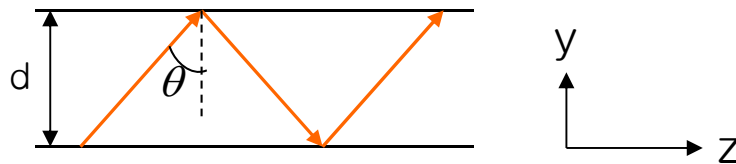


Lect. 12: Dielectric Waveguides (1)

Guidance condition in a waveguide

Metallic waveguide (TE, TM)



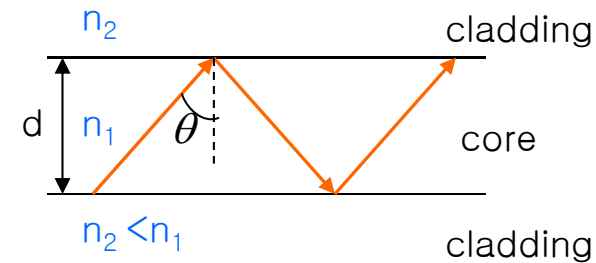
$$e^{-jk_y d} (-1) e^{-jk_y d} (-1) = e^{-j2k_y d} = 1$$

$$\therefore 2k_y d = 2m\pi \text{ and } k_y = \frac{m\pi}{d}$$

$$\beta (=k_z) = \sqrt{(nk_0)^2 - \left(\frac{m\pi}{d}\right)^2}$$

$$\tan \theta = \frac{\beta}{k_y}$$

Dielectric waveguide (TIR)



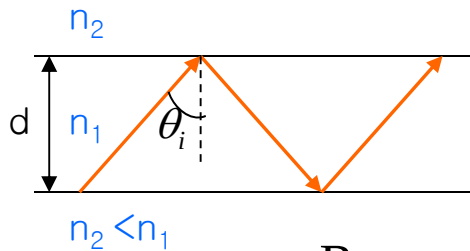
$$e^{-jk_y d} r_{\perp, //} e^{-jk_y d} r_{\perp, //} = 1$$

$$\text{Since } r_{\perp, //} = e^{j\phi_{\perp, //}}, \quad e^{-j2k_y d} e^{j2\phi_{\perp, //}} = 1$$

$$\therefore 2k_y d - 2\phi_{\perp, //} = 2m\pi$$

$$\text{Or } k_y d - \phi_{\perp, //} = m\pi$$

Lect. 12: Dielectric Waveguides (1)



$$k_y d - \phi_{\perp, //} = m\pi$$

Remember from Lect. 5 ($n = \frac{n_2}{n_1}$)

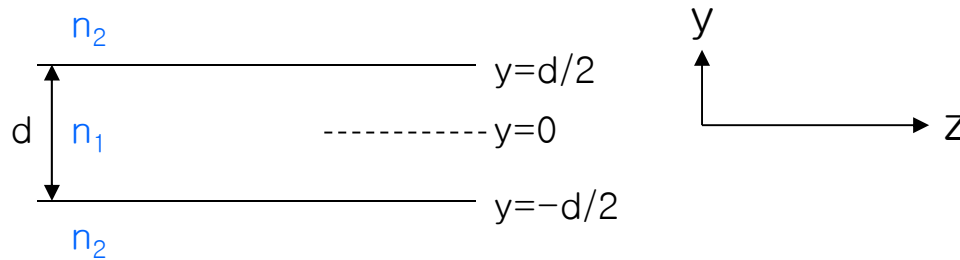
$$\phi_{\perp} : \tan\left(\frac{\phi_{\perp}}{2}\right) = \frac{(\sin^2 \theta_i - n^2)^{1/2}}{\cos \theta_i} \quad (\text{TE})$$

$$\phi_{//} : \tan\left(\frac{\phi_{//} + \pi}{2}\right) = \frac{(\sin^2 \theta_i - n^2)^{1/2}}{n^2 \cos \theta_i} \quad (\text{TM})$$

Numerically solve for $\phi_{\perp}$ and $\phi_{//}$ $\rightarrow k_y$ for each mode

$$\beta = \sqrt{\omega^2 \mu \epsilon - k_y^2} = \sqrt{(n_1 k_0)^2 - k_y^2}$$

Lect. 12: Dielectric Waveguides (1)



What are field profiles for guided modes?

