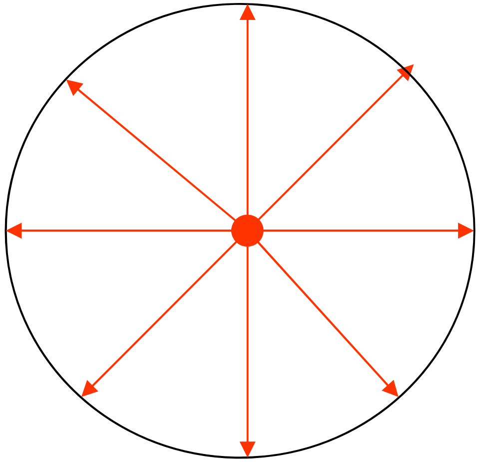
Lect. 6: Interference

Consider isotropic EM wave radiation by a point source.

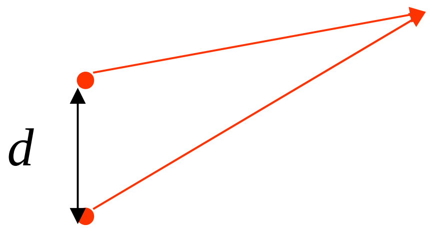


$$E \sim \frac{1}{R} e^{-jkR} \text{ (Spherical wave)}$$

Why $\frac{1}{R}$ dependence?

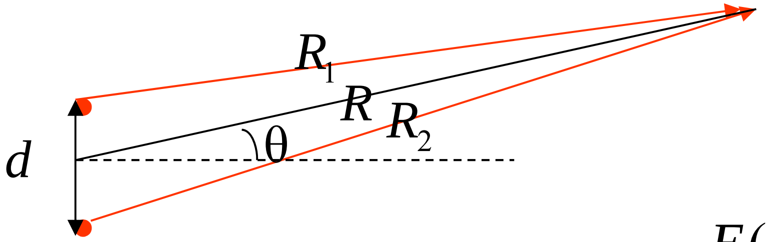
because $\int |E|^2 R^2 \sin\theta d\theta d\phi$ should be constant.

Two point sources separated by d



$$E_{\text{total}} = ?$$

Lect. 6: Interference



$$E(R,\theta,\phi) = E_1 + E_2$$

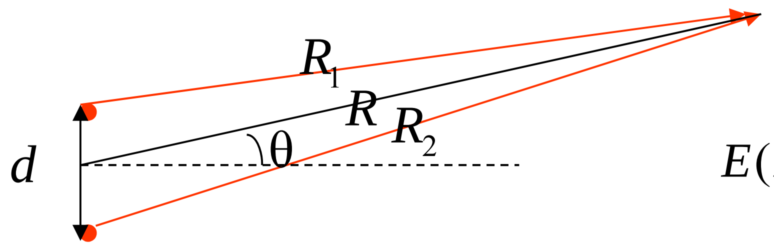
$$= \frac{A}{R_1} e^{-jkR_1} + \frac{A}{R_2} e^{-jkR_2}$$

Assume $R >> d$ $\quad R_1 \approx R - \frac{d}{2}\sin\theta \qquad R_2 \approx R + \frac{d}{2}\sin\theta$

$$\therefore E(R,\theta) \approx \frac{A}{R} e^{-jk(R-\frac{d}{2}\sin\theta)} + \frac{A}{R} e^{-jk(R+\frac{d}{2}\sin\theta)}$$

$$= \frac{A}{R} e^{-jkR} (e^{jk\frac{d}{2}\sin\theta} + e^{-jk\frac{d}{2}\sin\theta}) \quad = \frac{2A}{R} e^{-jkR} \cos(k\frac{d}{2}\sin\theta)$$

Lect. 6: Interference



$$E(R,\theta) \approx \frac{2A}{R} e^{-jkR} \cos(k\frac{d}{2}\sin\theta)$$

$$I\text{(Intensity)} : |E|^2 = 4(\frac{A}{R})^2 \cos^2(k\frac{d}{2}\sin\theta)$$

