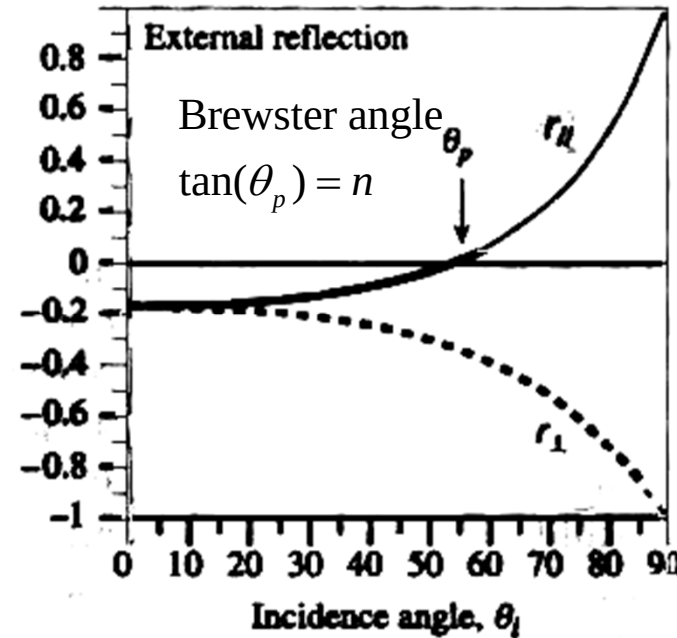
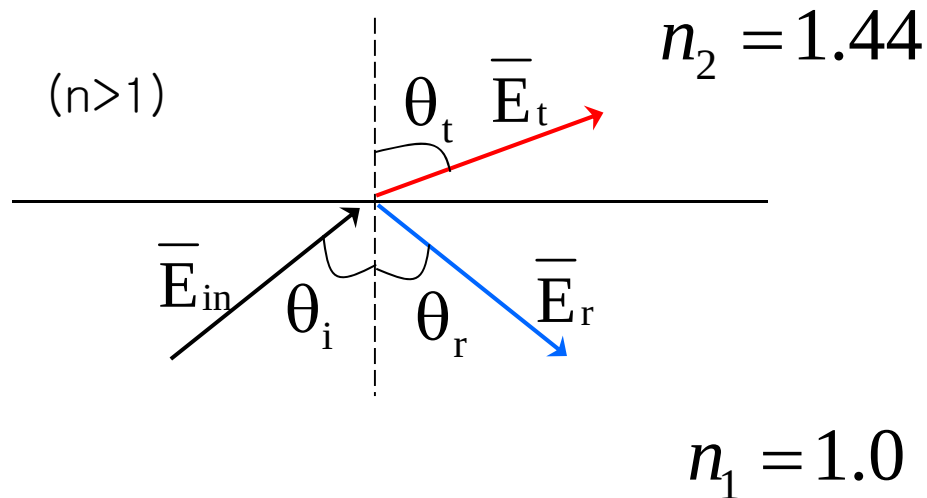