Full analysis starting from wave equations:

$$\nabla^2 \bar{E}(y, z, t) = \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2}(y, z, t)$$

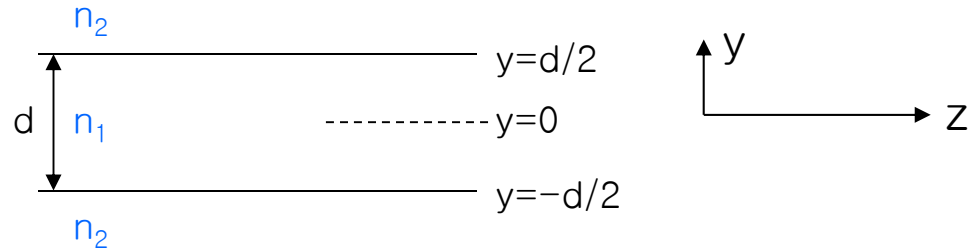
Assuming $\bar{E}(y, z, t) = \bar{E}(y) \cdot e^{-j\beta z} \cdot e^{j\omega t}$,

$$\frac{d^2}{dy^2} \bar{E}(y) + (k^2(y) - \beta^2) \bar{E}(y) = 0 \text{ where } k^2(y) = \omega^2 \mu\epsilon(y)$$

$$k(y) = n_2 k_0 \text{ for } |y| > \frac{d}{2}; \text{ cladding}$$

$$k(y) = n_1 k_0 \text{ for } |y| < \frac{d}{2}; \text{ core}$$

Lect. 12: Dielectric Waveguides (1)



$$\frac{d^2}{dy^2} \bar{E}(y) + (k^2(y) - \beta^2) \bar{E}(y) = 0, \quad k^2(y) = \omega^2 \mu \epsilon(y)$$

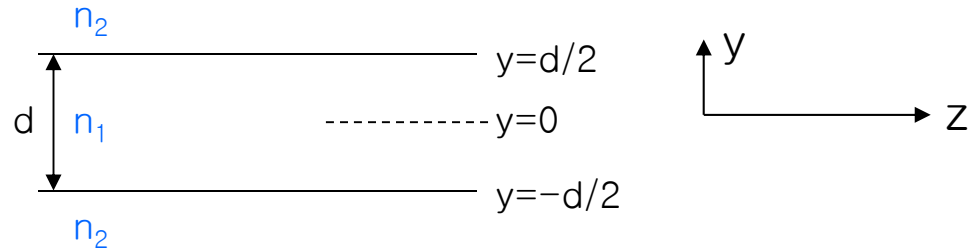
Consider TE Solution

$$\bar{E}(y) = \bar{x} E(y)$$

$$\frac{d^2 E(y)}{dy^2} + (k^2(y) - \beta^2) E(y) = 0$$

Solve for β and $E(y)$ (Eigen value equation)

Lect. 12: Dielectric Waveguides (1)



$$\frac{d^2 E(y)}{dy^2} + (k^2(y) - \beta^2) E(y) = 0$$

Sign of $(k^2(y) - \beta^2)$ determines the solution type

We know $n_1 k_0 > \beta > n_2 k_0$

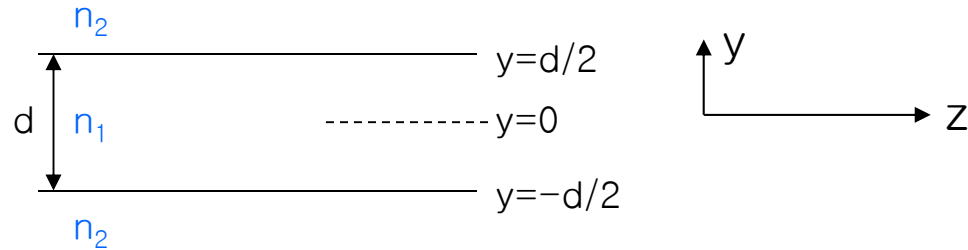
In core, $k^2(y) - \beta^2 > 0 \Rightarrow E(y) \sim \sin(k_y y)$ or $\cos(k_y y)$

