Test 2

June 7, 2016
E&M II
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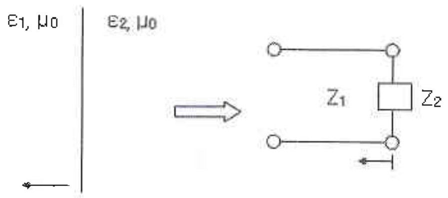
Prob. 1(30)

The reflection coefficient, Γ, for a normally incident EM wave at the dielectric interface can be determined by converting the problem into a transmission line problem. An example is shown below.



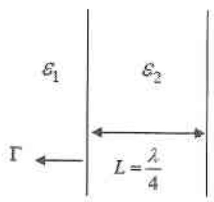
$$Z_1 \sim \sqrt{\frac{\mu_0}{\varepsilon_1}}$$

$$Z_2 \sim \sqrt{\frac{\mu_0}{\varepsilon_2}}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{\frac{1}{\sqrt{\varepsilon_2}} - \frac{1}{\sqrt{\varepsilon_1}}}{\frac{1}{\sqrt{\varepsilon_2}} + \frac{1}{\sqrt{\varepsilon_1}}} = \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$$

Determine Γ for each of the following cases using the transmission line theory. Assume there is no loss and all the media have the same permeability of μ_0. You answers should be expressions involving ε_1 and ε_2 .

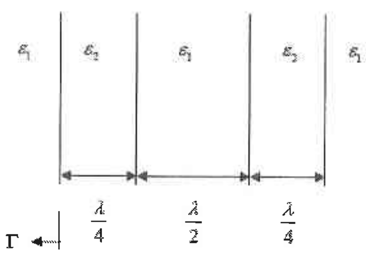
(a)(10)



λ: The wavelenght of the input EM wave

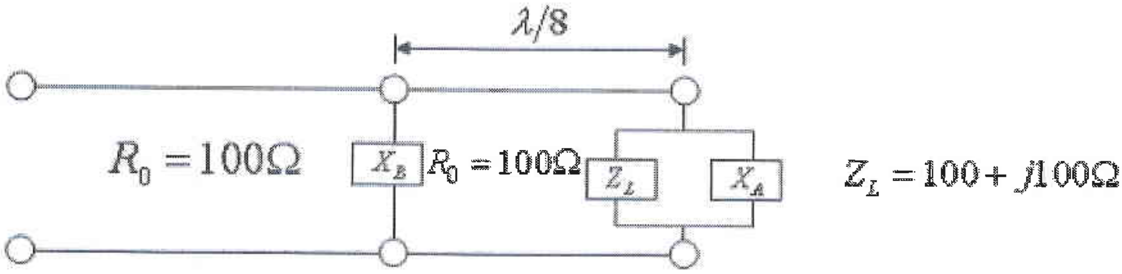
(b)(10) Same as (a) except L = λ/2.

(c)(10)

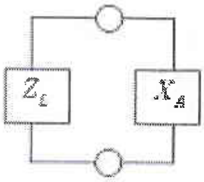


Prob. 2(35)

We would like to achieve impedance matching for a transmission line circuit by attaching reactive elements (either a capacitor or an inductor) at two locations as shown below. Use the provided Smith Chart in answering following questions. Make sure the points of importance are clearly marked on the chart.



(a)(10) For the reactive element (X_A) in parallel with the load (see the figure below), what are the values of **normalized admittance** that is required for impedance matching (more than one answer possible)?



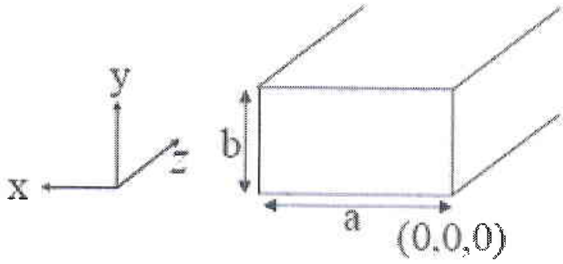
(b)(5) Should X_A be a capacitor or an inductor? Explain why?

(c)(10) What is the numerical value of capacitance or inductance obtained in (b). Assume the source frequency is 10MHz. If more than one answer is possible, give the smallest value.

(d)(10) For the reactive element (X_B) attached to the transmission line at $\lambda/8$ from the load, determine the required value of capacitance or inductance. If more than two values are possible, give the smallest one.

Prob. 3(35)

Determine the TM mode solutions in a 2-D rectangular metallic waveguide shown below by extending what you learned in the class for 1-D waveguide solutions.



(a)(5) Derive $\nabla^2_{xy}E_z(x,y)+h^2E_z(x,y)=0$ from $\nabla^2 E_z(x,y,z,t)-\mu\varepsilon\frac{\partial^2}{\partial t^2}E_z(x,y,z,t)=0$ assuming $E_z(x,y,z,t)=E_z(x,y)\cdot e^{-\beta z}\cdot e^{j\omega t}$. What is the expression for h^2?

(b)(10) Assume $E_z(x,y)=X(x)\cdot Y(y)$ and derive following two equations, each involving only one variable, x or y. What is the relationship between h, k_x, k_y?

$$\text{Eq. (1): } \frac{d^2}{dx^2}X(x)+k_x^{\,2}X(x)=0,\quad \text{Eq. (2): } \frac{d^2}{dx^2}Y(y)+k_y^{\,2}Y(y)=0$$

(c)(5) Solve Eq.(1) given above using the boundary conditions. Specify the required condition on k_x. Use index m in your answer.

(d)(5) Solve Eq.(2) given above using the boundary conditions. Specify the required condition on k_y. Use index n in your answer.

(e)(10) What is the cut-off frequency for $TM_{m,n}$ mode in Hz?