Lect. 9: Propagation through Plasma

(Cheng 8-3.3, 8-4)

- Plasma: Ionized gases with equal electron and ion densities

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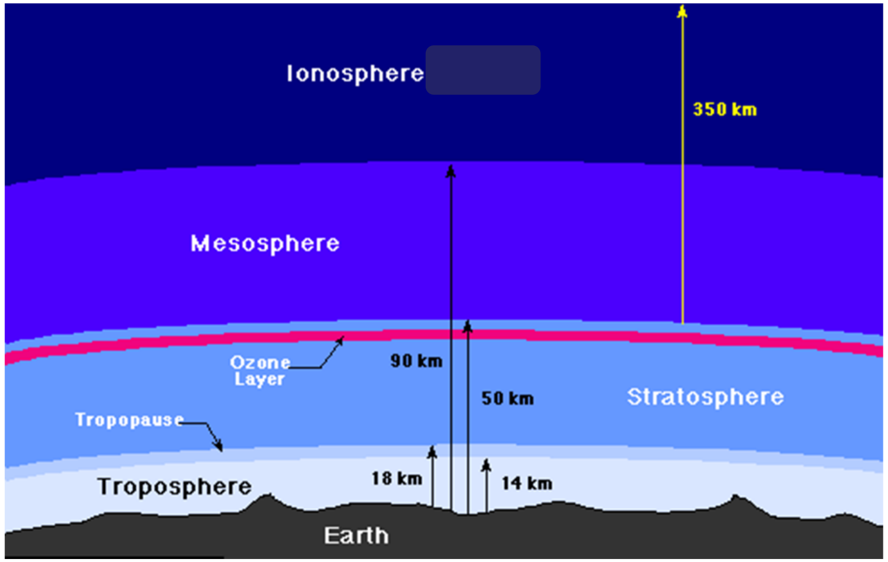
- Plasma: Ionized gases with equal electron and ion densities

– Plasma formation in ionosphere



– Air molecules located 50~500km above earth are ionized by solar radiation

– Plasma in ionosphere greatly affects EM wave propagation

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- Propagation through plasma

Simple modeling with P-vector

$$\bar{D} = \varepsilon \bar{E} = \varepsilon_0 \bar{E} + \bar{P}$$

$\bar{P} = N\bar{p}$ ($\bar{p}$: dipole moment, N: density) $\bar{p} = -q\bar{x}$ (minus for electron)

Determine P-vector in plasma

Electrons in plasma experience force with EM waves

$$\bar{F} = -q\bar{E} = m\bar{a} = m\frac{d^2\bar{x}}{dt^2} \Longrightarrow -m\omega^2\bar{x} \quad \text{(assume } e^{j\omega t} \text{ dependence)}$$

$$\bar{x} = \frac{q}{m\omega^2}\bar{E} \qquad \bar{p} = -q\bar{x} = \frac{-q^2}{m\omega^2}\bar{E}$$

$$\bar{P} = N\bar{p} = -\frac{Nq^2}{m\omega^2}\bar{E}$$

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$$\bar{P} = -\frac{Nq^2}{m\omega^2}\bar{E}$$

$$\bar{D} = \varepsilon_0 \bar{E} + \bar{P} \;\; = \varepsilon_0 \bar{E} - \frac{Nq^2}{m\omega^2}\bar{E} \;\; = \varepsilon_0 \left(1 - \frac{Nq^2}{m\omega^2 \varepsilon_0} \right) \bar{E}$$

$$= \varepsilon_0 \left(1 - \frac{\omega_p{}^2}{\omega^2} \right) \bar{E}$$

$$\omega_p = \sqrt{\frac{Nq^2}{m\varepsilon_0}} \quad (rad/s, \text{ plasma angular frequency})$$

➔ $\varepsilon = 0$ when $\omega = \omega_p$

W.-Y. Choi

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Propagation characteristics with $\varepsilon = \varepsilon_0\left(1-\frac{\omega_p{}^2}{\omega^2}\right)\overline{E}$

- If $\omega < \omega_p$ $\quad \varepsilon < 0$

$k = \omega\sqrt{\mu\varepsilon} = -j\alpha$: *purely imaginary*

No propagation!