$$k_y^2 = k(y)^2 - \beta^2, \quad k_y = \sqrt{(n_1 k_0)^2 - \beta^2}$$

In cladding, $k^2(y) - \beta^2 < 0 \Rightarrow E(y) \sim \exp(\alpha y)$ or $\exp(-\alpha y)$

$$\alpha^2 = \beta^2 - k(y)^2, \quad \alpha = \sqrt{\beta^2 - (n_2 k_0)^2}$$

Lect. 12: Dielectric Waveguides (1)



Solutions

$$y > \frac{d}{2} : E(y) = A \exp(\alpha y) + B \exp(-\alpha y)$$

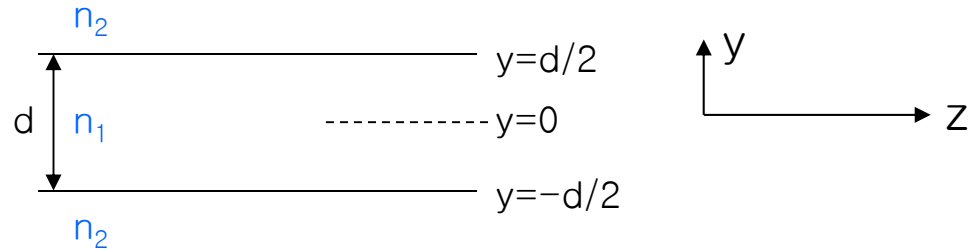
$$|y| < \frac{d}{2} : E(y) = C \sin(k_y y) + D \cos(k_y y)$$

$$y < -\frac{d}{2} : E(y) = E \exp(\alpha y) + F \exp(-\alpha y)$$

$A=0$ and $F=0$

For easier analysis, divide the solutions into even and odd solutions

Lect. 12: Dielectric Waveguides (1)



Even Solutions

$$y > \frac{d}{2} : E(y) = B \exp(-\alpha y)$$

$$|y| < \frac{d}{2} : E(y) = D \cos(k_y y)$$

$$y < -\frac{d}{2} : E(y) = B \exp(\alpha y)$$

$$(E = B)$$

Odd Solutions

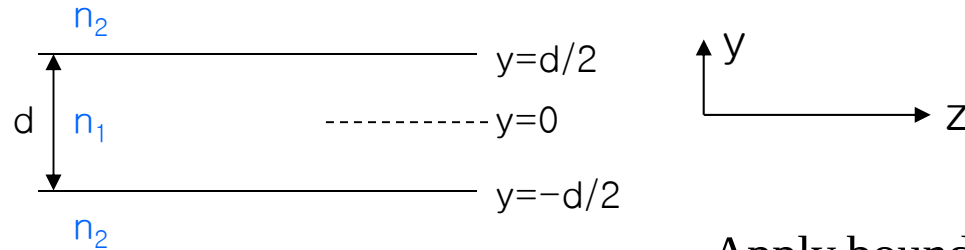
$$y > \frac{d}{2} : E(y) = B \exp(-\alpha y)$$

$$|y| < \frac{d}{2} : E(y) = D \sin(k_y y)$$

$$y < -\frac{d}{2} : E(y) = -B \exp(\alpha y)$$

$$(E = -B)$$

Lect. 12: Dielectric Waveguides (1)



Apply boundary conditions:

Even Solutions

$$y > \frac{d}{2} : E(y) = B \exp(-\alpha y)$$

$$|y| < \frac{d}{2} : E(y) = D \cos(k_y y)$$

$$y < -\frac{d}{2} : E(y) = B \exp(\alpha y)$$

$$(E = B)$$

$$E(y) \text{ is continuous at } y = \pm \frac{d}{2}$$

$$H_{\text{tan}} \text{ should be continuous at } y = \pm \frac{d}{2}$$

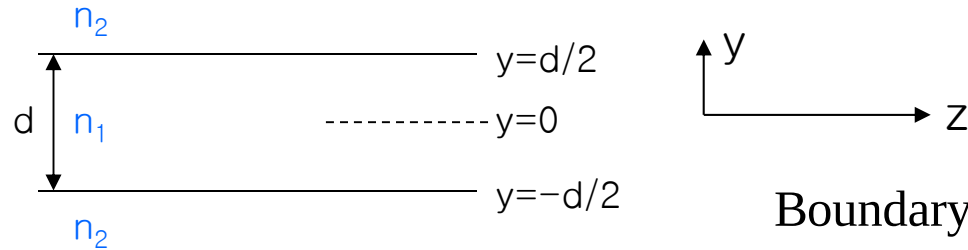
$$\text{For } \bar{E}(y, z) = \bar{x}E(y)e^{-j\beta z}$$

$$\bar{H} \text{ has } \bar{y} \text{ and } \bar{z} \text{ components, where } H_z \sim \frac{dE(y)}{dy}$$

H_z should be continuous across the boundary

$$\therefore E(y) \text{ and } \frac{dE(y)}{dy} \text{ are continuous at } y = \pm \frac{d}{2}$$

Lect. 12: Dielectric Waveguides (1)



Even Solutions

$$y > \frac{d}{2} : E(y) = B \exp(-\alpha y)$$

$$|y| < \frac{d}{2} : E(y) = D \cos(k_y y)$$

$$y < -\frac{d}{2} : E(y) = B \exp(\alpha y)$$

$$(E = B)$$

Boundary conditions:

$$E(y) \text{ and } \frac{dE(y)}{dy} \text{ are continuous at } y = \pm \frac{d}{2}$$

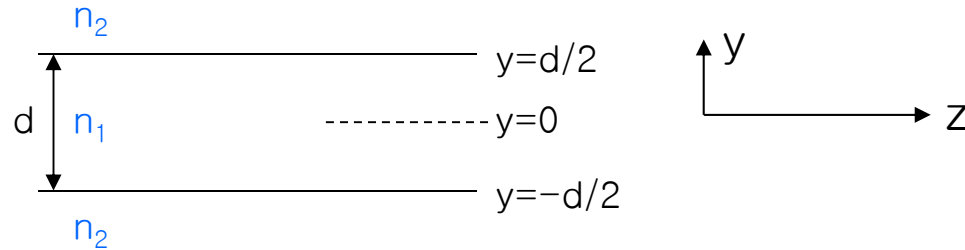
$$\text{At } y = \frac{d}{2},$$

$$B \exp(-\alpha \frac{d}{2}) = D \cos(k_y \frac{d}{2}) \quad \text{----- (1)}$$

$$-\alpha B \exp(-\alpha \frac{d}{2}) = -k_y D \sin(k_y \frac{d}{2}) \quad \text{----- (2)}$$

$$\frac{(2)}{(1)} : \alpha = k_y \tan(k_y \frac{d}{2})$$

Lect. 12: Dielectric Waveguides (1)



Apply boundary conditions.

