Lect. 2: Light as EM Waves (Chap. 8 in Cheng)

In the beginning, God said

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

$$\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t}$$

$$\nabla \bullet \overline{D} = \rho \qquad \begin{cases} \overline{D} = \varepsilon \overline{E} \\ \overline{B} = \mu \overline{H} \end{cases}$$

$$\nabla \bullet \overline{B} = 0$$

Then, there was light!

➔ Maxwell's Equations

Material Parameters $\begin{cases} \varepsilon : permittivity \\ \mu : permeability \end{cases}$

$$\nabla^2 \overline{E} = \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2} \qquad \nabla^2 \overline{H} = \mu\varepsilon \frac{\partial^2 \overline{H}}{\partial t^2}$$

Wave Equations

(source free, uniform medium)

➔ light propagation

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Solutions for Wave Equations

$$\nabla^2 \overline{E} = \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2}$$

Guess $\overline{E} = \overline{x} E_0 \cos(\omega t - kz)$

$$\nabla^2 \overline{E} = \overline{x}(-k^2) E_0 \cos(\omega t - kz)$$

$$\mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2} = \overline{x}\mu\varepsilon(-\omega^2) E_0 \cos(\omega t - kz)$$

Our guess is correct if $k^2 = \mu\varepsilon\omega^2$

Or $k = \omega\sqrt{\mu\varepsilon}$

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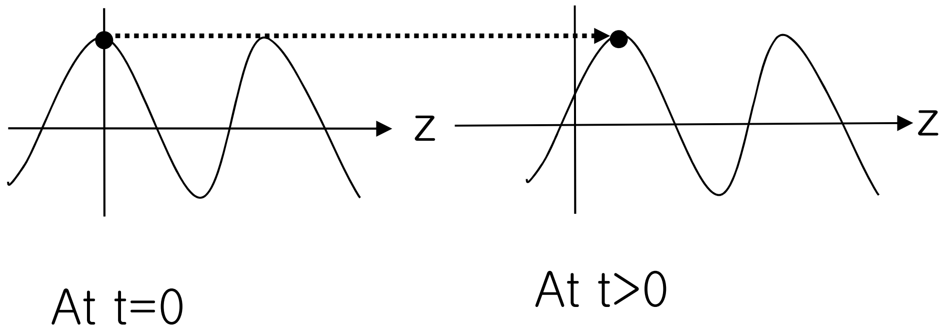
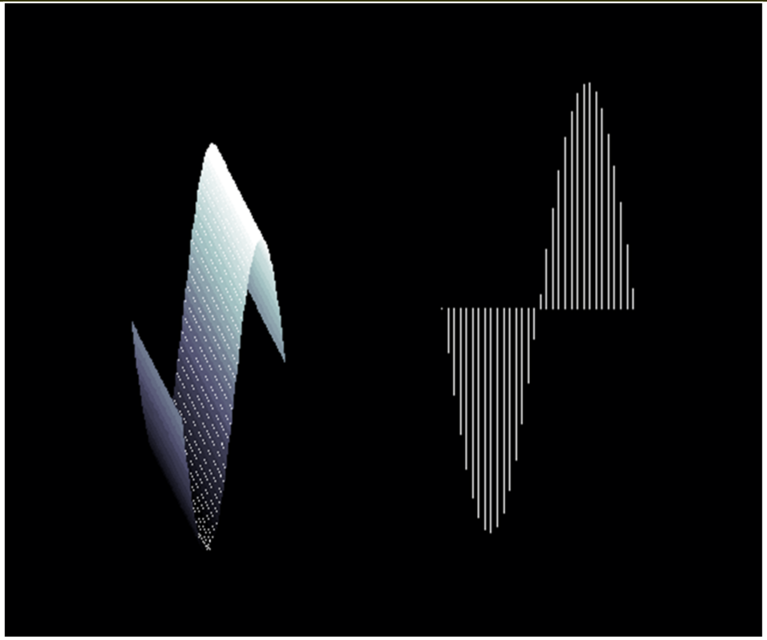
How does it look?

