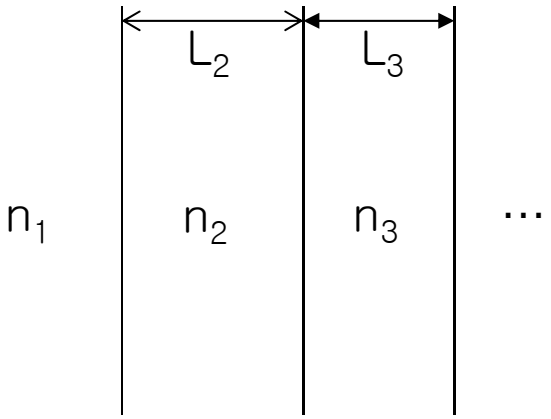
Lect. 7: Multiple Dielectric Interface



Complex Problem:

Requires an advanced technique

Consider two special cases:

1) $L_i = m\dfrac{\lambda}{2n_i}$ (Half Wavelength)

2) $L_i = (m+\dfrac{1}{2})\dfrac{\lambda}{2n_i}$ (Quarter Wavelength)

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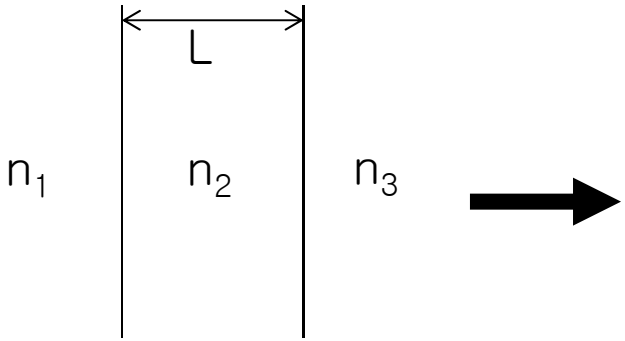
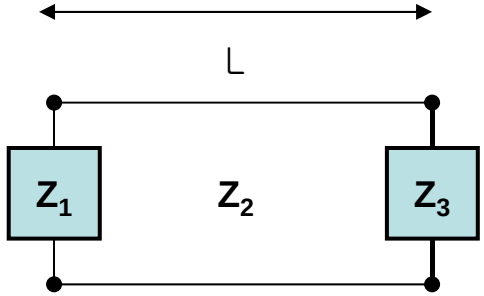
Model dielectric layers with impedance and transmission line (E&M 2)



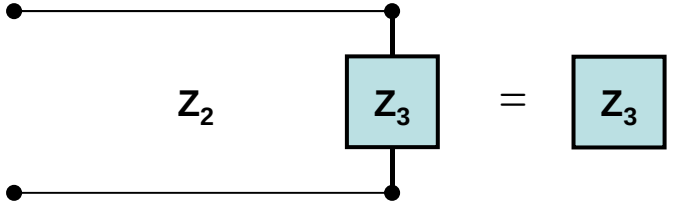


$$Z_i = \sqrt{\frac{\mu}{\varepsilon_i}} = \frac{1}{n_i}\sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$= \frac{1}{n_i}\eta_0$$

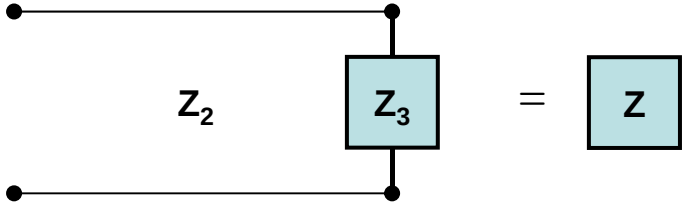
If L: half wavelength

(One rotation on Smith Chart!)