Lect. 5: Total Internal Reflection



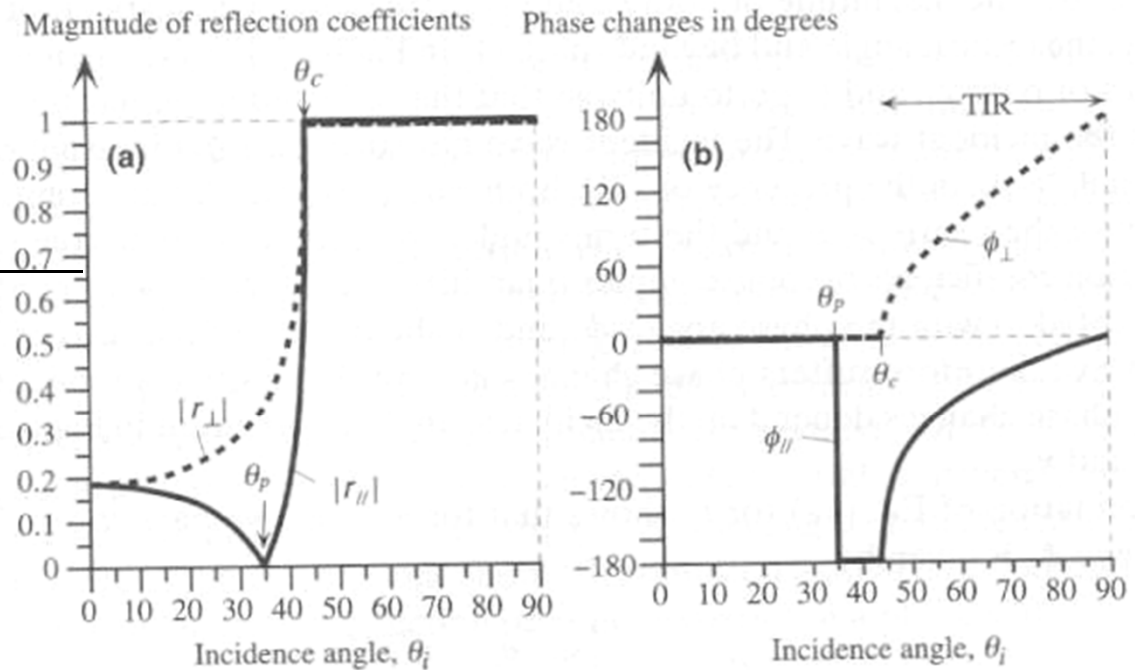
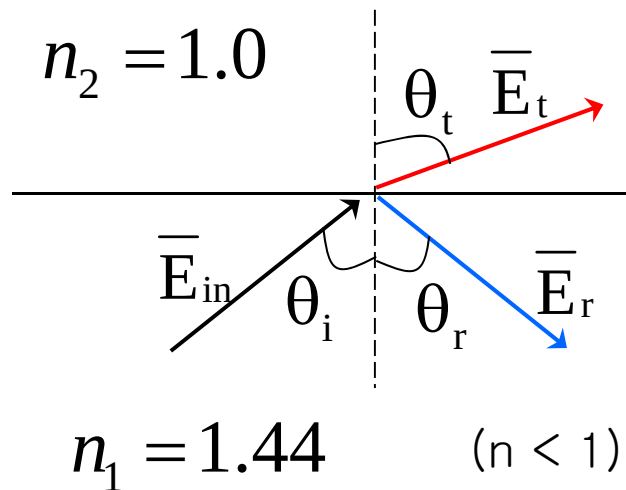
$$r_{\perp} = \frac{E_r}{E_i} = \frac{\cos \theta_i - [n^2 - \sin^2 \theta_i]^{1/2}}{\cos \theta_i + [n^2 - \sin^2 \theta_i]^{1/2}}$$

$$r_{\parallel} = \frac{[n^2 - \sin^2 \theta_i]^{1/2} - n^2 \cos \theta_i}{[n^2 - \sin^2 \theta_i]^{1/2} + n^2 \cos \theta_i}$$

$$t_{\perp} = \frac{E_t}{E_i} = \frac{2 \cos \theta_i}{\cos \theta_i + [n^2 - \sin^2 \theta_i]^{1/2}} \quad (n = \frac{n_2}{n_1})$$

$$t_{\parallel} = \frac{2n \cos \theta_i}{[n^2 - \sin^2 \theta_i]^{1/2} + n^2 \cos \theta_i}$$

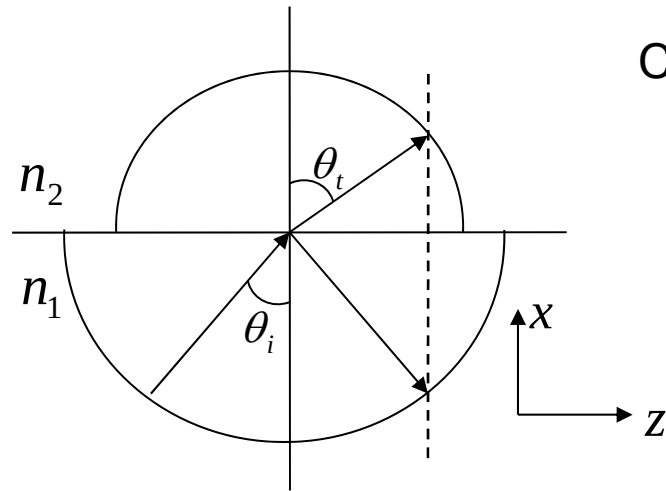
Lect. 5: Total Internal Reflection



What is θ_c (critical angle)?