=> There exist max. and min. intensity conditions:

For max., $k\frac{d}{2}\sin\theta = m\pi \Rightarrow I = \left(\frac{2A}{R}\right)^2$

phase difference $=2m\pi$; In Phase

length difference: $d\sin\theta = m\lambda$

➔ Constructive interference

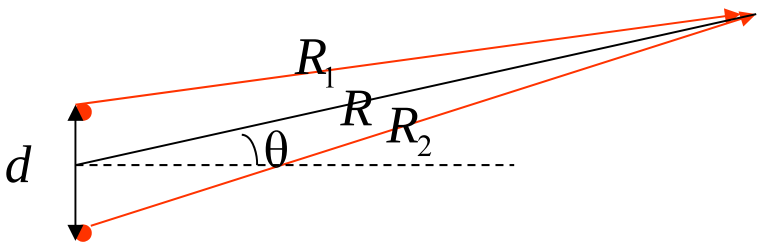
For min., $k\frac{d}{2}\sin\theta = (m+\frac{1}{2})\pi \Rightarrow I = 0$

Phase difference $=(2m+1)\pi$; Out of Phase

length difference: $d\sin\theta = (m+\frac{1}{2})\lambda$

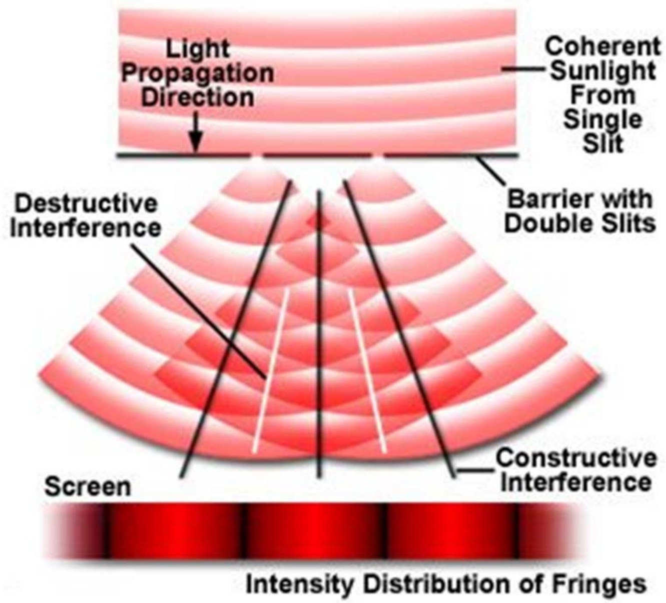
➔ Destructive interference

Lect. 6: Interference

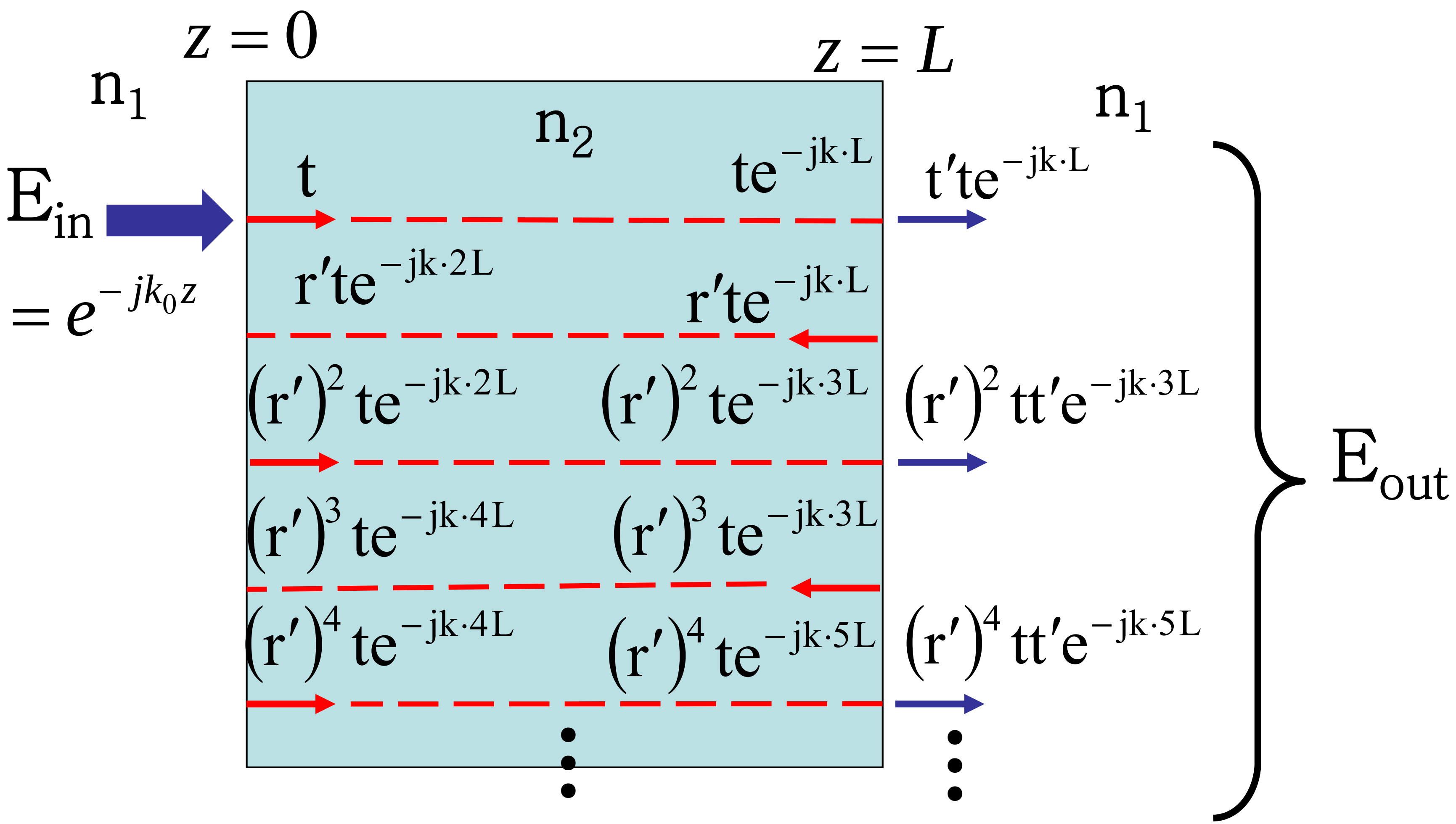


$$I\,(\text{Intensity}) = |E|^2 = 4(\frac{A}{R})^2 \cos^2(k\frac{d}{2}\sin\theta)$$

