Lect. 8: Plane Waves in Lossy Media (Cheng, 8-3)

Maxwell's Equations

$$\nabla \cdot \overline{D} = \rho$$

$$\nabla \times \overline{E} = -j\omega \overline{B}$$

$$\nabla \cdot \overline{B} = 0$$

$$\nabla \times \overline{H} = \overline{J} + j\omega \overline{D}$$

$$\overline{D} = \varepsilon \overline{E}$$

$$\overline{B} = \mu \overline{H}$$

We assumed $\overline{J} = 0$

for EM wave equation derivation

What if E-field generates currents?

$\overline{J} = \sigma \cdot \overline{E}, \ \sigma \neq 0$

➔ Lossy medium

W.-Y. Choi

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$$\overline{J} = \sigma \cdot \overline{E}, \ \sigma \neq 0$$

$$\nabla \times \overline{H} = \sigma\overline{E} + j\omega\varepsilon\overline{E}$$

$$= j\omega(\varepsilon - j\frac{\sigma}{\omega})\overline{E} = j\omega\varepsilon_c\overline{E}$$

➔ Complex ε!

$$\varepsilon_c \equiv \varepsilon - j\frac{\sigma}{\omega}$$

$$\varepsilon_c = \varepsilon' - j\varepsilon'', \ \varepsilon'' = \frac{\sigma}{\omega}$$

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Wave number for $\varepsilon_c = \varepsilon' - j\varepsilon''$?

$$k = \omega\sqrt{\mu\varepsilon_c} = \omega\sqrt{\mu\varepsilon'}\left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{\frac{1}{2}} = \beta - j\alpha, \quad \rightarrow \begin{cases} \alpha : \text{attenuation constant} \\ \beta : \text{propagation constant} \end{cases}$$

What does complex k mean?

Consider plane wave solution:

$$\overline{E} = \overline{x}E_0 e^{-jkz} = \overline{x}E_0 e^{-j(\beta - j\alpha)z} = \overline{x}E_0 e^{-j\beta z} e^{-\alpha z}$$ Exponential Decay!

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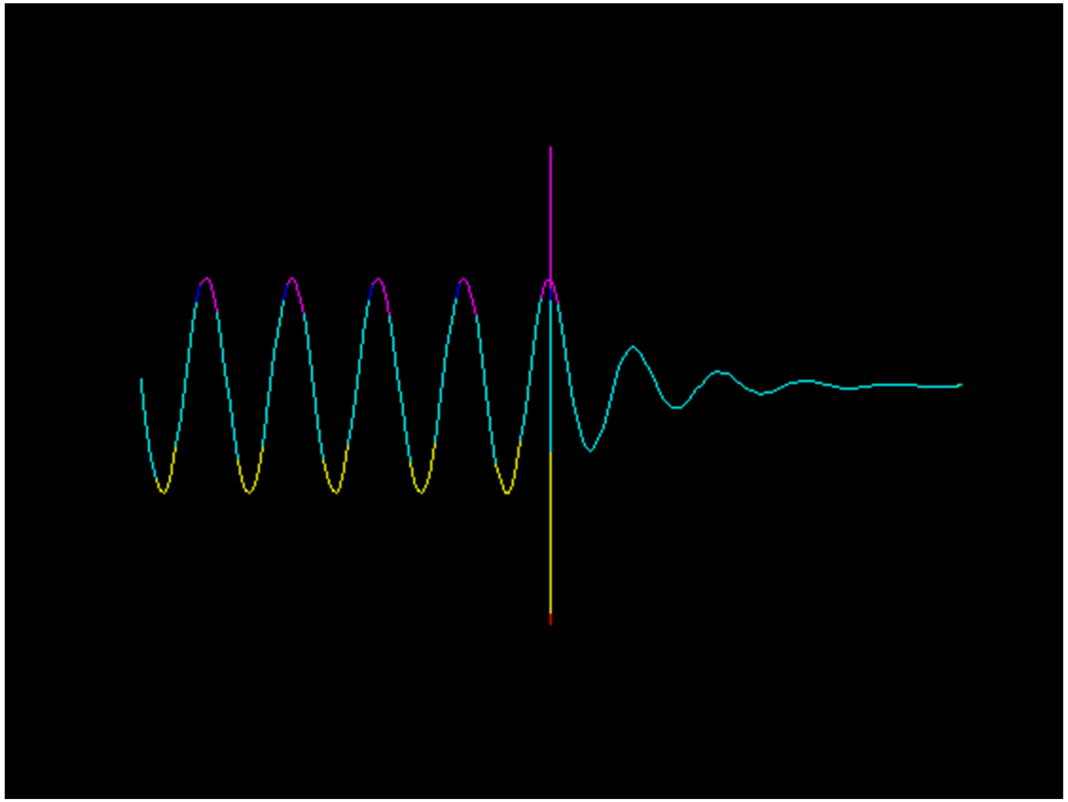
$$\overline{E} = \overline{x} E_0 e^{-jkz} = \overline{x} E_0 e^{-j(\beta - j\alpha)z} = \overline{x} E_0 e^{-j\beta z} e^{-\alpha z}$$ Exponential Decay!



