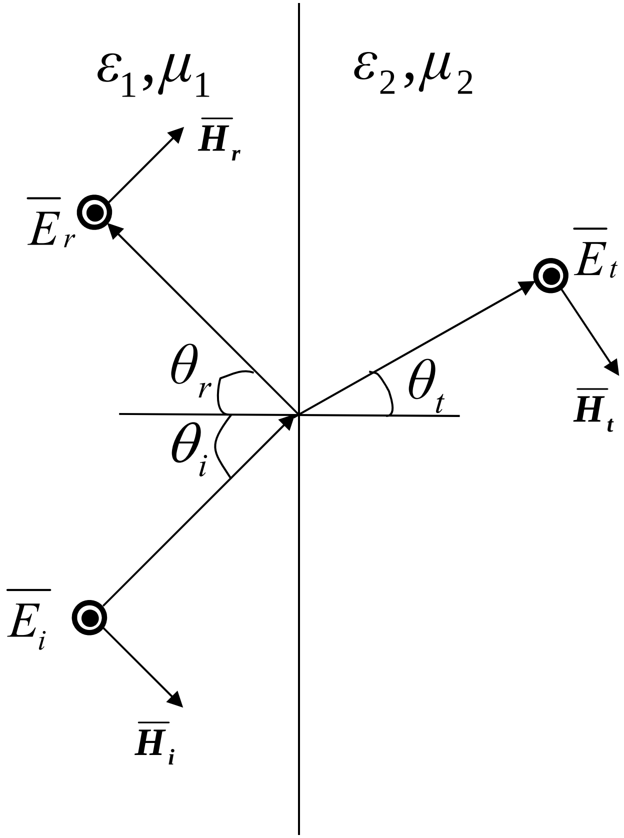
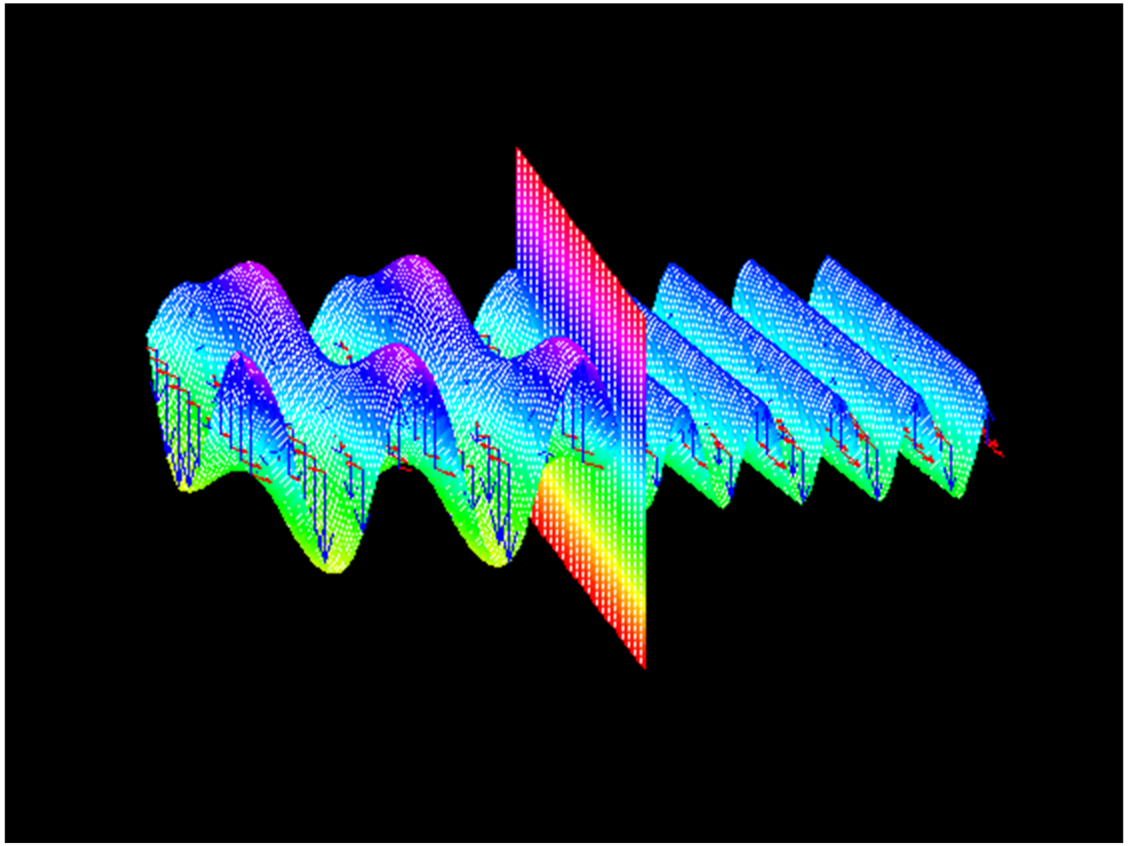
Lect. 4: Dielectric Interface (Cheng 8-10)