If L: quarter wavelength

(Half rotation on Smith Chart!)



$$Z_2 \frac{1}{Z_3 / Z_2} = \frac{Z_2^{\,2}}{Z_3}$$

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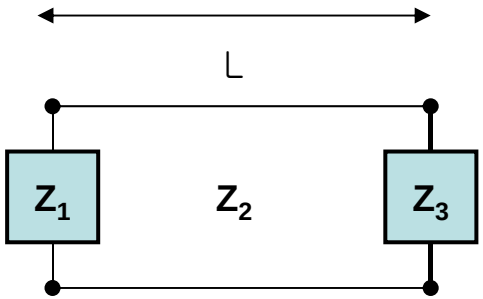

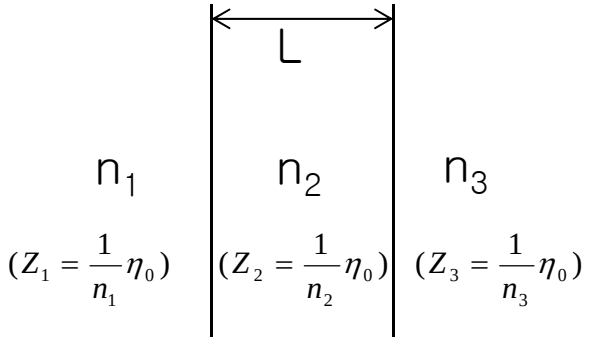

$$L = \frac{\lambda}{2n_2} \quad = \quad Z_3$$

n_1 | n_3

$$\therefore\ r = \frac{n_1 - n_3}{n_1 + n_3},\ t = \frac{2n_1}{n_1 + n_3}$$

$$L = \frac{\lambda}{4n_2} \quad = \quad Z \quad \frac{Z_2^{\,2}}{Z_3}$$

n_1 | $n = \frac{n_2^{\,2}}{n_3}$

$$r = \frac{n_1 - n_2^{\,2}/n_3}{n_1 + n_2^{\,2}/n_3},\ t = \frac{2n_1}{n_1 + n_2^{\,2}/n_3}$$

$$\therefore\ r = \frac{n_1 n_3 - n_2^{\,2}}{n_1 n_3 + n_2^{\,2}},\ t = \frac{2n_1 n_3}{n_1 n_3 + n_2^{\,2}}$$

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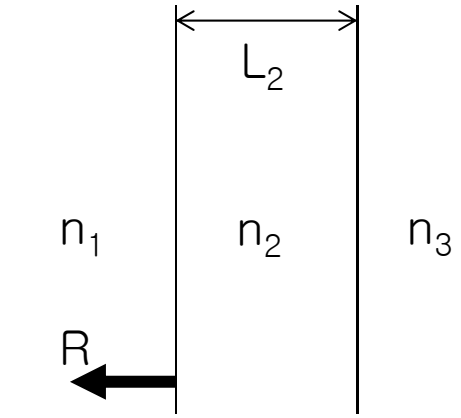
Anti-Reflection coating: Determine L_2 and n_2 so that R=0



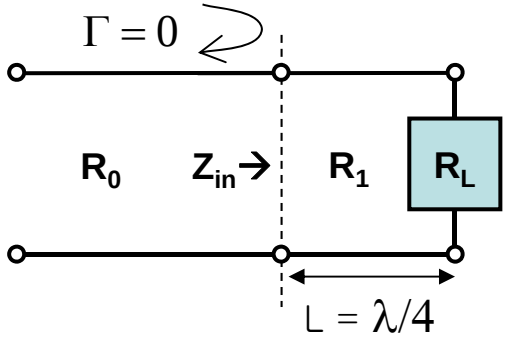
With $L_2 = \frac{\lambda}{4n_2}$

n_1 | $n = \frac{n_2^2}{n_3}$

Since $r = \frac{n_1 n_3 - n_2^2}{n_1 n_3 + n_2^2}$,

For r=0, $n_1 n_3 - n_2^2 = 0$

$n_2 = \sqrt{n_1 n_3}$



$R_1 = \sqrt{R_0 R_L}$

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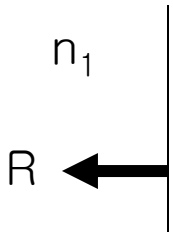
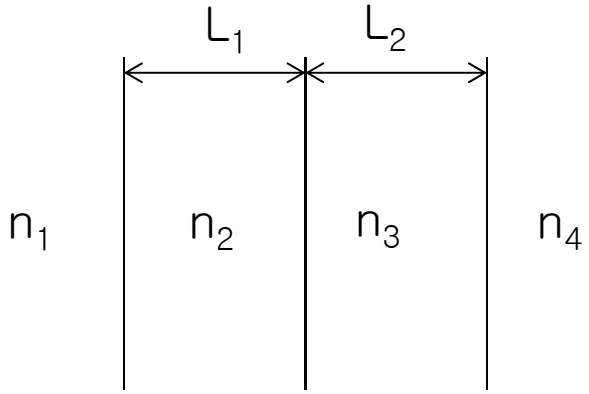
High-Reflection Coating

