Lect. 2: Electric Fields

(Chap. 3 in Cheng)

- Force between charges

q_1 q_2

- Coulomb force

$$\overline{F} = \overline{a}\frac{q_1 \cdot q_2}{4\pi\varepsilon_0 R^2}$$

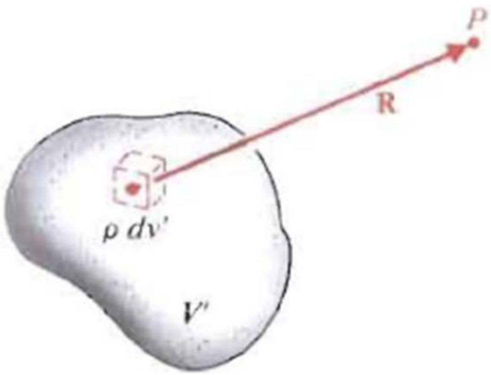
- Charge produces electric field

q_1

- Electric field

$$\overline{E} = \overline{a}\frac{q_1}{4\pi\varepsilon_0 R^2}$$

- Continuous charge distribution



$$d\overline{E} = \overline{a}\frac{\rho dv'}{4\pi\varepsilon_0 R^2}$$

$$\overline{E} = \frac{1}{4\pi\varepsilon_0}\int_{v'} \overline{a}\frac{\rho(v')dv'}{R^2}$$

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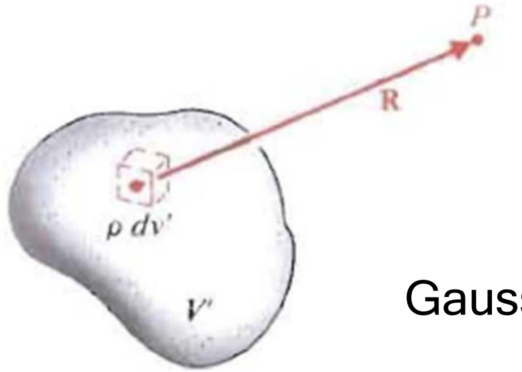

$$\overline{E} = \frac{1}{4\pi\varepsilon_0} \int_{v'} \overline{a} \frac{\rho(v')dv'}{R^2}$$

Gauss's Law $$\oint_S \overline{E} \bullet d\overline{s} = \frac{Q}{\varepsilon_0}$$

$$\oint_S \overline{E} \bullet d\overline{s} = \int_v \nabla \bullet \overline{E}\, dv = \frac{1}{\varepsilon_0} \int_v \rho(v) dv$$

$$\nabla \cdot \overline{E} = \frac{\rho}{\varepsilon_0}$$

➔ Charges produce E-field

- Magnitude, Direction

- Unit for E-field? $\overline{E} = -\nabla V$

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Gauss's Law is equivalent to Coulomb's Law

Deriving Gauss's law from Coulomb's law

$$\mathbf{E}(\mathbf{r}) = \frac{q}{4\pi\varepsilon_0}\frac{\mathbf{e_r}}{r^2}$$

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\mathbf{s})(\mathbf{r}-\mathbf{s})}{|\mathbf{r}-\mathbf{s}|^3}\, d^3\mathbf{s}$$

$$\nabla\cdot\left(\frac{\mathbf{r}}{|\mathbf{r}|^3}\right) = 4\pi\delta(\mathbf{r})$$

$$\nabla\cdot\mathbf{E}(\mathbf{r}) = \frac{1}{\varepsilon_0}\int \rho(\mathbf{s})\ \delta(\mathbf{r}-\mathbf{s})\, d^3\mathbf{s}$$

$$\nabla\cdot\mathbf{E}(\mathbf{r}) = \frac{\rho(\mathbf{r})}{\varepsilon_0},$$

Deriving Coulomb's law from Gauss's law

$$\oint_S \mathbf{E}\cdot d\mathbf{A} = \frac{Q}{\varepsilon_0}$$

$$4\pi r^2\hat{\mathbf{r}}\cdot\mathbf{E}(\mathbf{r}) = \frac{Q}{\varepsilon_0}$$

$$\mathbf{E}(\mathbf{r}) = \frac{Q}{4\pi\varepsilon_0}\frac{\hat{\mathbf{r}}}{r^2}$$