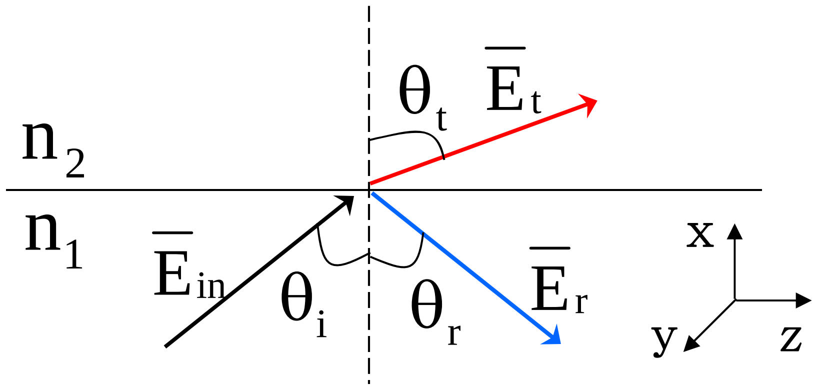




$4\varepsilon_1 = \varepsilon_2,\ \mu_1 = \mu_2$ and $\theta_i = 20^\circ$

Lect. 4: Dielectric Interface

(Assume E_{in} has only y-component: Perpendicular Polarization)



$$\overline{E}_{in} = \overline{y} E_i e^{-jk_x \cdot x} e^{-jk_z \cdot z}$$

$$\overline{E}_r = \overline{y} E_r e^{jk_{rx} \cdot x} e^{-jk_{rz} \cdot z}$$

$$\overline{E}_t = \overline{y} E_t e^{-jk_{tx} \cdot x} e^{-jk_{tz} \cdot z}$$

$$k_x = n_1 k_0 \cos\theta_i,\ k_z = n_1 k_0 \sin\theta_i$$

$$k_{rx} = n_1 k_0 \cos\theta_r, k_{rz} = n_1 k_0 \sin\theta_r$$

$$k_{tx} = n_2 k_0 \cos\theta_t, k_{tz} = n_2 k_0 \sin\theta_t$$

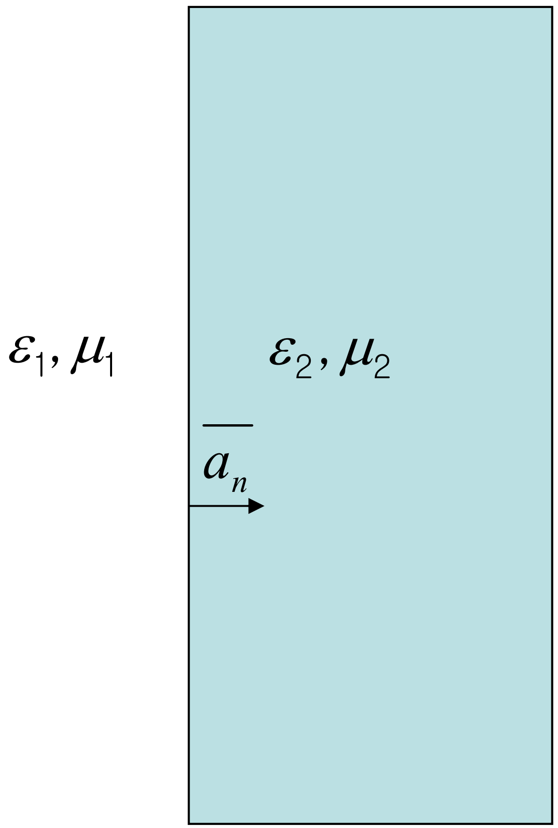
Unknowns?

$$\theta_r, \theta_t, E_r, E_t$$

How to solve for Unknows?