Double Slit Interference



Interference in a dielectric slab (etalon)



Lect. 6: Interference

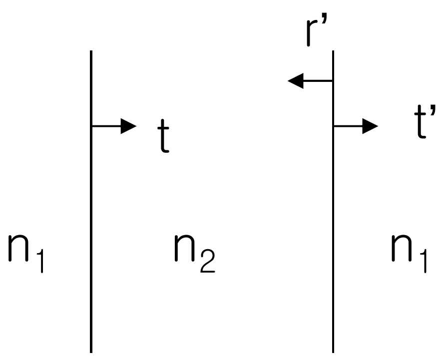
$$E_{out} = E_{t,total} = tt'e^{-jk\cdot L} + (r')^2 tt'e^{-jk\cdot 3L} + (r')^4 tt'e^{-jk\cdot 5L} + \bullet\bullet\bullet = \frac{tt'e^{-jk\cdot L}}{1-(r')^2 e^{-j2kL}}$$

$$T = \frac{|E_t|^2}{|E_i|^2} = \frac{(tt')^2}{[1-(r')^2 e^{-j2kL}][1-(r')^2 e^{j2kL}]} = \frac{(tt')^2}{[1-(r')^2]^2 + 4r'^2 \sin^2(kL)}$$

$$\begin{aligned}[1-(r')^2 e^{-j2kL}][1-(r')^2 e^{j2kL}] &= 1-(r')^2 e^{j2kL} - (r')^2 e^{-j2kL} + (r')^4 \\ &= 1-2(r')^2\cos(2kL) + (r')^4 \\ &= 1-2(r')^2(1-2\sin^2 kL) + (r')^4 \\ &= [1-(r')^2]^2 + 4(r')^2\sin^2(kL)\end{aligned}$$

