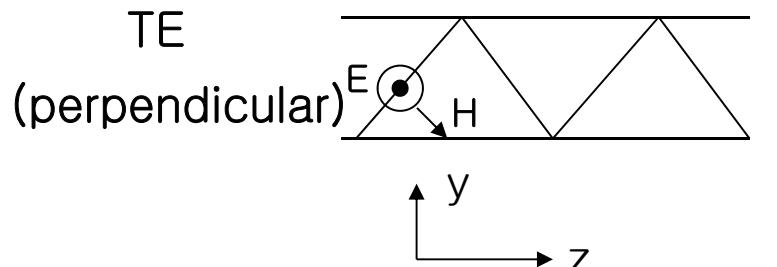


TE_1



TM_1

Lect. 24 TE waves



$$\beta = \sqrt{k^2 - h^2} = \sqrt{\omega^2 \mu \epsilon - \frac{n^2 \pi^2}{b^2}} = \omega \sqrt{\mu \epsilon} \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$f_c = \frac{n}{2b\sqrt{\mu \epsilon}}$$

Cut-off when $f = f_c$ or $\beta = 0$

With plane wave decomposition,

$$\begin{aligned} \sin(hy) &\sim e^{jhy} - e^{-jhy} \\ \cos(hy) &\sim e^{jhy} + e^{-jhy} \end{aligned} \Bigg) e^{-j\beta z}$$

$$e^{-j(2hb)} = 1 \quad 2hb = 2n\pi \quad h = \frac{n\pi}{b}$$

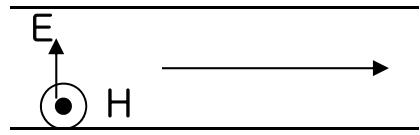
$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu \epsilon}} \frac{1}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$v_g = \frac{1}{d\beta} = \frac{1}{\sqrt{\mu \epsilon}} \cdot \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

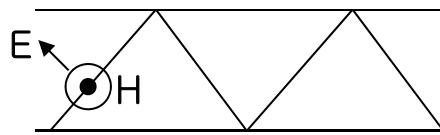
Same as TM

Lect. 24 TE waves

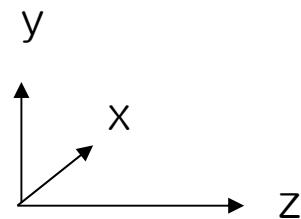
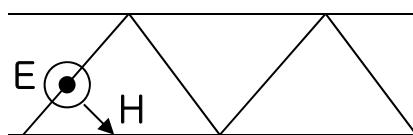
TEM



TM



TE



$$Z_{TEM} = -\frac{E_y}{H_x}$$

$$Z_{TM} = -\frac{E_y}{H_x}$$

$$Z_{TE} = \frac{E_x}{H_y}$$

Wave impedance (Z) is defined as a ratio of orthogonal E-field and H-field component with a right-hand rule for sign

$$\bar{H} = \frac{1}{Z} \bar{a}_z \times \bar{E}$$

$$\bar{E} = -Z (\bar{a}_z \times \bar{H})$$

Lect. 24 TE waves

- 6/2: No Class
- 6/7: Test #2 (From Lect. 15 to 24)
- 6/9: Test #2 Review, Project title and abstract due
- 6/14: Project Presentation (10 min. for each presentation)

Project:

Select a topic of your interest in the area of electromagnetic waves technology, investigate the selected topic, and make in-class oral presentations in English.

(Quiz: 25%, Tests: $25 \times 2 = 50\%$, Project: 15%, Attendance: 10%)