Lect. 4: Dielectric Interface

Boundary Conditions: Constraints on E,H fields at a boundary.
Each Maxwell's Eq. provides one constraint on E or H.



$$\nabla \bullet \overline{D} = \rho,$$

$$D_{2,n} - D_{1,n} = \rho_s \Leftrightarrow \boxed{\varepsilon_2 E_{2,n} - \varepsilon_1 E_{1,n} = \rho_s}$$

$$\nabla \times \overline{E} = -\frac{d\overline{B}}{dt},$$

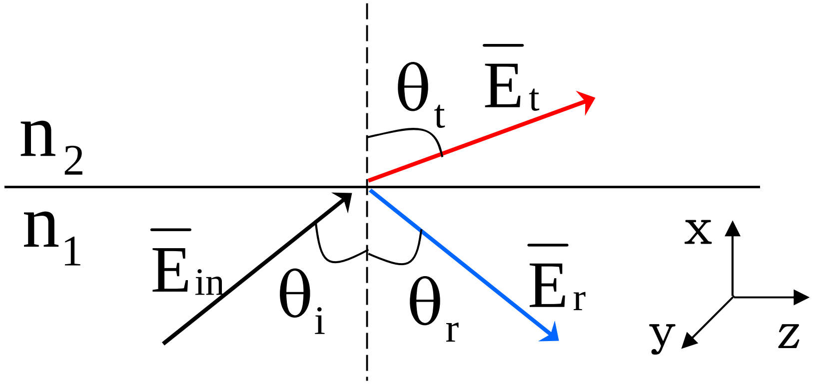
$$\boxed{E_{2,t} - E_{1,t} = 0}$$

$$\nabla \bullet \overline{B} = 0,$$

$$B_{2,n} - B_{1,n} = 0 \Leftrightarrow \boxed{\mu_2 H_{2,n} - \mu_1 H_{1,n} = 0}$$

$$\nabla \times \overline{H} = \overline{J} + \frac{d\overline{D}}{dt},$$

$$\boxed{\overline{a}_n \times (\overline{H}_2 - \overline{H}_1) = \overline{J}_s} \Leftrightarrow H_{2,t} - H_{1,t} = J_s$$