$N \approx 3\times10^{11}$ m^{-3} for ionosphere

$f_p \approx 5$ MHz

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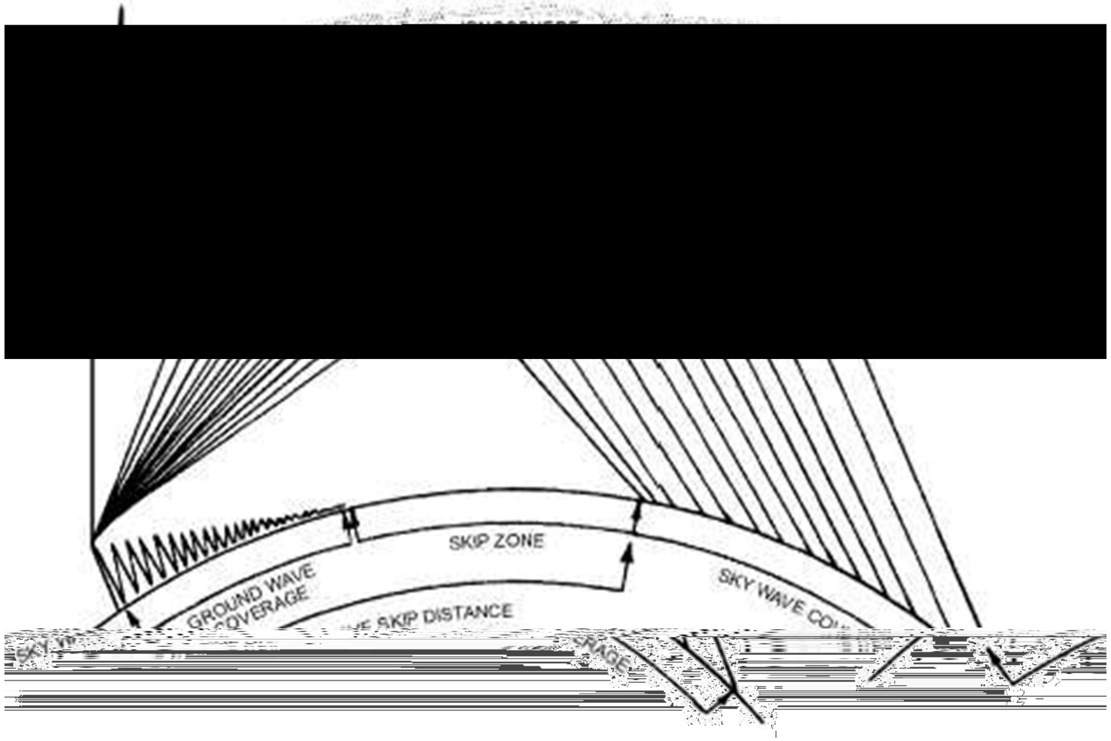
Very long distance communication using ionoshere: Amateur (Ham) radio



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Propagation characteristics with $\varepsilon = \varepsilon_0\left(1-\frac{\omega_p{}^2}{\omega^2}\right)\overline{E}$

If $\omega > \omega_p$

$\varepsilon > 0$ Propagation possible

Consider phase velocity

$$v = \frac{\omega}{k} = \frac{\omega}{\omega\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}\left(1-\frac{\omega_p{}^2}{\omega^2}\right)^{\frac{1}{2}}} > \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c$$

Greater than speed of light?

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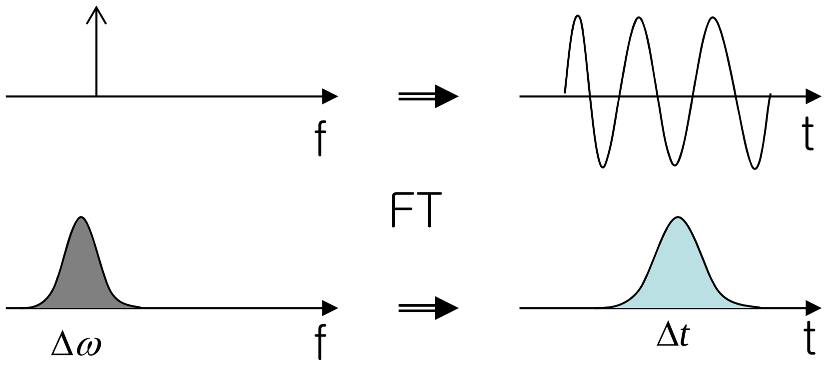
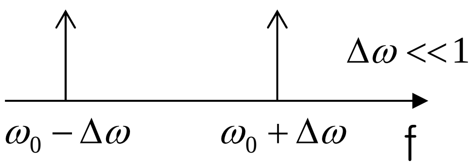
✓ In order to transport information (energy), more than one frequency is needed



Infinitely long sine function
(no information delivery)