Lect. 6: Interference

$$T = \frac{|E_t|^2}{|E_i|^2} = \frac{(tt')^2}{[1-(r')^2]^2 + 4r'^2 \sin^2(kL)}$$



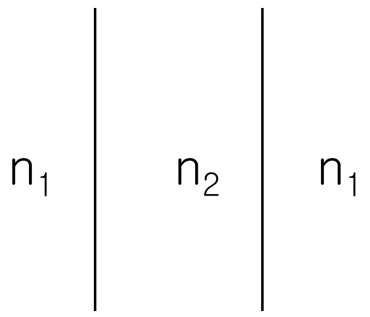
$$t = \frac{2n_1}{n_1 + n_2}, \quad r' = \frac{n_2 - n_1}{n_2 + n_1}, \quad t' = \frac{2n_2}{n_1 + n_2}$$

$$\therefore tt' = \frac{4n_1 n_2}{(n_1 + n_2)^2}, \quad 1 - r'^2 = \frac{4n_1 n_2}{(n_2 + n_1)^2}$$

$$\text{Let } R = r'^2 = r^2$$

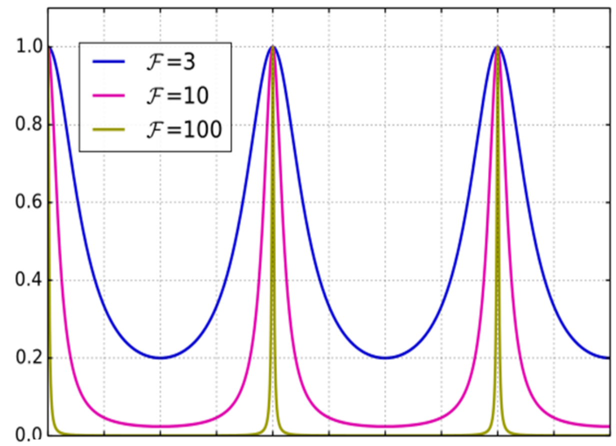
$$T = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(kL)} = \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2(kL)}$$

Lect. 6: Interference



$$T = \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2(kL)}$$

Fabry-Perot Interference



$$\text{Max. Transmission: } \sin(kL) = 0 \quad => T = 1$$

$$kL = m\pi; \; n_2 \frac{2\pi}{\lambda} L = m\pi \qquad => L = m\frac{\lambda}{2n_2} \; \text{(half wavelength)}$$

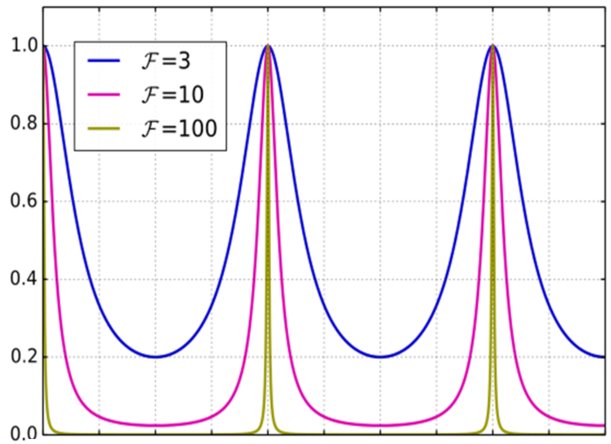
$$\text{Min. Transmission: } \sin(kL) = 1$$

$$kL = (m + \frac{1}{2})\pi; \; n_2 \frac{2\pi}{\lambda} L = (m + \frac{1}{2})\pi \qquad => L = \frac{\lambda}{2n_2}(m + \frac{1}{2}) \; \text{(quarter wavelength)}$$