$$\overline{E} = \overline{x}E_0 \cos(\omega t - kz)$$





- Moving to the right direction maintaining its shape ➔ Wave
- The wave is periodic in space with periodicity $2\pi / k = \lambda$
- The wave is periodic in time with periodicity $2\pi / \omega = T = 1/f$

$$k = \omega\sqrt{\mu\varepsilon} \qquad \frac{\lambda}{T} = \frac{1}{\sqrt{\mu\varepsilon}}$$

EM wave propagates with velocity of $\dfrac{1}{\sqrt{\mu\varepsilon}}$

Solutions for Wave Equations

$$\nabla^2 \overline{E} = \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2}$$

More generally, $\overline{E} = \overline{x} E_0 e^{j(\omega t - kz)}$

$$\nabla^2 \overline{E} = \overline{x}(-k^2) E_0 e^{j(\omega t - kz)}$$

$$\mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2} = \overline{x}\mu\varepsilon(-\omega^2) E_0 e^{j(\omega t - kz)}$$

$$\therefore k = \omega\sqrt{\mu\varepsilon}$$

➔ Plane-wave solutions

(exponential solutions, phasor notation, …)

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How about H-field?

$$\nabla^2 \overline{E} = \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2} \qquad \overline{E} = \overline{x} E_0 e^{j(\omega t - kz)}$$

It can be shown from Maxwell's Equations,

$$\overline{H} = \overline{y} H_0 e^{j(\omega t - kz)}$$

Direction of E, H fields?

Direction of propagation?

Speed of propagation?

$$\frac{E_0}{H_0} = \sqrt{\frac{\mu}{\varepsilon}} = \eta\ [\Omega]$$ (377Ω for vacuum)

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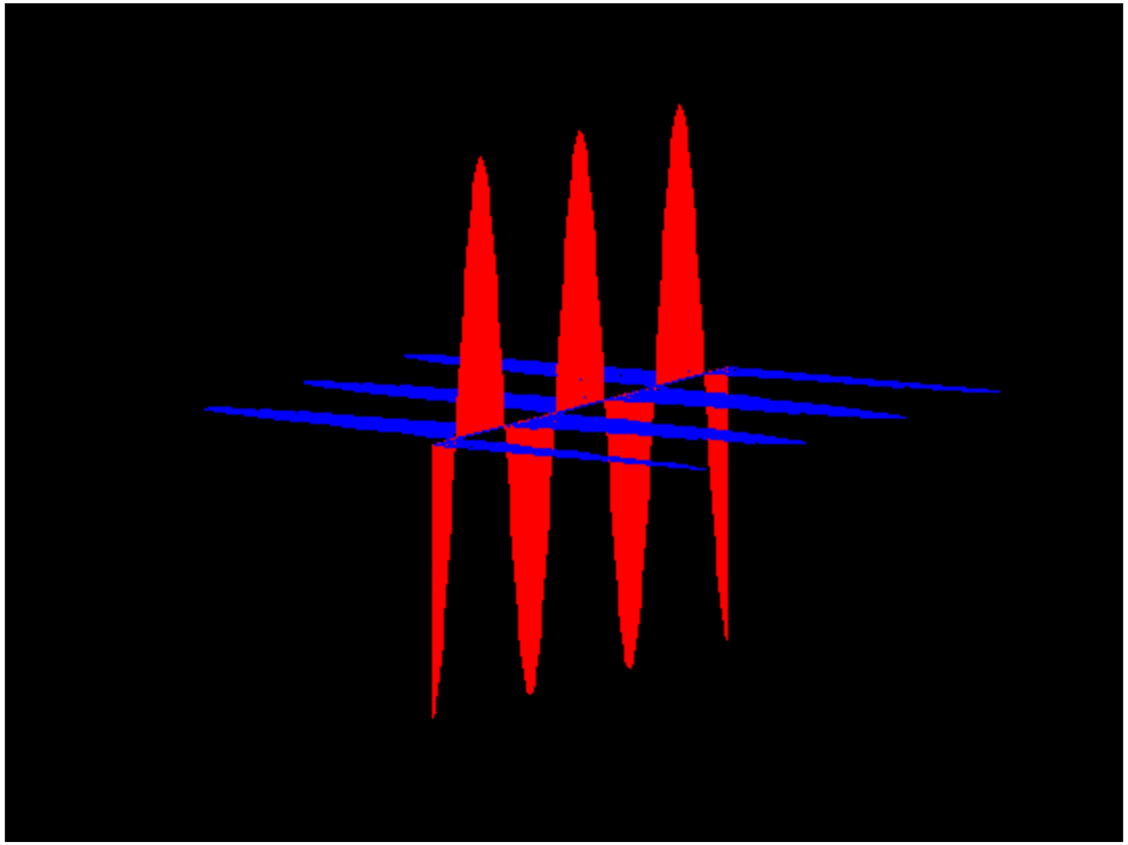
How does the plane-wave solution look like?