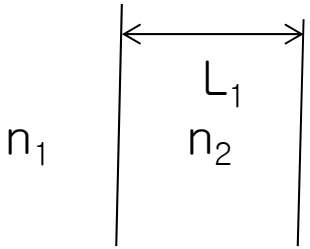


$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \qquad R \to 1 \text{ if } n_1 >> n_2 \text{ or } n_1 << n_2$$

Use quarter-wavelength layers

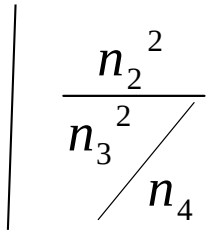


With $L_2 = \frac{\lambda}{4n_3}$



$$n = \frac{n_3^{\,2}}{n_4}$$

With $L_1 = \frac{\lambda}{4n_2}$



$$R = \left(\frac{n_1 - \left(\frac{n_2}{n_3}\right)^2 n_4}{n_1 + \left(\frac{n_2}{n_3}\right)^2 n_4}\right)^2$$

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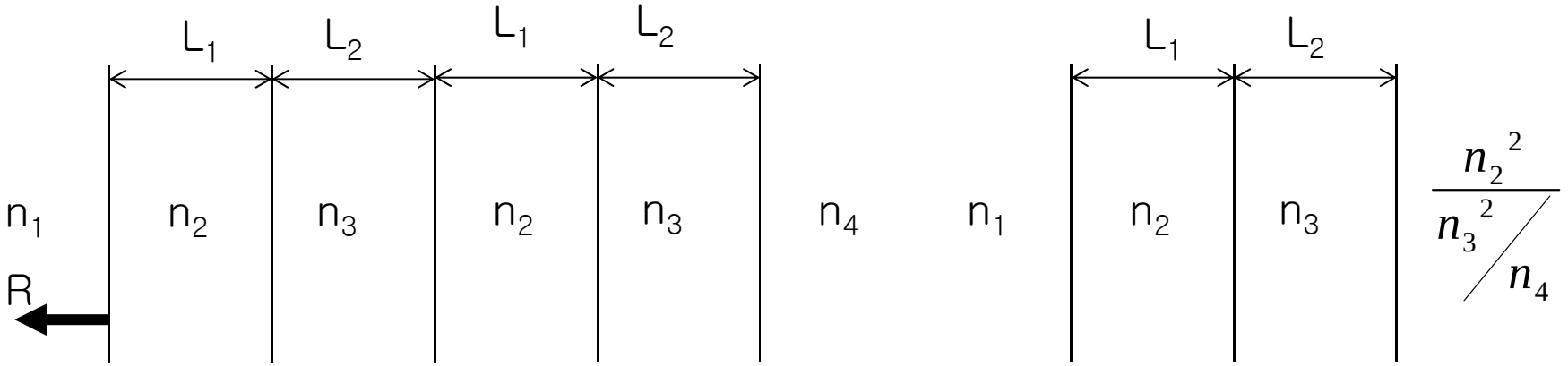
High-Reflection Coating

Repeat the quarter-wavelength pair



Substitute $\frac{n_2^{\,2}}{n_3^{\,2}/n_4}$ for n_4 in $R = \left(\frac{n_1 - \left(\frac{n_2}{n_3}\right)^2 n_4}{n_1 + \left(\frac{n_2}{n_3}\right)^2 n_4} \right)^2$

Then, $R = \left(\frac{n_1 - \left(\frac{n_2}{n_3}\right)^4 n_4}{n_1 + \left(\frac{n_2}{n_3}\right)^4 n_4} \right)^2$