- How does α affect wave propagation?

- How much does the wave propagate into the medium?

$$\delta = \frac{1}{\alpha}$$ Penetration depth, Or Skin depth

- What is wavelength in lossy medium?

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In medium where currents can flow (non-zero σ)

➔ Complex permittivity: $\varepsilon_c \equiv \varepsilon - j\dfrac{\sigma}{\omega} = \varepsilon' - j\varepsilon''$

➔ Complex wave number: $k = \beta - j\alpha,$

➔ Wave experiences loss: Lossy Medium

- How much loss does the medium have?

$\dfrac{1}{\alpha}$: penetration depth, $\dfrac{\varepsilon''}{\varepsilon'} = \tan\delta_c$: loss angle, loss tangent

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$$k = \omega\sqrt{\mu\varepsilon_c} = \omega\sqrt{\mu\varepsilon'}\left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{\frac{1}{2}} = \beta - j\alpha,\ \varepsilon'' = \frac{\sigma}{\omega}$$ ➔ Complicated Expression

Use approximation: If σ<<1 ➔ ε''<<1

Using $(1+x)^{1/2} \approx 1 + \frac{1}{2}x - \frac{1}{8}x^2$ $\quad k = \omega\sqrt{\mu\varepsilon'}\left[1 - j\frac{1}{2}\left(\frac{\varepsilon''}{\varepsilon'}\right) + \frac{1}{8}\left(\frac{\varepsilon''}{\varepsilon'}\right)^2 + ...\right]$

$$\beta \approx \omega\sqrt{\mu\varepsilon'}\left[1 + \frac{1}{8}\left(\frac{\varepsilon''}{\varepsilon'}\right)^2\right]$$

$$\alpha \approx \omega\sqrt{\mu\varepsilon'}\frac{\varepsilon''}{2\varepsilon'} = \frac{1}{2}\sqrt{\frac{\mu}{\varepsilon'}}\sigma$$

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$$k = \omega\sqrt{\mu\varepsilon'}\left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{\frac{1}{2}} = \beta - j\alpha,\ \ \varepsilon'' = \frac{\sigma}{\omega}$$

$$\begin{cases} \beta \approx \omega\sqrt{\mu\varepsilon'}\left[1 + \dfrac{1}{8}\left(\dfrac{\varepsilon''}{\varepsilon'}\right)^2\right] \\ \alpha \approx \omega\sqrt{\mu\varepsilon'}\dfrac{\varepsilon''}{\varepsilon'} = \dfrac{1}{2}\sqrt{\dfrac{\mu}{\varepsilon'}}\sigma \end{cases}$$

Phase velocity? $v_p = \dfrac{\omega}{\beta}$

Impedance? $\eta = \sqrt{\dfrac{\mu}{\varepsilon_c}} = \sqrt{\dfrac{\mu}{\varepsilon'}}\left(1 - j\dfrac{\varepsilon''}{\varepsilon'}\right)^{-\frac{1}{2}} \approx \sqrt{\dfrac{\mu}{\varepsilon'}}\left(1 + j\dfrac{\varepsilon''}{2\varepsilon'}\right)$

Complex impedance?