EM wave animation available at tera.yonsei.ac.kr
(Classes ➔ any 전자기학2 link ➔ Demonstration of EM waves)

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When a wave is propagating into   Plane wave solutions

+z direction: $e^{j(\omega t - kz)}$

-z direction: $e^{j(\omega t + kz)}$

+y direction: $e^{j(\omega t - ky)}$

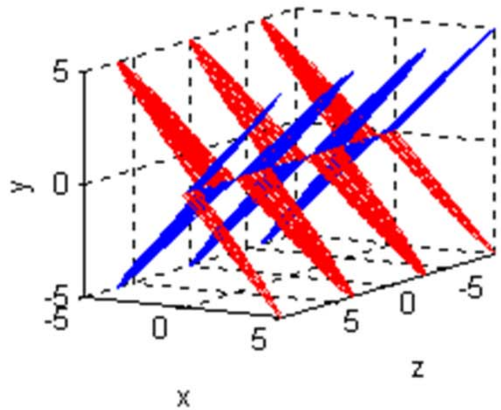
Any direction? $e^{j\omega t} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z} = e^{j(\omega t - \bar{k} \bullet \bar{R})}$

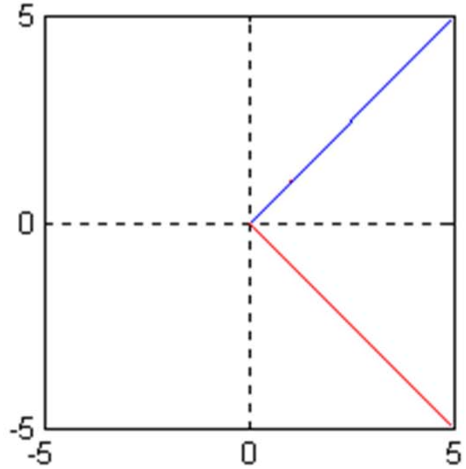
$$\bar{k} = \bar{x}k_x + \bar{y}k_y + \bar{z}k_z \quad \bar{R} = \bar{x}x + \bar{y}y + \bar{z}z$$

$\left|\bar{k}\right| = \dfrac{2\pi}{\lambda}$, $\angle\bar{k}$: direction of propagation

Polarization: Change of E-field direction with time

Linear Polarization $\overline{E} = \left(\overline{x}E_0 + \overline{y}E_0\right)e^{j\omega t}e^{jkz}$





