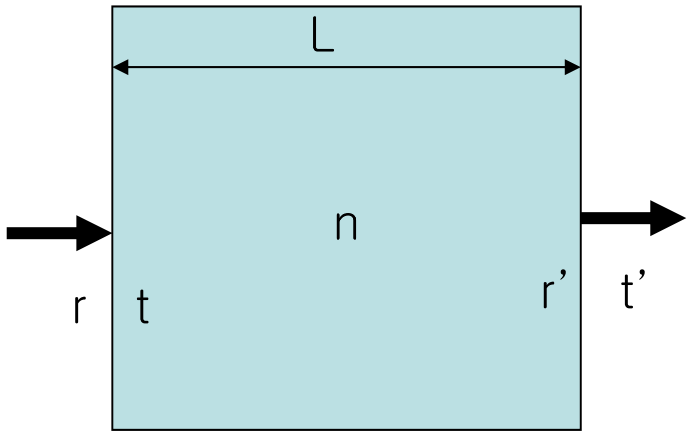
Lect. 8: Interferometers



$$T = \frac{I_{out}}{I_{in}} = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(kL)} \quad (R = r'^2)$$

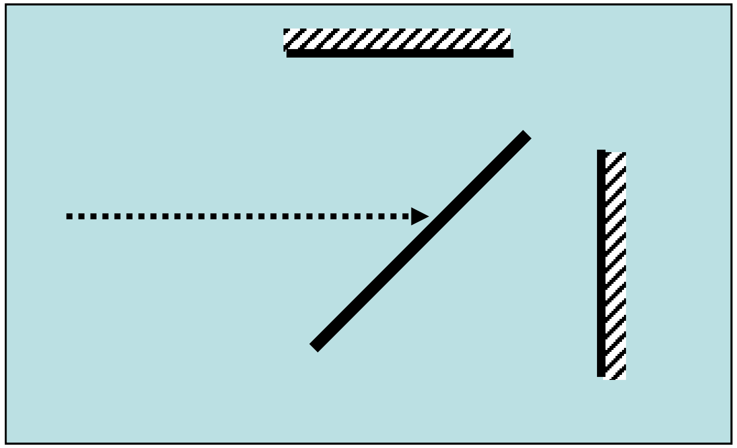
Fabry-Perot Interferometer

Charles Fabry
(1867-1945)

Alfred Perot
(1863-1925)

FP Interferometer can be also realized with two parallel partially reflecting mirrors

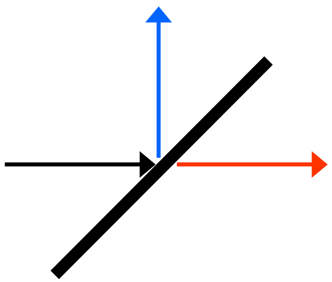
Michelson Interferometer



Albert Michelson
(1852~1931)
Nobel Prize in Physics in 1907

Two mirrors and one beam splitter

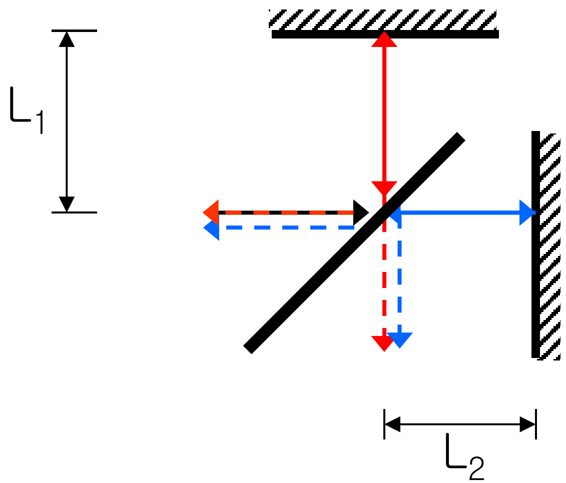
Beam Splitter



$$R = \frac{1}{2}, \quad T = \frac{1}{2}$$

$$r = ?, \quad t = ?$$

Michelson Interferometer:



Four outputs:

Side	1: r r	
	2: t t	
Bottom	1: r t	
	2: t r	

Assuming $E_{in} = 1$,

$$E_{out,\,side} = re^{-jkl_1}(-1)e^{-jkl_1}r + te^{-jkl_2}(-1)e^{-jkl_2}t = -r^2e^{-j2kl_1} - t^2e^{-j2kl_2}$$

$$E_{out,\,bottom} = re^{-jkl_1}(-1)e^{-jkl_1}t + te^{-jkl_2}(-1)e^{-jkl_2}r = -rte^{-j2kl_1} - rte^{-j2kl_2}$$

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With $r = t = \frac{1}{\sqrt{2}}$,

$$E_{out,\,side} = -r^2 e^{-j2kl_1} - t^2 e^{-j2kl_2}$$

$$= -\frac{1}{2}\left(e^{-j2kl_1} + e^{-j2kl_2}\right) = -\frac{1}{2} e^{-jk(l_1+l_2)}\left(\mathrm{e}^{-jk(l_1-l_2)} + \mathrm{e}^{jk(l_1-l_2)}\right)$$

$$I_{out,\,side} = \left|E_{out,\,side}\right|^2 = \cos^2[k(l_1 - l_2)]$$

$$E_{out,\,bottom} = -rte^{-j2kl_1} - rte^{-j2kl_2}$$

$$= -\frac{1}{2}\left(e^{-j2kl_1} + e^{-j2kl_2}\right)$$

$$I_{out,\,bottom} = \left|E_{out,\,bottom}\right|^2 = \cos^2[k(l_1 - l_2)]$$

Against energy conservation!

With $r = \frac{1}{\sqrt{2}},\ t = j\frac{1}{\sqrt{2}}$

(r and t should have $\pi/2$ phase difference in order to satisfy energy conservation)

$$E_{out,side} = -r^2 e^{-j2kl_1} - t^2 e^{-j2kl_2}$$

$$= -\frac{1}{2}\left(e^{-j2kl_1} - e^{-j2kl_2}\right) = -\frac{1}{2}e^{-jk(l_1+l_2)}\left(\mathrm{e}^{-jk(l_1-l_2)} - \mathrm{e}^{jk(l_1-l_2)}\right)$$

$$I_{out,side} = \left|E_{out,side}\right|^2 = \sin^2[k(l_1 - l_2)]$$

$$E_{out,bottom} = -rte^{-j2kl_1} - rte^{-j2kl_2}$$

$$= -\frac{j}{2}\left(e^{-j2kl_1} + e^{-j2kl_2}\right)$$

$$I_{out,bottom} = \left|E_{out,bottom}\right|^2 = \cos^2[k(l_1 - l_2)]$$

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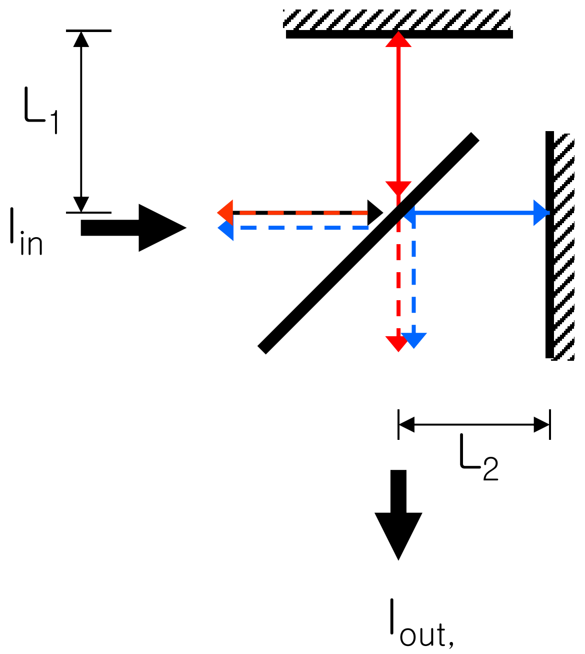
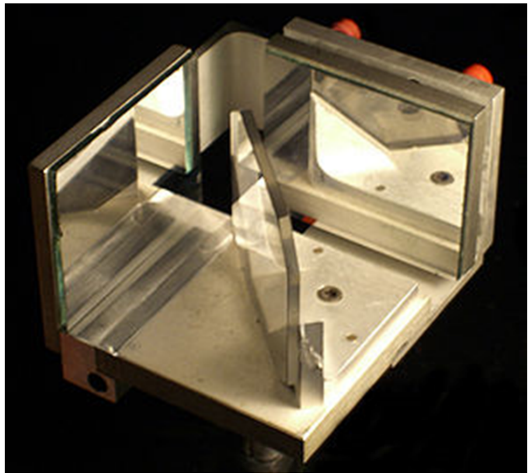
Michelson Interferometer:



$$\frac{I_{out,bottom}}{I_{in}} = \cos^2[k(l_1 - l_2)]$$



$k(l_1 - l_2)$ can be measured very precisely

➔ Speed of light

Michelson-Morley experiment:

Speed of light is same in all direction everywhere

➔ Aether does not exist, Special relativity

Recently, used for detecting gravitational wave!

LIGO : Laser Interferometer Gravitational-Wave Observatory

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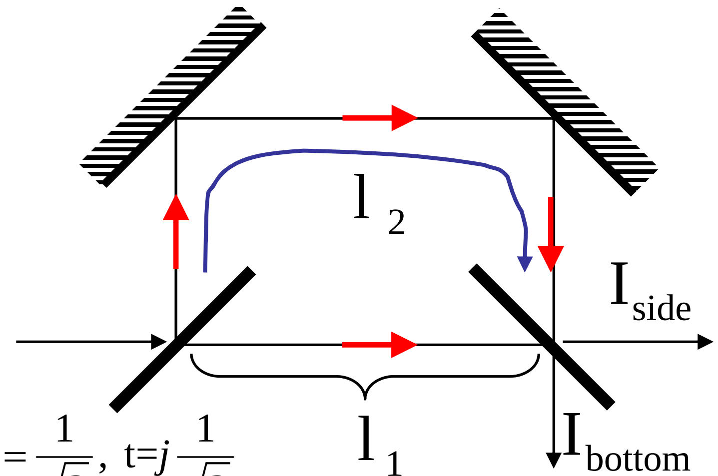
Mach-Zehnder Interferometer:



$r = \frac{1}{\sqrt{2}}$, t=$j\frac{1}{\sqrt{2}}$

Assuming $E_{in} = 1$,

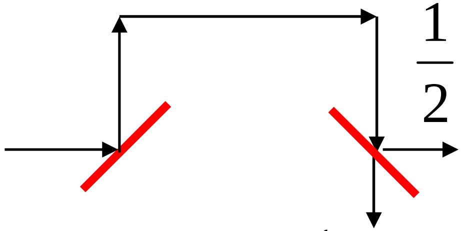
$$E_{out,\,side} = \frac{1}{2}\left(e^{-jkl_2} - e^{-jkl_1}\right) = \frac{1}{2}e^{-jk\frac{l_2+l_1}{2}}\left(e^{-jk\frac{l_2-l_1}{2}} - e^{jk\frac{l_2-l_1}{2}}\right)$$

$$I_{out,\,side} = \sin^2\left(k\frac{l_1-l_2}{2}\right)$$

$$E_{out,\,bottom} = \frac{j}{2}\left(e^{-jkl_1} + e^{-jkl_2}\right) = \frac{j}{2}e^{-jk\frac{l_1+l_2}{2}}\left(e^{-jk\frac{l_1-l_2}{2}} + e^{jk\frac{l_1-l_2}{2}}\right)$$

$$I_{out,\,bottom} = \cos^2\left(k\frac{l_1-l_2}{2}\right)$$

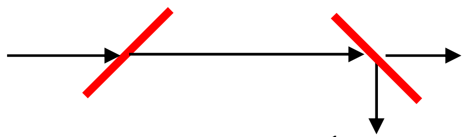
Case#1



$\frac{1}{2}e^{-jkl_2}$

$j\frac{1}{2}e^{-jkl_2}$

Case#2



$-\frac{1}{2}e^{-jkl_1}$

$j\frac{1}{2}e^{-jkl_1}$

Lect. 8: Interferometers

Can any EM wave cause interference?

➔ Only EM waves having clear phase relationship experience interference: coherent

How large (l_1-l_2) can be?

➔ As long as two separated waves are coherent or within coherent length
Separated waves become incoherent due to intrinsic phase noises

Homework: Determine I_{out}/I_{in} for an interferometer shown below. It is made of two beam splitters, two mirrors and a block of material having index n and length l_1 placed in one arm. Assume there is no reflection from this block of material.