$E(y)$ and $\frac{dE(y)}{dy}$ are continuous at $y = \pm \frac{d}{2}$

Odd Solutions

$$y > \frac{d}{2} : E(y) = B \exp(-\alpha y)$$

$$|y| < \frac{d}{2} : E(y) = D \sin(k_y y)$$

$$y < -\frac{d}{2} : E(y) = -B \exp(\alpha y)$$

$$(E = -B)$$

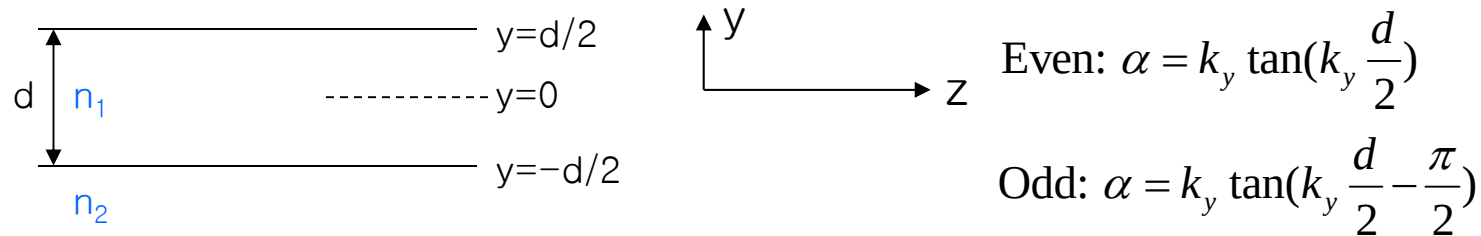
$$\text{At } y = \frac{d}{2},$$

$$B \exp(-\alpha \frac{d}{2}) = D \sin(k_y \frac{d}{2}) \quad \text{----- (1)}$$

$$-\alpha B \exp(-\alpha \frac{d}{2}) = k_y D \cos(k_y \frac{d}{2}) \quad \text{----- (2)}$$

$$\frac{(2)}{(1)} : \alpha = -k_y \cot(k_y \frac{d}{2}) = k_y \tan(k_y \frac{d}{2} - \frac{\pi}{2})$$

Lect. 12: Dielectric Waveguides (1)



Determine k_y and α that satisfy above conditions.

For easier solutions, do following normalization:

Let $X = k_y \frac{d}{2}$, $Y = \alpha \frac{d}{2}$

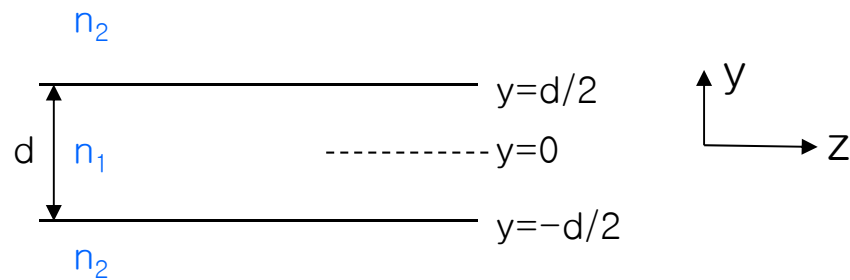
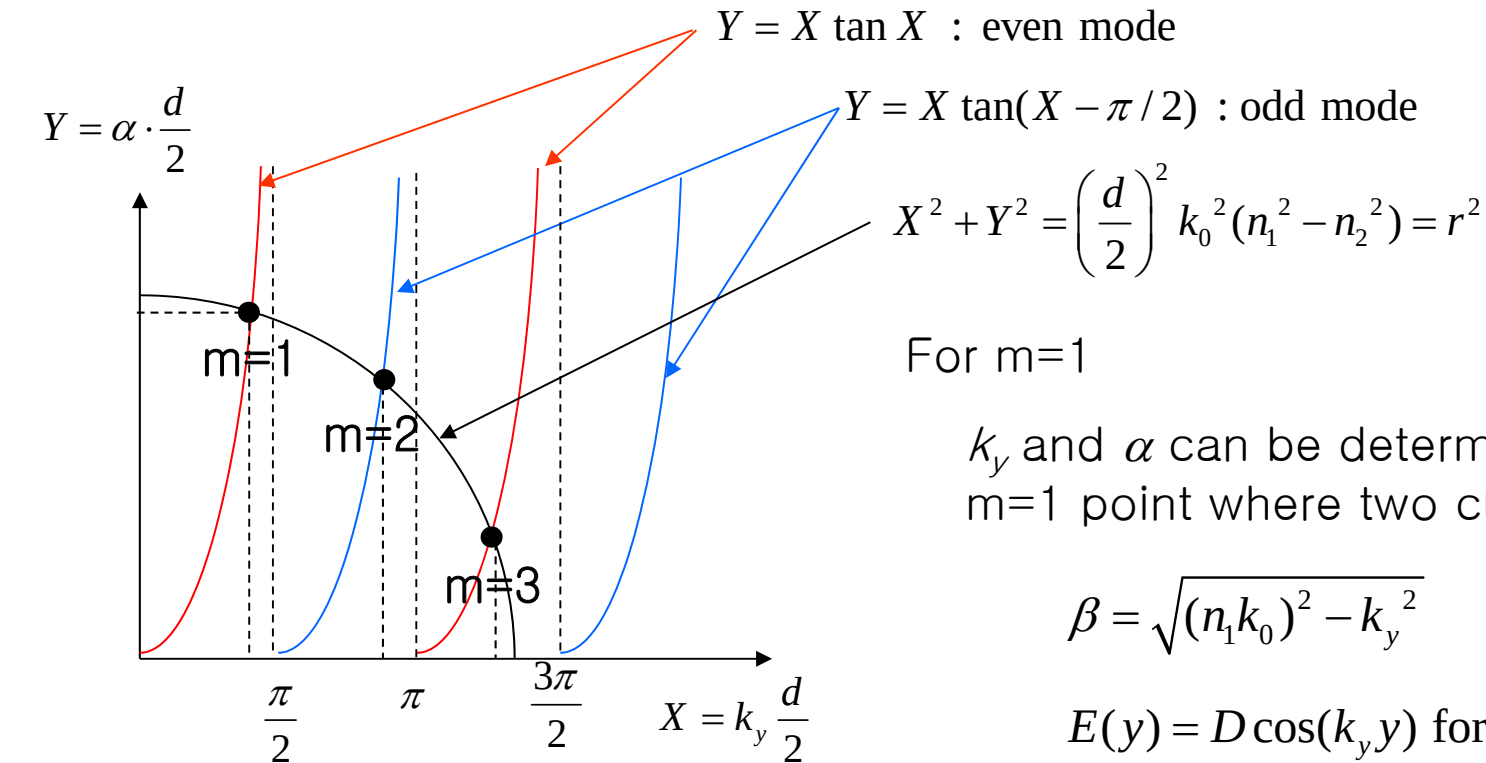
Then, $Y = X \tan X$ for even (1) $Y = X \tan(X - \frac{\pi}{2})$ for odd (2)

$$X^2 + Y^2 = \left(\frac{d}{2}\right)^2 (k_y^2 + \alpha^2) = \left(\frac{d}{2}\right)^2 [(n_1 k_0)^2 - \beta^2] + [\beta^2 - (n_2 k_0)^2] = \left(\frac{d}{2}\right)^2 k_0^2 (n_1^2 - n_2^2)$$

$$X^2 + Y^2 = r^2 \quad (3)$$

Plot (1), (2), (3) on X-Y plane!

Lect. 12: Dielectric Waveguides (1)