$$\overline{E}_{in} = \overline{y}E_i e^{-jk_x \cdot x} e^{-jk_z \cdot z}$$

$$\overline{E}_r = \overline{y}E_r e^{jk_{rx} \cdot x} e^{-jk_{rz} \cdot z}$$

$$\overline{E}_t = \overline{y}E_t e^{-jk_{tx} \cdot x} e^{-jk_{tz} \cdot z}$$

$$k_z = n_1 k_0 \sin\theta_i$$

$$k_{rz} = n_1 k_0 \sin\theta_r$$

$$k_{tz} = n_2 k_0 \sin\theta_t$$

Applying BC on E at $x = 0$,

$$E_i e^{-jk_z \cdot z} + E_r e^{-jk_{rz} \cdot z} = E_t e^{-jk_{tz} \cdot z}$$

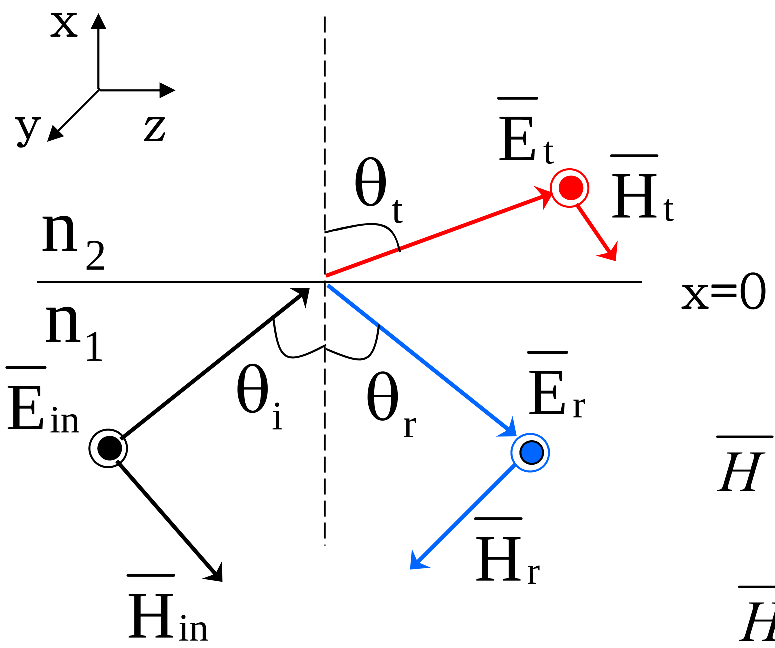
=> $k_z = k_{rz} = k_{tz}$ and $E_i + E_r = E_t$

$\therefore n_1 \sin\theta_i = n_1 \sin\theta_r = n_2 \sin\theta_t$

➔ Continuity of k_{tan}

=> $\boxed{\theta_i = \theta_r \text{ and } n_1 \sin\theta_i = n_2 \sin\theta_t \quad \text{(Snell's Law)}}$

Lect. 4: Dielectric Interface



$$\bar{E}_{in} = \bar{y}E_i e^{-jk_x \cdot x} e^{-jk_z \cdot z}$$

$$\bar{E}_r = \bar{y}E_r e^{jk_{rx} \cdot x} e^{-jk_{rz} \cdot z}$$

$$\bar{E}_t = \bar{y}E_t e^{-jk_{tx} \cdot x} e^{-jk_{tz} \cdot z}$$

$$\bar{H}_{in} = \left(-\bar{x}\sin\theta_i + \bar{z}\cos\theta_i\right) E_i/\eta_1 \; e^{-jk_x \cdot x} e^{-jk_z \cdot z}$$

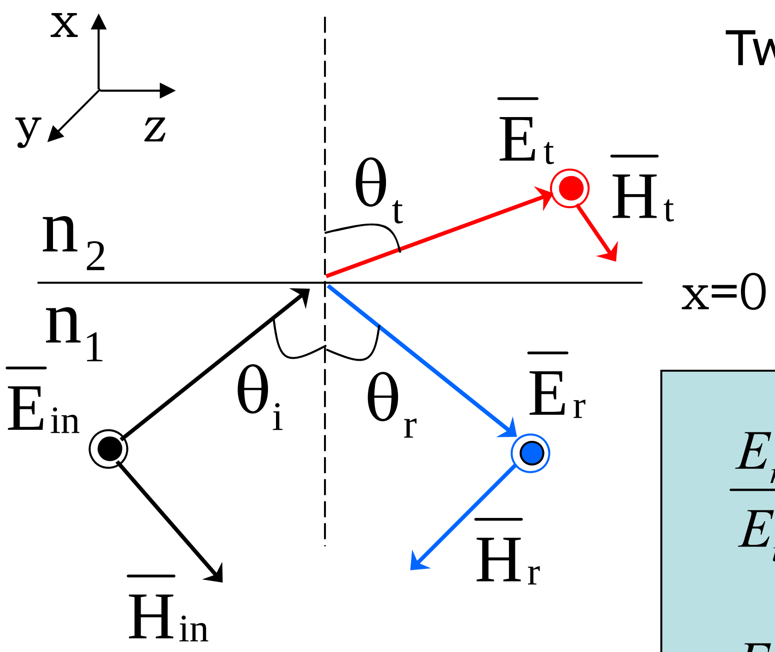
$$\bar{H}_r = \left(-\bar{x}\sin\theta_i - \bar{z}\cos\theta_i\right) E_r/\eta_1 \; e^{+jk_{rx} \cdot x} e^{-jk_{rz} \cdot z}$$

$$\bar{H}_t = \left(-\bar{x}\sin\theta_t + \bar{z}\cos\theta_t\right) E_t/\eta_2 \; e^{-jk_{tx} \cdot x} e^{-jk_{tz} \cdot z}$$