E_x and E_y in-phase

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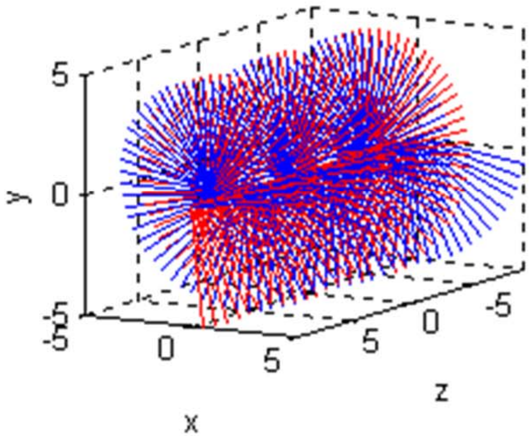
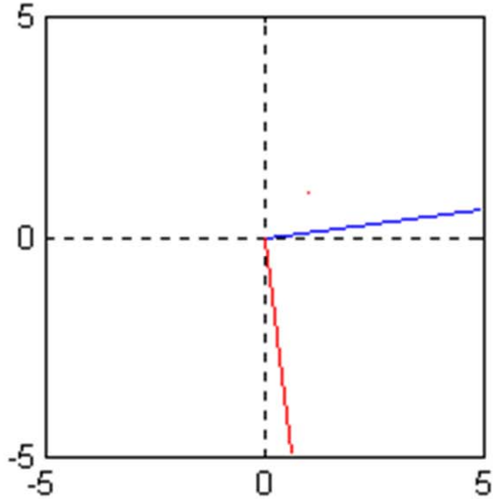
Circular Polarization

$$\overline{E} = \left(\overline{x} E_0 + \overline{y}\, j E_0 \right) e^{j\omega t} e^{jkz}$$

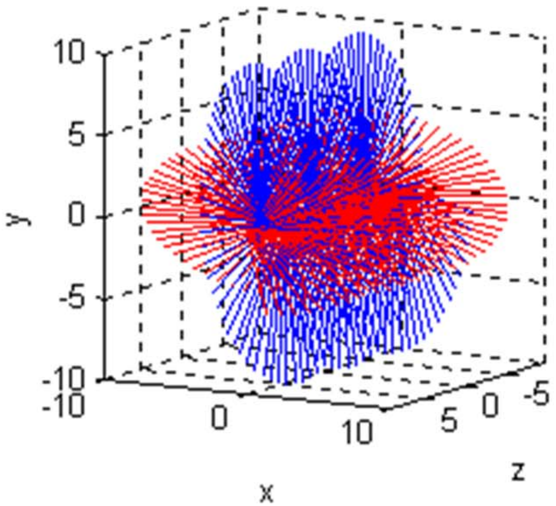


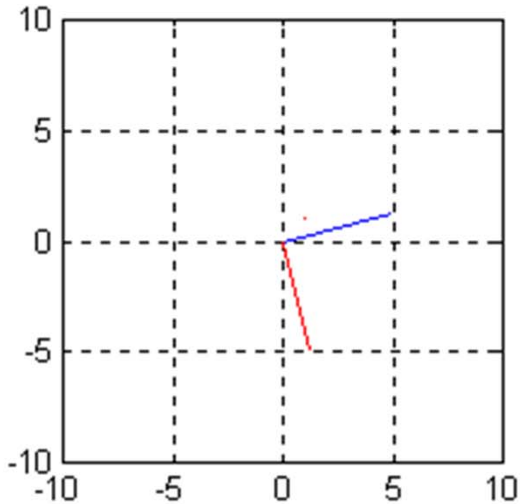


E_x and E_y out-of-phase   Handedness?

Elliptical Polarization

$$\overline{E} = \left(\overline{x} E_0 + \overline{y} j 2 E_0 \right) e^{j\omega t} e^{jkz}$$





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– Homeworks: Due on 9/7 in the class

(1) Derive the wave equation $\nabla^2 \overline{E} = \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2}$ from the Maxwell Equations

(2) A uniform plane wave propagating in a dielectric medium has the E-field given as

$$\overline{E}(t, z) = \overline{x}\, 2\cos(10^8 t - z/\sqrt{3}) + \overline{y}\sin(10^8 t - z/\sqrt{3})$$

(a) What is the dielectric constant of the medium?

(b) What type of polarization does the wave have?

(c) What is the corresponding H-field?