$$\beta = \sqrt{(n_1 k_0)^2 - k_y^2}$$

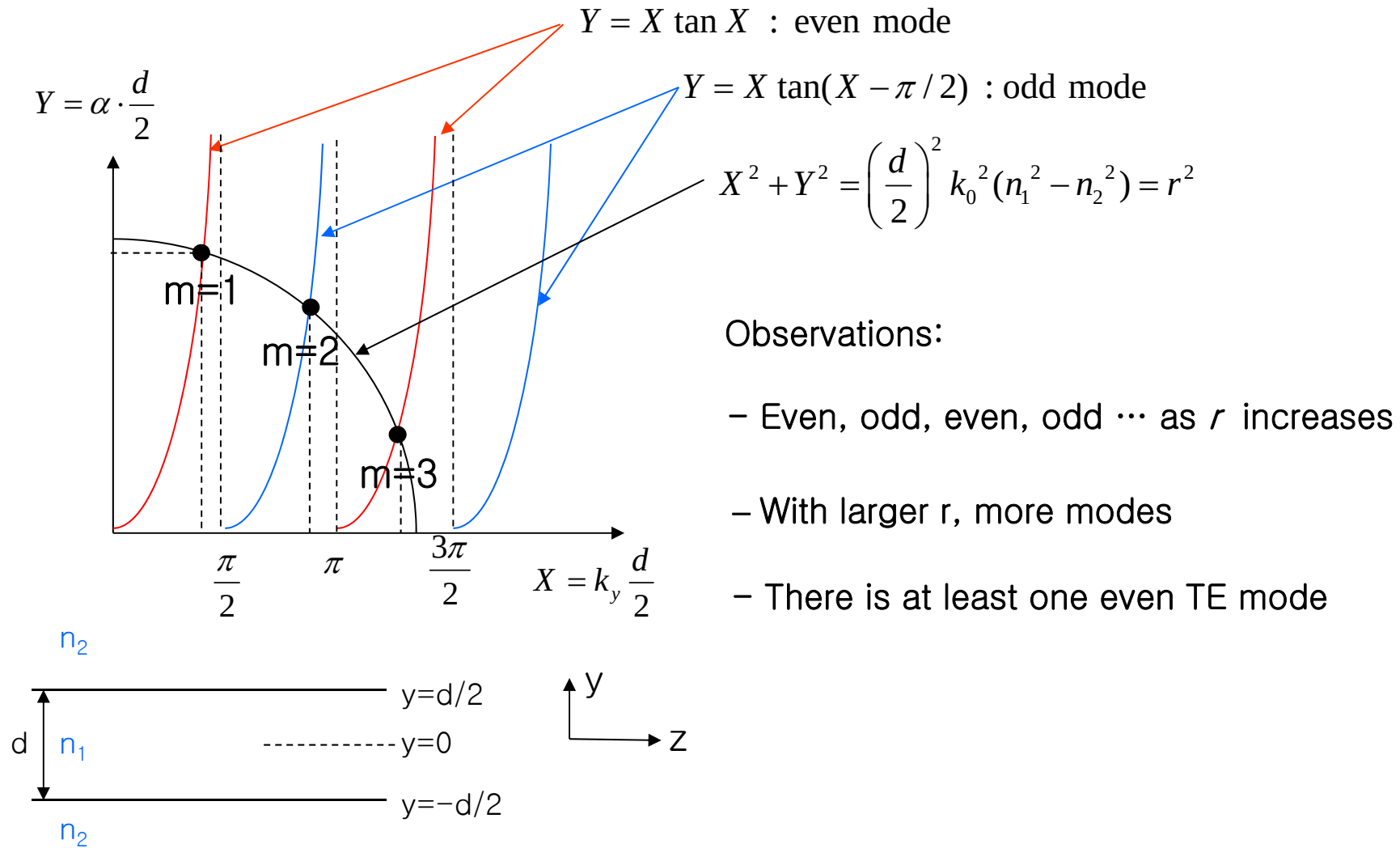
$$E(y) = D \cos(k_y y) \text{ for } |y| < \frac{d}{2}$$

$$= B \exp(-\alpha y) \text{ for } y > \frac{d}{2}$$

$$= B \exp(\alpha y) \text{ for } y < -\frac{d}{2}$$

$$E(y, z) = E(y) \cdot e^{-j\beta z} \text{ determined!}$$

Lect. 12: Dielectric Waveguides (1)



Lect. 12: Dielectric Waveguides (1)

Homework:

A symmetric three-layer waveguide is shown below. Consider only TE mode for this problem.

- (a) Determine how many modes the waveguide can support for $\lambda=1.0\mu\text{m}$.
- (b) Sketch the electric field intensity in the waveguide for each mode.
- (c) When λ is increased, the number of modes the waveguide can support may change. What is the largest wavelength for which the mode number remains the same as what was obtained in (a)?