Applying BC on H at x=0

$$\left(\cos\theta_i\right) E_i/\eta_1 + \left(-\cos\theta_i\right) E_r/\eta_1 = \left(\cos\theta_t\right) E_t/\eta_2$$

Lect. 4: Dielectric Interface



Two simultaneous equations:

$$E_i + E_r = E_t$$

$$(\cos\theta_i)\frac{E_i}{\eta_1} + (-\cos\theta_i)\frac{E_r}{\eta_1} = (\cos\theta_t)\frac{E_t}{\eta_2}$$

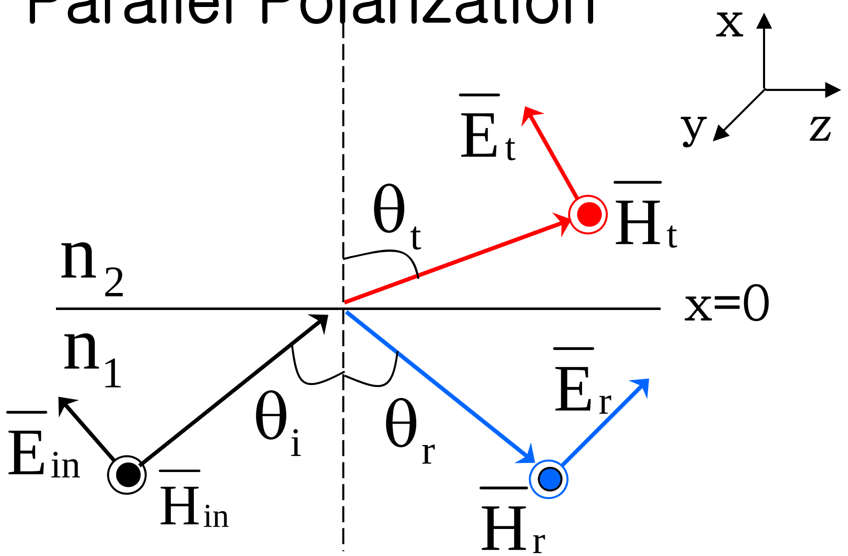
$$\frac{E_r}{E_i} = r_\perp = \frac{\cos\theta_i - \left[n^2 - \sin^2\theta_i\right]^{1/2}}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}}$$

$$\frac{E_t}{E_i} = t_\perp = \frac{2\cos\theta_i}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}} \qquad \left(n = \frac{n_2}{n_1}\right)$$

$\theta_r, \theta_t, E_r, E_t$ are all solved!

Lect. 4: Dielectric Interface

Parallel Polarization



$$\overline{H}_i = \overline{y} H_i e^{-jk_x \cdot x} e^{-jk_z \cdot z}$$

$$\overline{H}_r = \overline{y} H_r e^{+jk_{rx} \cdot x} e^{-jk_{rz} \cdot z}$$

$$\overline{H}_t = \overline{y} H_t e^{-jk_{tx} \cdot x} e^{-jk_{tz} \cdot z}$$

$$k_z = n_1 k_0 \sin\theta_i$$

$$k_{rz} = n_1 k_0 \sin\theta_r$$

$$k_{tz} = n_2 k_0 \sin\theta_t$$

$$\overline{E}_i = \left(\overline{x}\sin\theta_i - \overline{z}\cos\theta_i\right)\cdot \eta_1 H_i e^{-jk_x \cdot x} e^{-jk_z \cdot z}$$