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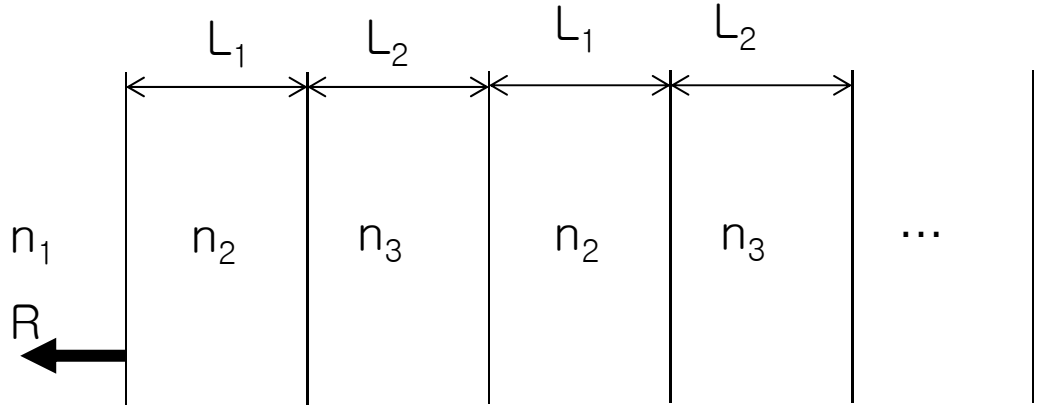
High-Reflection Coating

Repeat the quarter-wavelength pair m times.



$$R = \left(\frac{n_1 - \left(\frac{n_2}{n_3} \right)^{2m} n_4}{n_1 + \left(\frac{n_2}{n_3} \right)^{2m} n_4} \right)^2$$

When m is sufficiently large,

For $n_2 > n_3$, $R \sim \left(\frac{-(n_2/n_3)^{2m} n_4}{+(n_2/n_3)^{2m} n_4} \right)^2 = 1$

For $n_2 < n_3$, $R \sim \left(\frac{n_1}{n_1} \right)^2 = 1$

Multiple quarter-wavelength layers of two different materials

➔ Dielectric mirror with $r = 1$ or -1

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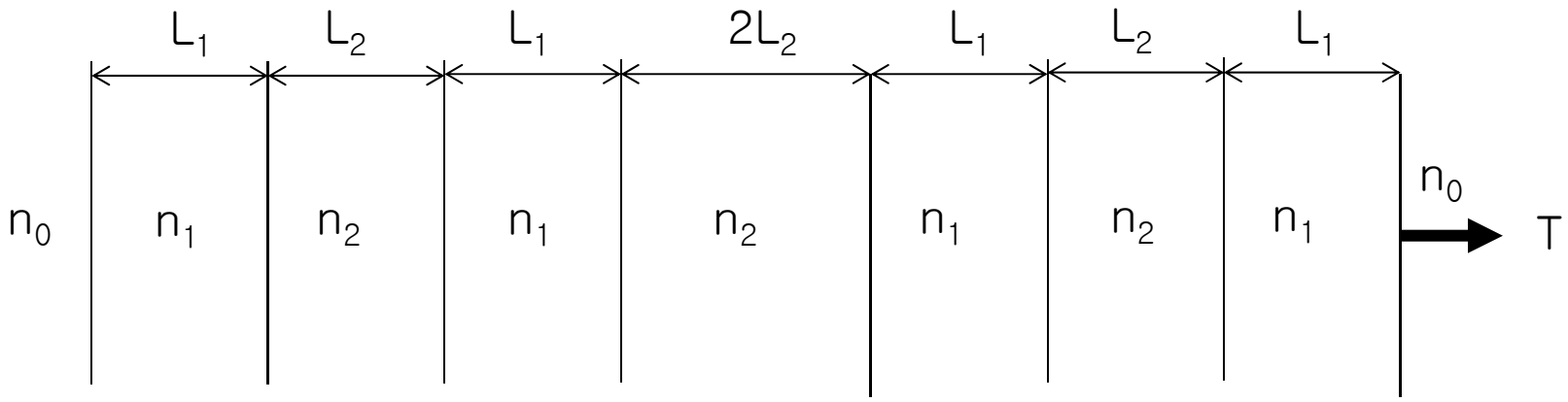
Homework:

Determine T for the following multiple dielectric layers.



$$n_1 = 1.35,\ n_2 = 2.3,\ L_{1,2} = \frac{\lambda}{4n_{1,2}}$$