Lect. 6: Interference

n$_1$ | n$_2$ | n$_1$

$$T = \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2(kL)}$$



Period? ➔ Free Spectral Range

$$\Delta kL = \pi \qquad \Delta k = \frac{\pi}{L}$$

$$\Delta\omega = ? \qquad k = n_2\frac{\omega}{c} \qquad \Delta\omega = \frac{c}{n_2}\Delta k = \frac{c}{n_2}\frac{\pi}{L} \qquad \Delta f = \frac{c}{2n_2L} = \frac{1}{T} \qquad T = \frac{2L}{c/n_2}$$

(round-trip time)

$$\Delta\lambda = ? \qquad \lambda = n_2\frac{2\pi}{k} \qquad \Delta\lambda = \frac{d\lambda}{dk}\Delta k = -n_2\frac{2\pi}{k^2}\Delta k = -\frac{\lambda^2}{2n_2L}$$

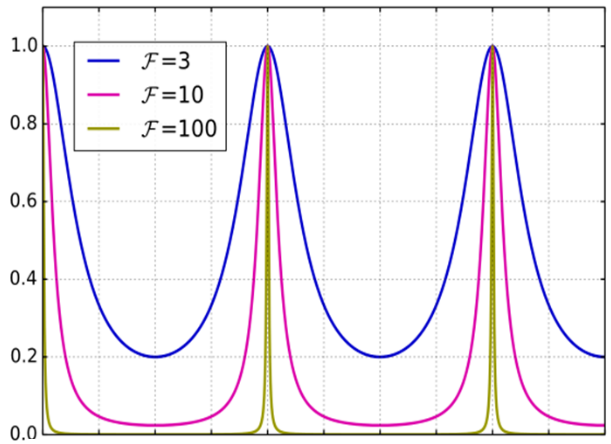
Lect. 6: Interference

$$T = \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2(kL)}$$

Sharpness (Linewidth)?

Determine k where $T = 0.5$

$$\frac{1}{2} = \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2(kL)} \qquad \frac{4R}{(1-R)^2}\sin^2(kL) = 1$$



$$kL = \sin^{-1}\sqrt{\frac{(1-R)^2}{4R}} = \sin^{-1}\frac{(1-R)}{2\sqrt{R}}$$

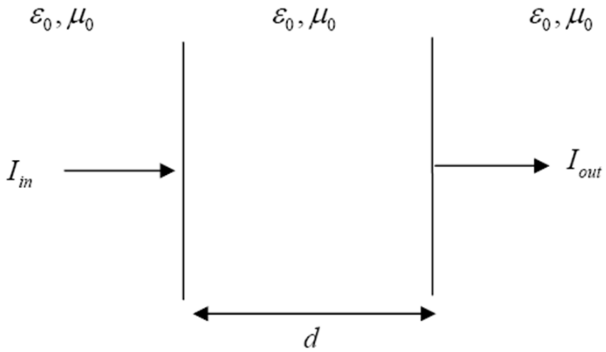
(If FWHM<<1)

FWHM (Full Width at Half Maximum) for $kL = 2\sin^{-1}\frac{(1-R)}{2\sqrt{R}} \sim \frac{(1-R)}{\sqrt{R}}$

As R increases, FWHM decreases => sharper response

Lect. 6: Interference

Homework: Consider a Fabry–Perot interferometer made of two identical partially reflecting/transmitting mirrors as shown below. The mirrors transmit and reflect half of the incident power. (Or it has r = 1/sqrt(2) and t = j/sqrt(2)).



(a) What is I_{out}/I_{in}? Give your answer as a function of sin(kd), where k is the wavenumber in the vacuum and d is the distance between two mirrors.

(b) If the output power is plotted as a function of the frequency of the input light, what is the frequency separation between two adjacent peaks? Express your answer interms of c, speed of light, d, and other fundamental parameters if required.

(c) What is the finesse of this interferometer? Give a numerical answer.

[Finesse = (Free Spectral Range)/FWHM]