What happens when $\theta_i > \theta_c$?

Lect. 5: Total Internal Reflection



Consider k-vector diagram:

- Direction: Direction of wave propagation
- Magnitude: $\omega\sqrt{\mu\epsilon}$
- As θ_i changes, k_i , k_r , k_t traces on a circle

From Lect. 4, $k_{i,z} = k_{r,z} = k_{t,z}$

$$n_1 k_0 \sin \theta_i = n_2 k_0 \sin \theta_t \quad \rightarrow \quad n_1 \sin \theta_i = n_2 \sin \theta_t$$

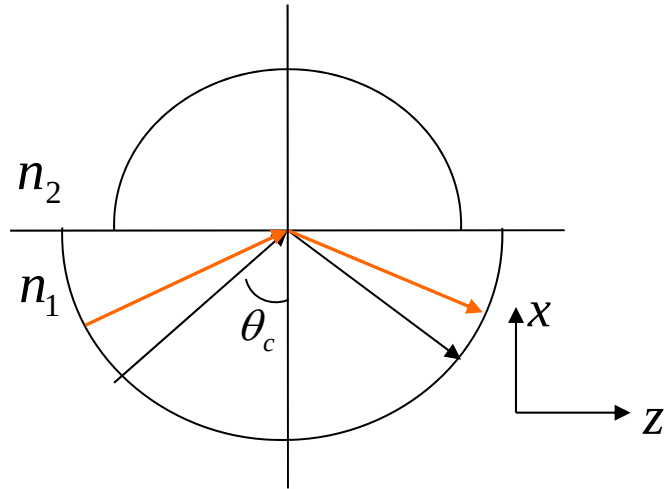
As θ_i increases, θ_t increases

θ_t reaches 90 deg before θ_i

$$n_1 \sin \theta_i = n_2 \quad \theta_c \text{ (critical angle)} = \sin^{-1} \left(\frac{n_2}{n_1} \right) = \sin^{-1}(n)$$

What if $\theta_i > \theta_c$?

Lect. 5: Total Internal Reflection



What happens if $\theta_i > \theta_c$?

$$k_t^2 = k_{t,z}^2 + k_{t,x}^2$$

$$k_{t,x}^2 = k_t^2 - k_{t,z}^2 = (n_2 k_0)^2 - (n_1 k_0 \sin \theta_i)^2 < 0$$

$$(\because k_{t,z} = k_{i,z} = n_1 k_0 \sin \theta_i)$$

$$\therefore k_{t,x} = -j\alpha$$

Attenuation in x-direction!

$$\therefore E_t = tE_i e^{-jk_{tx} \cdot x} e^{-jk_{tz} \cdot z} = tE_i e^{(-\alpha x)} e^{(-jk_{t,z} z)}$$