Consider the simplest case of having only two frequencies



$$\beta = \omega\sqrt{\mu\varepsilon}$$

$$\omega_0 \pm \Delta\omega \Rightarrow \beta_0 \pm \Delta\beta$$

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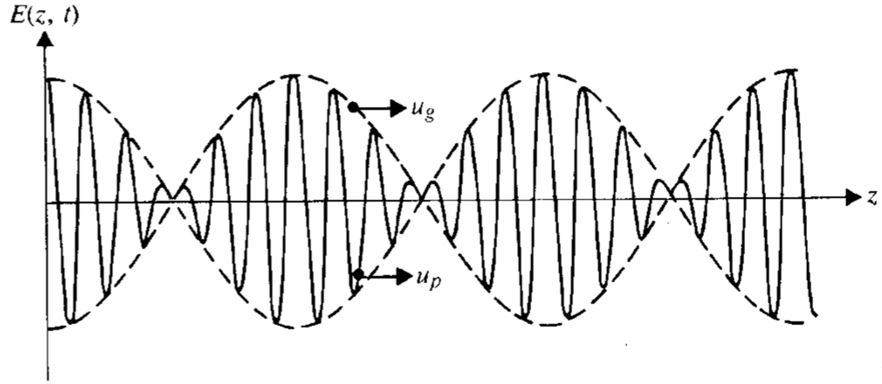
$$E(z,t) = e^{j\{(\omega_0+\Delta\omega)t-(\beta_0+\Delta\beta)z} + e^{j\{(\omega_0-\Delta\omega)t-(\beta_0-\Delta\beta)z}$$

$$= e^{j(\omega_0 t-\beta_0 z)}\left[e^{j(\Delta\omega t-\Delta\beta z)} + e^{-j(\Delta\omega t-\Delta\beta z)}\right]$$

$$= 2e^{j(\omega_0 t-\beta_0 z)}\cos(\Delta\omega t - \Delta\beta z)$$



Can information be delivered?

What is the velocity of information delivery?

Two types of velocity

Phase velocity $\dfrac{\omega}{\beta}$ Group velocity: $v_g = \dfrac{\Delta\omega}{\Delta\beta}$ => $\dfrac{d\omega}{d\beta}$

➔ Phase and group velocities can have different values!

W.-Y. Choi

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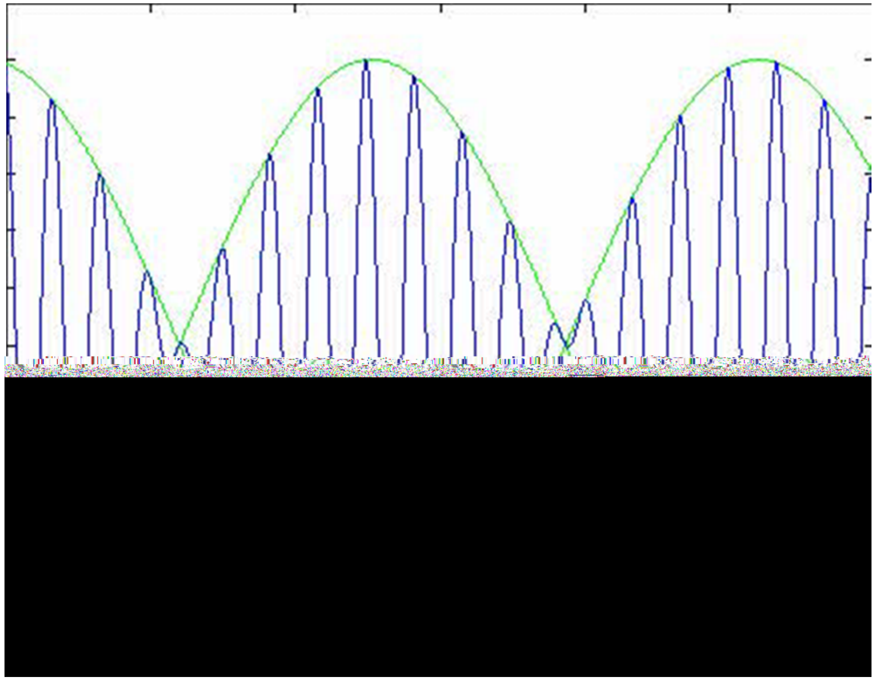
Phase Velocity vs Group Velocity



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In plasma

$$k = \beta = \omega\sqrt{\mu_0\varepsilon_0}\left(1-\frac{\omega_p{}^2}{\omega^2}\right)^{\frac{1}{2}} \;,\; \omega > \omega_p$$

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}\left(1-\frac{\omega_p{}^2}{\omega^2}\right)^{\frac{1}{2}}} > \frac{1}{\sqrt{\mu\varepsilon_0}} = c$$

$$v_g = \frac{d\omega}{d\beta} = \frac{1}{d\beta / d\omega} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}\left(1-\frac{\omega_p{}^2}{\omega^2}\right)^{\frac{1}{2}} < c$$

Group velocity is less than c!