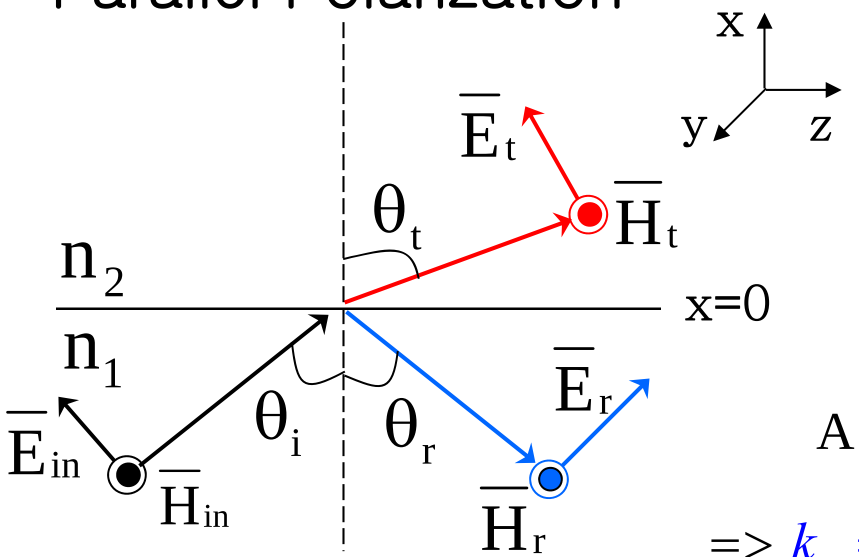
$$\theta_r, \theta_t, H_r, H_t = ???$$

$$\overline{E}_r = \left(\overline{x}\sin\theta_r + \overline{z}\cos\theta_r\right)\cdot \eta_1 H_r e^{+jk_{rx} \cdot x} e^{-jk_{rz} \cdot z}$$

$$\overline{E}_t = \left(\overline{x}\sin\theta_t - \overline{z}\cos\theta_t\right)\cdot \eta_2 H_t e^{-jk_{tx} \cdot x} e^{-jk_{tz} \cdot z}$$

Lect. 4: Dielectric Interface

Parallel Polarization



$$\overline{H}_i = \overline{y} H_i e^{-jk_x \cdot x} e^{-jk_z \cdot z}$$

$$\overline{H}_r = \overline{y} H_r e^{+jk_{rx} \cdot x} e^{-jk_{rz} \cdot z}$$

$$\overline{H}_t = \overline{y} H_t e^{-jk_{tx} \cdot x} e^{-jk_{tz} \cdot z}$$

Applying BC on H at $x = 0$,

=> $k_z = k_{rz} = k_{tz}$ and $H_i + H_r = H_t$

$$k_z = n_1 k_0 \sin\theta_i$$

$$k_{rz} = n_1 k_0 \sin\theta_r$$

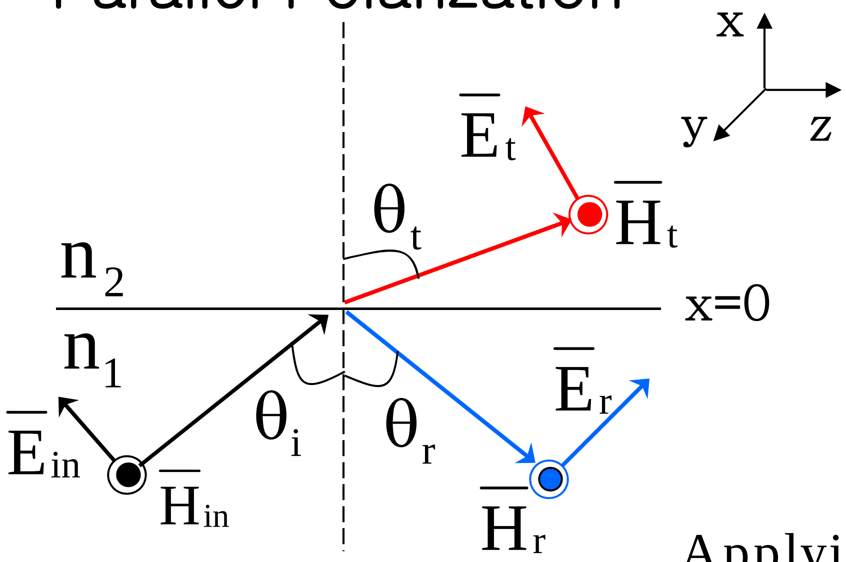
$$k_{tz} = n_2 k_0 \sin\theta_t$$

$$\therefore n_1 \sin\theta_i = n_1 \sin\theta_r = n_2 \sin\theta_t$$

=> $\boxed{\theta_i = \theta_r \text{ and } n_1 \sin\theta_i = n_2 \sin\theta_t \text{ (Snell's Law)}}$