→ Total internal reflection

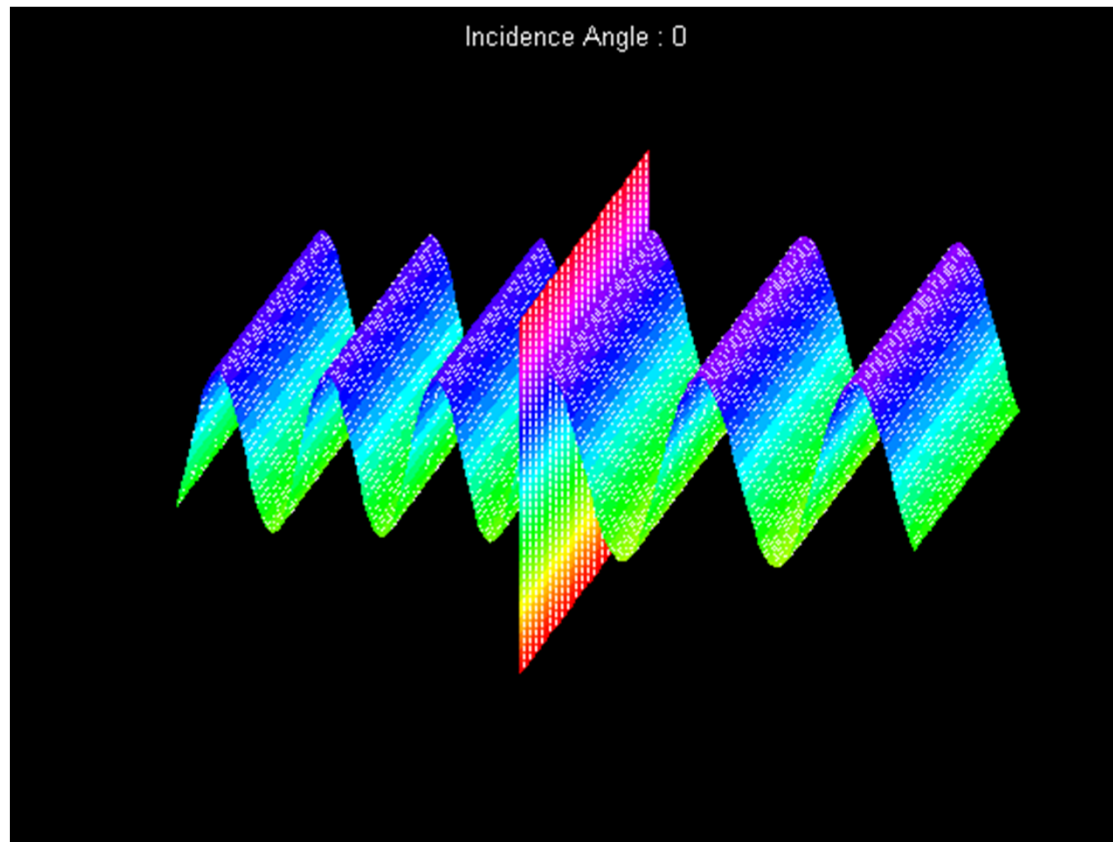
$$\alpha = ? \quad \text{From } k_t^2 = k_{t,z}^2 + k_{t,x}^2, \quad (n_2 k_0)^2 = (n_1 k_0 \sin \theta_i)^2 - \alpha^2$$

$$\alpha^2 = (n_1 k_0 \sin \theta_i)^2 - (n_2 k_0)^2 \quad \therefore \alpha = k_0 [(n_1 \sin \theta_i)^2 - n_2^2]^{\frac{1}{2}}$$

Lect. 5: Total Internal Reflection

$\varepsilon_1 = 2\varepsilon_2$, $\mu_1 = \mu_2$ and θ_i : from 0° to 90°

Incident and Transmitted Waves for perpendicular polarization



Lect. 5: Total Internal Reflection

What is $r_{\perp}$, $t_{\perp}$? for total internal reflection

$$\text{From } r_{\perp} = \frac{\cos \theta_i - [n^2 - \sin^2 \theta_i]^{\frac{1}{2}}}{\cos \theta_i + [n^2 - \sin^2 \theta_i]^{\frac{1}{2}}}$$

$$\sin \theta_i > n \quad (\because \sin \theta_i > \sin \theta_c = n) \quad \text{Let } [n^2 - \sin^2 \theta_i]^{\frac{1}{2}} = -j[\sin^2 \theta_i - n^2]^{\frac{1}{2}}$$

$$r_{\perp} = \frac{\cos \theta_i + j[\sin^2 \theta_i - n^2]^{\frac{1}{2}}}{\cos \theta_i - j[\sin^2 \theta_i - n^2]^{\frac{1}{2}}} = |r_{\perp}| e^{j\phi_{\perp}}$$

$$|r_{\perp}| = 1 \quad \phi_{\perp} = \tan^{-1}\left(\frac{(\sin^2 \theta_i - n^2)^{\frac{1}{2}}}{\cos \theta_i}\right) - (-\tan^{-1}\left(\frac{(\sin^2 \theta_i - n^2)^{\frac{1}{2}}}{\cos \theta_i}\right)) = 2\tan^{-1}\left(\frac{(\sin^2 \theta_i - n^2)^{\frac{1}{2}}}{\cos \theta_i}\right)$$

