Lect. 4: Time-varying Fields

(Cheng 7.2, 7.3)

Static Maxwell's Equations

$$\nabla \cdot \overline{D} = \rho \qquad \nabla \cdot \overline{B} = 0$$

$$\nabla \times \overline{E} = 0 \qquad \nabla \times \overline{H} = \overline{J}$$

➔ In static conditions, electric and magnetic fields are independent of each other

But

- Time-varying B can produce E

- Time varying D can produce H

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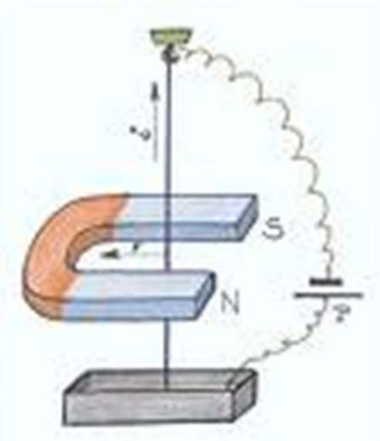
- Time-varying B can produce E



$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$ Faraday's Law

- In integral form $$\oint \overline{E} \cdot d\bar{l} = -\frac{d}{dt}\int \overline{B} \cdot d\bar{s}$$

Voltage (emf) = - time change rate of magnetic flux density

➔ Electromagnetic induction

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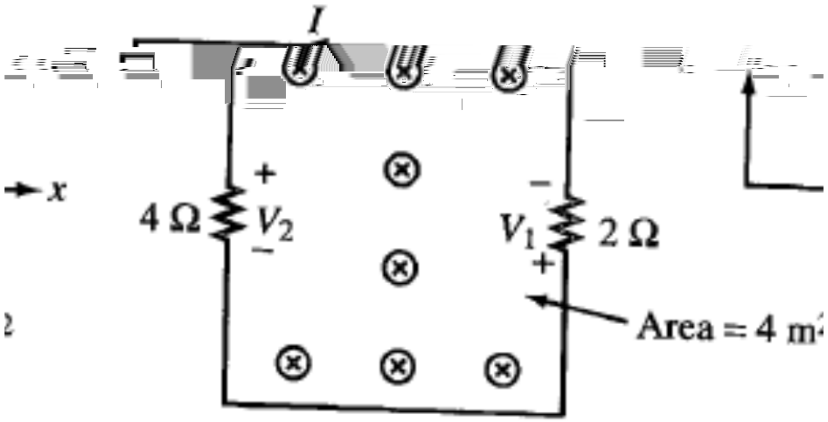
Determine V_1, V_2 when $\overline{B} = -\overline{z}\ 0.3t$

$$\oint \overline{E}\cdot d\overline{l} = -\frac{d}{dt}\int \overline{B}\cdot d\overline{s}$$



$$\Phi = \int_S \mathbf{B}\cdot d\mathbf{s} = \int_S (-\hat{\mathbf{z}}0.3t)\cdot\hat{\mathbf{z}}\,ds$$
$$= -0.3t \times 4 = -1.2t$$

$$V_{\text{emf}}^{\text{tr}} = -\frac{d\Phi}{dt} = 1.2$$

$$I = \frac{V_{\text{emf}}^{\text{tr}}}{R_1 + R_2} = \frac{1.2}{2+4} = 0.2 \text{ A},$$

$V_1 = IR_1 = 0.2 \times 2 = 0.4$ V,

$V_2 = IR_2 = 0.2 \times 4 = 0.8$ V.

Sign?

Lenz's Law:
Currents in the loop flow in the direction that opposes the change of magnetic flux.

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$$\nabla \cdot \overline{D} = \rho$$

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

$$\nabla \cdot \overline{B} = 0$$

$$\nabla \times \overline{H} = \overline{J} \quad \Rightarrow \quad \nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t}$$

- Time-varying D can produce H

Why?

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$$\nabla \cdot \overline{D} = \rho$$

Assume dD/dt does not produce H

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

$$\nabla \cdot \overline{B} = 0$$

$$\nabla \times \overline{H} = \overline{J}$$

We know $$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$$ (Equation of continuity)

But from above equations, $$\nabla \cdot \mathbf{J} = \qquad = 0$$

We need an additional term!

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Assume $$\nabla \times \overline{H} = \overline{J} + \overline{\mathrm{X}}$$

$$\nabla \cdot (\nabla \times \overline{H}) = \nabla \cdot \overline{J} + \nabla \cdot \overline{\mathrm{X}} = 0$$

$$\nabla \cdot \overline{J} = - \nabla \cdot \overline{\mathrm{X}} = -\frac{\partial \rho}{\partial t}$$

$$\nabla \cdot \overline{\mathrm{X}} = \frac{\partial \rho}{\partial t} = \frac{\partial \nabla \cdot \overline{D}}{\partial t} = \nabla \cdot \frac{\partial \overline{D}}{\partial t}$$

$$\therefore \overline{\mathrm{X}} = \frac{\partial \overline{D}}{\partial t} \rightarrow \nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t}$$

Unit?

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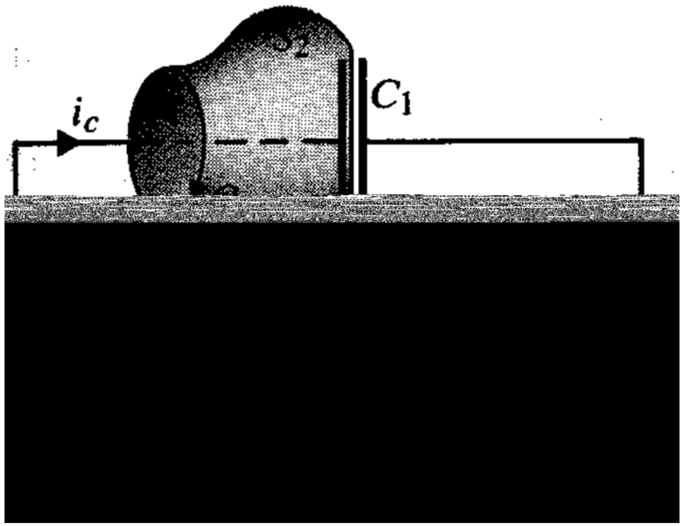
- Example 7-5 in Cheng   Determine current passing through the capacitor (displacement current)



$v_c = V_o \sin(\omega t)$

Maxwell's Equations

$$\nabla \cdot \overline{D} = \rho$$

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

$$\nabla \cdot \overline{B} = 0$$

$$\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t}$$

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