Lect. 4: Dielectric Interface

Parallel Polarization



$$\overline{E}_{in} = \left(\overline{x}\sin\theta_i - \overline{z}\cos\theta_i\right)\cdot\eta_1 H_i e^{-jk_x\cdot x} e^{-jk_z\cdot z}$$

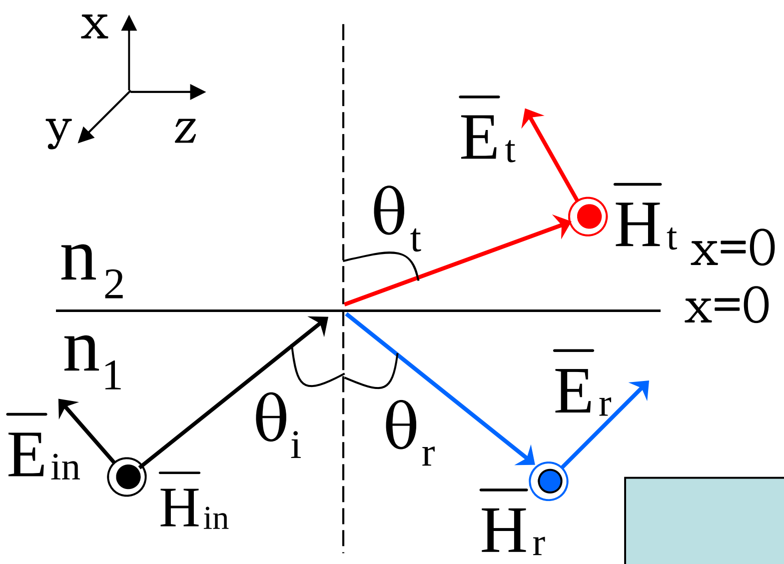
$$\overline{E}_{r} = \left(\overline{x}\sin\theta_i + \overline{z}\cos\theta_i\right)\cdot\eta_1 H_r e^{+jk_{rx}\cdot x} e^{-jk_{rz}\cdot z}$$

$$\overline{E}_{t} = \left(\overline{x}\sin\theta_t - \overline{z}\cos\theta_t\right)\cdot\eta_2 H_t e^{-jk_{tx}\cdot x} e^{-jk_{tz}\cdot z}$$

Applying BC on E at $x = 0$,

$$-\cos\theta_i\cdot\eta_1 H_i + \cos\theta_i\cdot\eta_1 H_r = -\cos\theta_t\cdot\eta_2 H_t$$

Lect. 4: Dielectric Interface



Two simultaneous equations:

$$H_i + H_r = H_t$$

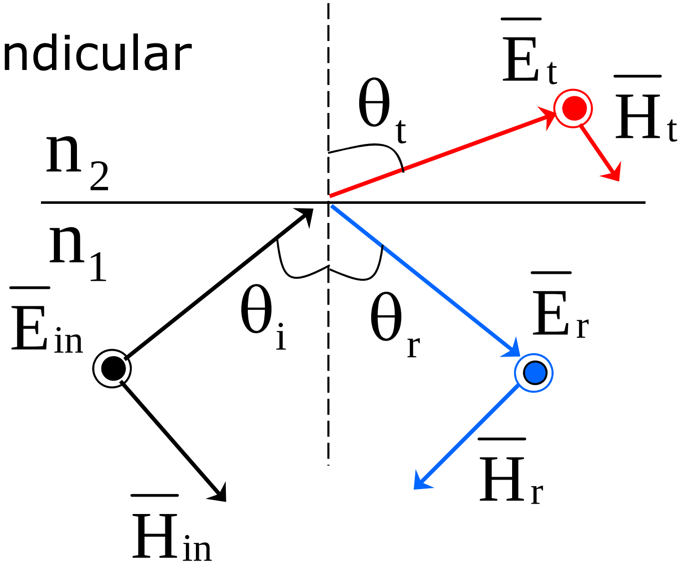
$$-\cos\theta_i \cdot \eta_1 H_i + \cos\theta_i \cdot \eta_1 H_r = -\cos\theta_t \cdot \eta_2 H_t$$

$$r_\parallel = -\frac{H_r}{H_i} = \frac{\left[n^2 - \sin^2\theta_i\right]^{1/2} - n^2\cos\theta_i}{\left[n^2 - \sin^2\theta_i\right]^{1/2} + n^2\cos\theta_i}$$