For $t_{\perp}$, use $E_i + E_r = E_t$ or $1 + r_{\perp} = t_{\perp}$

$$\Rightarrow t_{\perp} = 1 + r_{\perp}$$

Lect. 5: Total Internal Reflection

What is $r_{\parallel}$, $t_{\parallel}$? for total internal reflection

$$\text{From } r_{\parallel} = \frac{[n^2 - \sin^2 \theta_i]^{1/2} - n^2 \cos \theta_i}{[n^2 - \sin^2 \theta_i]^{1/2} + n^2 \cos \theta_i}$$

$$\sin \theta_i > n \quad (\because \sin \theta_i > \sin \theta_c = n) \quad \text{Let } [n^2 - \sin^2 \theta_i]^{1/2} = -j[\sin^2 \theta_i - n^2]^{1/2}$$

$$r_{\parallel} = \frac{-j[\sin^2 \theta_i - n^2]^{1/2} - n^2 \cos \theta_i}{-j[\sin^2 \theta_i - n^2]^{1/2} + n^2 \cos \theta_i} = |r_{\parallel}| e^{j\phi_{\parallel}}$$

$$|r_{\parallel}| = 1 \text{ and } \phi_{\parallel} = -\pi + \tan^{-1}\left(\frac{(\sin^2 \theta_i - n^2)^{1/2}}{n^2 \cos \theta_i}\right) - (-\tan^{-1}\left(\frac{(\sin^2 \theta_i - n^2)^{1/2}}{n^2 \cos \theta_i}\right)) = -\pi + 2\tan^{-1}\left(\frac{(\sin^2 \theta_i - n^2)^{1/2}}{n^2 \cos \theta_i}\right)$$

$$\text{From } H_i + H_r = H_t \Rightarrow 1 + \frac{H_r}{H_i} = \frac{H_t}{H_i} \quad \text{Since } r_{\parallel} = -\frac{H_r}{H_i} \text{ and } t_{\parallel} = \frac{\eta_1 H_t}{\eta_2 H_i} \text{ (Lect. 4)}$$

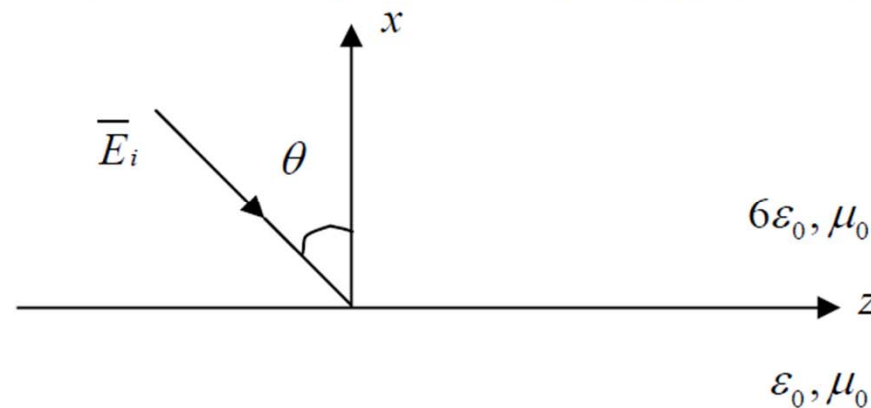
$$1 - r_{\parallel} = \frac{\eta_2}{\eta_1} t_{\parallel} = nt_{\parallel} \quad \therefore 1 - r_{\parallel} = nt_{\parallel}$$

Lect. 5: Total Internal Reflection

Homework:

A plane wave is incident at an angle $\theta=30$ degrees on a dielectric boundary as shown below. Its electric field is given as

$$\bar{E}_i = E_0 (\bar{x} \sin\theta + \bar{y} + \bar{z} \cos\theta) \exp(jk_x x - jk_z z).$$



- (a) What is the polarization of the incident wave?
- (b) It is observed that the polarization of the reflected wave polarization is circular. Explain how this can happen.