$\hat{\mathbf{r}}$

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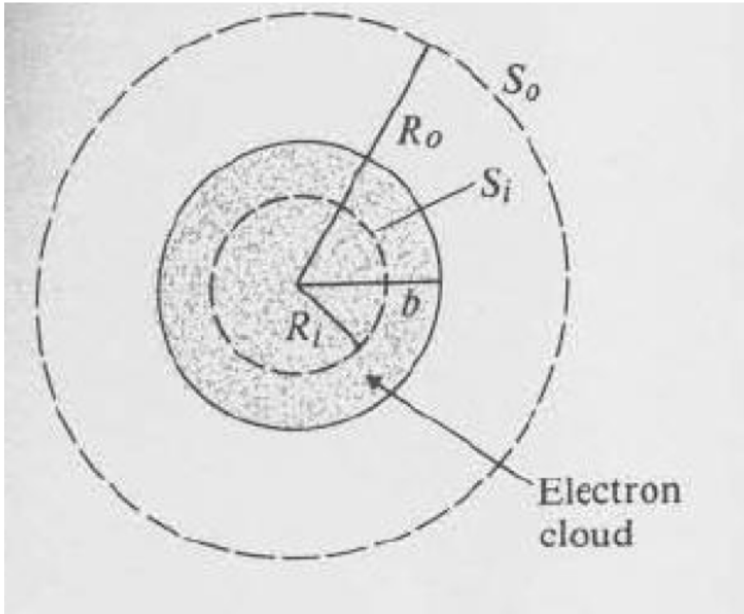
- Example 3-7 in Cheng. With charge density $-\rho_0$, E=?



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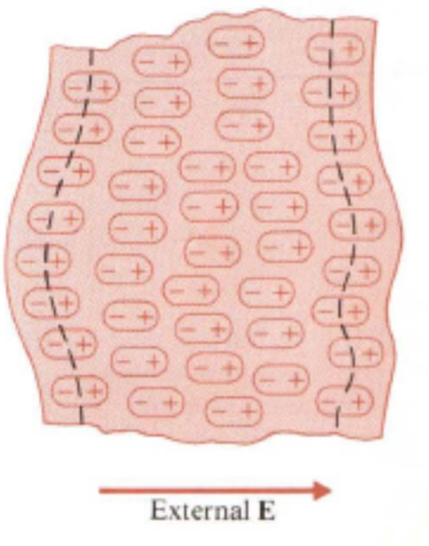
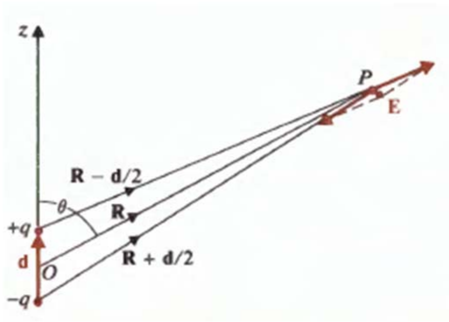
- How do ***dielectric*** materials response to electric field?



- Polarized
- Dipoles are produced



$$\mathbf{E} = \frac{p}{4\pi\epsilon_0 R^3}(\mathbf{a}_R 2\cos\theta + \mathbf{a}_\theta \sin\theta) \quad \text{(V/m)}.$$ (p = qd)

- Define Polarization Vector $$\mathbf{P} = \lim_{\Delta v \to 0} \frac{\sum_{k=1}^{n\Delta v} \mathbf{p}_k}{\Delta v} \quad (\mathrm{C/m^2}),$$

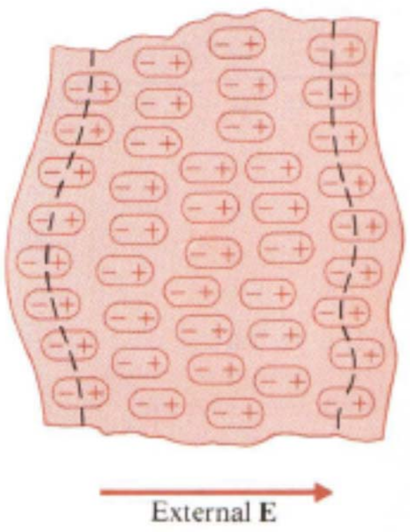
Often $\overline{P} = \chi_e \varepsilon_0 \overline{E}$ χ_e : (electric susceptibility)

$-\nabla \cdot \overline{P}$: charge density

Modified Gauss's Law: $$\nabla \cdot \overline{E} = \frac{\rho - \nabla \cdot \overline{P}}{\varepsilon_0}$$

Lect. 2: Electric Fields

- How do *dielectric* materials response to electric field?



$$\overline{P} = \chi_e \varepsilon_0 \overline{E} \qquad \nabla \cdot \overline{E} = \frac{\rho - \nabla \cdot \overline{P}}{\varepsilon_0}$$

- Define Displacement Vector $\overline{D} = \varepsilon_0 \overline{E} + \overline{P}$

$$\overline{D} = (1 + \chi_e)\varepsilon_0 \overline{E} = \varepsilon_r \varepsilon_0 \overline{E} = \varepsilon \overline{E}$$

ε : permittivity ε_r : relative permittivity or dielectric constant

$$\nabla \cdot \overline{D} = \rho$$

Unit for *D*: C/m^2 Unit for ε: F/m

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Example 3-12 in Cheng, Determine D, E, P