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When σ≫1

$$k = \omega\sqrt{\mu\varepsilon_c} = \omega\sqrt{\mu\varepsilon'}\left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{\frac{1}{2}}, \quad \frac{\varepsilon''}{\varepsilon'} >> 1 \quad (\varepsilon'' = \frac{\sigma}{\omega})$$

$$\approx \omega\sqrt{\mu\varepsilon'\left(\frac{\varepsilon''}{\varepsilon'}\right)}(-j)^{\frac{1}{2}} = \omega\sqrt{\mu\varepsilon''}(1-j)\frac{1}{\sqrt{2}}$$

$$= \sqrt{\frac{\mu\omega\sigma}{2}}(1-j) = \sqrt{\pi f \mu\sigma}(1-j) = \beta - j\alpha$$

$$\Longrightarrow \alpha = \beta = \sqrt{\pi f \mu\sigma}$$

W.-Y. Choi

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When $\sigma \gg 1$

$$k = \omega\sqrt{\mu\varepsilon_c} = \beta - j\alpha;\ \alpha = \beta = \sqrt{\pi f \mu\sigma}$$

- α increases with frequency

➔ Higher frequency signals have larger loss in good conductors

– β increases with frequency

➔ Higher frequency signals have smaller wavelength in good conductors

- Penetration depth?

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When $\sigma \gg 1$, Impedance?

$$\eta = \sqrt{\frac{\mu}{\varepsilon_c}} \approx \sqrt{\frac{\mu}{-j\varepsilon''}} = \sqrt{\frac{\mu\omega}{\sigma}}\left(\frac{1}{-j}\right)^{\frac{1}{2}} = \sqrt{\frac{\mu\omega}{\sigma}}\frac{1}{\sqrt{2}}(1+j)$$

$$= \sqrt{\frac{\pi f \mu}{\sigma}}(1+j)$$

➔ E and H have phase difference of 45°

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Penetration depths for various materials

Material	Conductivity (S/m)	f = 60(Hz)	1(MHz)	1(GHz)
Silver	6.17×10^7	8.27(mm)	0.064(mm)	0.0020(mm)
Copper	5.80×10^7	8.53	0.066	0.0021
Gold	4.10×10^7	10.14	0.079	0.0025
Aluminum	3.54×10^7	10.92	0.084	0.0027
Iron ($\mu^r \cong 10^3$)	1.00×10^7	0.65	0.005	0.0016
Seawater	4	32(m)	0.25(m)	

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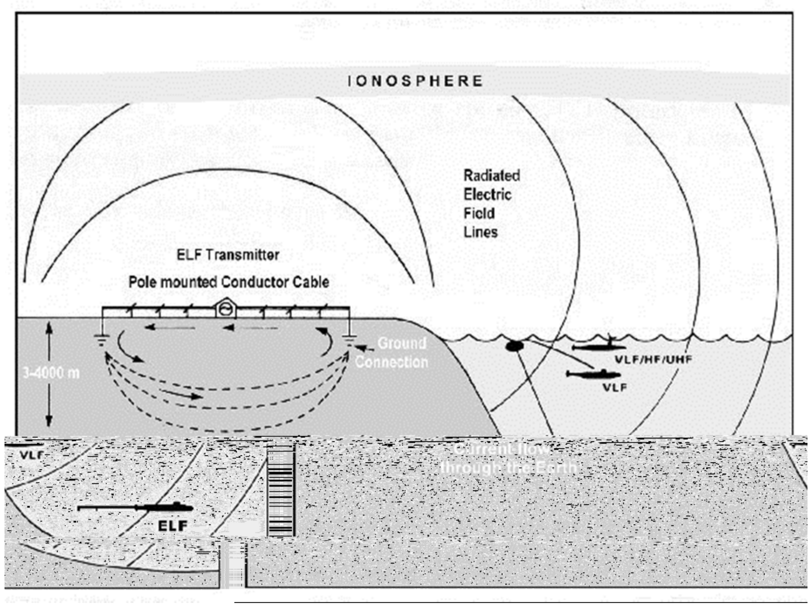
Question: How do submarines inside deep sea communicate?

-Extremely Low Frequency Communication: 76, 82Hz

➔ Wavelength: ~ 3,600 km

-Transmitter Antenna size: several tens of km!

- Not much bandwidth



W.-Y. Choi