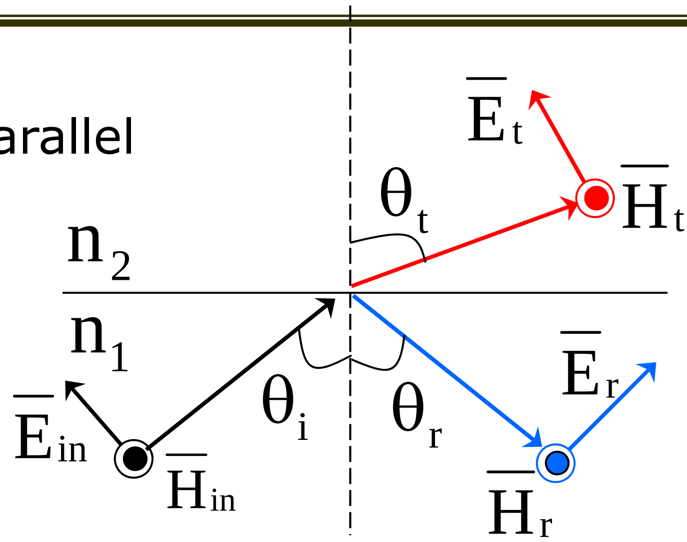
$$t_\parallel = \frac{\eta_2 H_t}{\eta_1 H_i} = \frac{2n\cos\theta_i}{\left[n^2 - \sin^2\theta_i\right]^{1/2} + n^2\cos\theta_i} \qquad (n = \frac{n_2}{n_1})$$

Lect. 4: Dielectric Interface

Perpendicular



Parallel



$$r_\perp = \frac{\cos\theta_i - \left[n^2 - \sin^2\theta_i\right]^{1/2}}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}}$$

$$t_\perp = \frac{2\cos\theta_i}{\cos\theta_i + \left[n^2 - \sin^2\theta_i\right]^{1/2}}$$

For $\theta_i = 0$,

$$r_\perp = r_\parallel = \frac{1-n}{1+n}$$

$$t_\perp = t_\parallel = \frac{2}{1+n}$$

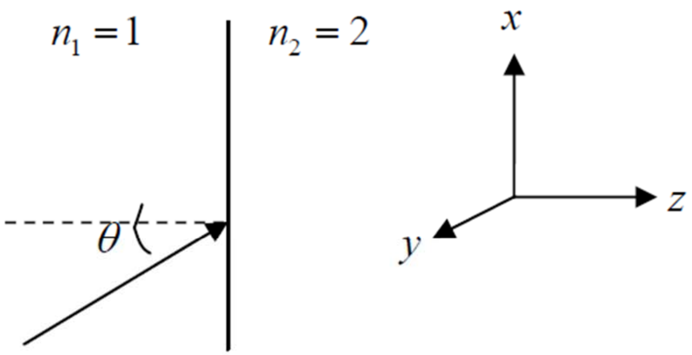
$$r_\parallel = \frac{\left[n^2 - \sin^2\theta_i\right]^{1/2} - n^2\cos\theta_i}{\left[n^2 - \sin^2\theta_i\right]^{1/2} + n^2\cos\theta_i}$$

$$t_\parallel = \frac{2n\cos\theta_i}{\left[n^2 - \sin^2\theta_i\right]^{1/2} + n^2\cos\theta_i}$$

Lect. 4: Dielectric Interface

Homework

A plane wave whose electric field is given as $\overline{E} = (\bar{x} + \bar{y} - \bar{z})$ exp(−jk_xx −jk_zz) is incident on a dielectric interface as shown below.



(a) Show that the incident angle q is 45 degrees. (Hint: Use Maxwell's Equations)

(b) Determine the expression for the reflected E-field.

(Hint: Decompose the electric field into the perpendicular component and the parallel component)