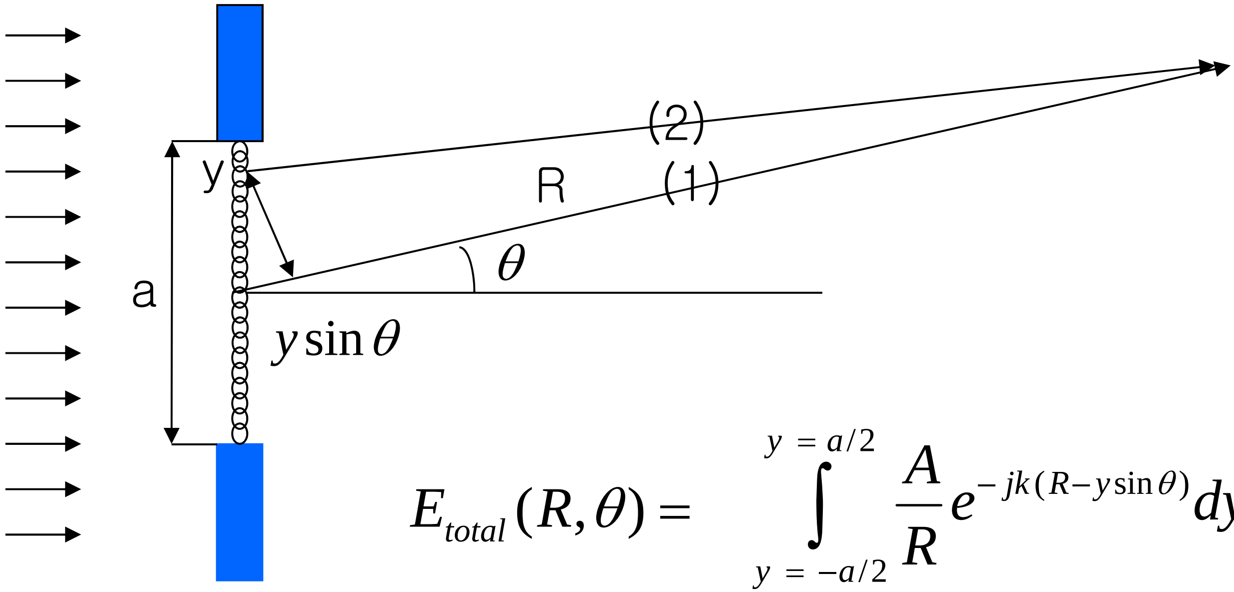
Lect. 9: Diffraction



(1) $\frac{A}{R}\exp(-jkR)$

(2) $\frac{A}{R}\exp[-jk(R-y\sin\theta)]$

$$E_{total}(R,\theta) = \int_{y=-a/2}^{y=a/2} \frac{A}{R} e^{-jk(R-y\sin\theta)} dy = \frac{A}{R} e^{-jkR} \int_{y=-a/2}^{y=a/2} e^{jky\sin\theta} dy$$

Since interference is determined by ***phase difference*** ,
the constant phase term can be ignored without affecting the final result.

$$E_{total}(R,\theta) = \frac{A}{R} \int_{y=-a/2}^{y=a/2} e^{jky\sin\theta} dy$$

Evaluate $E_{total}(R,\theta) = \frac{A}{R}\int_{-a/2}^{a/2} e^{jky\sin\theta} dy$

Let $y' = jky\sin\theta$ => $dy' = jk\sin\theta dy$

$$E_{total}(R,\theta) = \frac{A}{R}\int_{y'=-jk\frac{a}{2}\sin\theta}^{y'=jk\frac{a}{2}\sin\theta} e^{y'} \frac{dy'}{jk\sin\theta} = \frac{A}{R}\frac{1}{jk\sin\theta}\left(e^{jk\frac{a}{2}\sin\theta} - e^{-jk\frac{a}{2}\sin\theta}\right)$$

$$= \frac{A}{R}\frac{2j}{jk\sin\theta}\sin(k\frac{a}{2}\sin\theta)$$

$$= \frac{2A}{R}\frac{\sin(k\frac{a}{2}\sin\theta)}{k\sin\theta}$$

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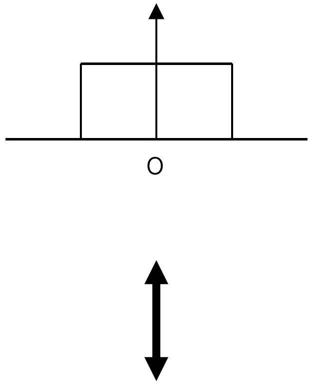
$$E_{total}(R,\theta) = \frac{2A}{R}\frac{\sin(k\frac{a}{2}\sin\theta)}{k\sin\theta}$$

$$\frac{E_{total}(R,\theta)}{E_{total}(R,0)} = ?$$

$$E_{total}(R,0) = \frac{2A}{R}\frac{\cos(k\frac{a}{2}\sin\theta)k\frac{a}{2}\cos\theta\Big|_{\theta=0}}{k\cos\theta\Big|_{\theta=0}} = \frac{2A}{R}\frac{a}{2}$$

$$\therefore \frac{E_{total}(\theta)}{E_{total}(0)} = \frac{\frac{2A}{R}\frac{\sin(k\frac{a}{2}\sin\theta)}{k\sin\theta}}{\frac{2A}{R}\frac{a}{2}} = \frac{\sin(k\frac{a}{2}\sin\theta)}{k\frac{a}{2}\sin\theta} = \frac{\sin(k_y\frac{a}{2})}{k_y\frac{a}{2}}$$



sinc function

FT relationship

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$$E_{total}(R,\theta) = \frac{A}{R}\int_{y=-a/2}^{y=a/2} e^{jky\sin\theta}dy \quad \Rightarrow \quad E_{total}(R,k_y) = \frac{1}{R}\int_{y=-\infty}^{y=\infty} A(y)e^{jk_y y}dy$$

$$\left(k_y = k\sin\theta\right)$$

From Signals and Systems, inverse FT is defined as

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega$$

$$\mathrm{f}(t) \quad \Leftrightarrow \quad F(\omega)$$

$$E_{total}(k_y) \Leftrightarrow A(y)$$

Far-field diffraction field $E(k_y)$ is (inverse) F.T. of $A(y)$ within the constant factor!

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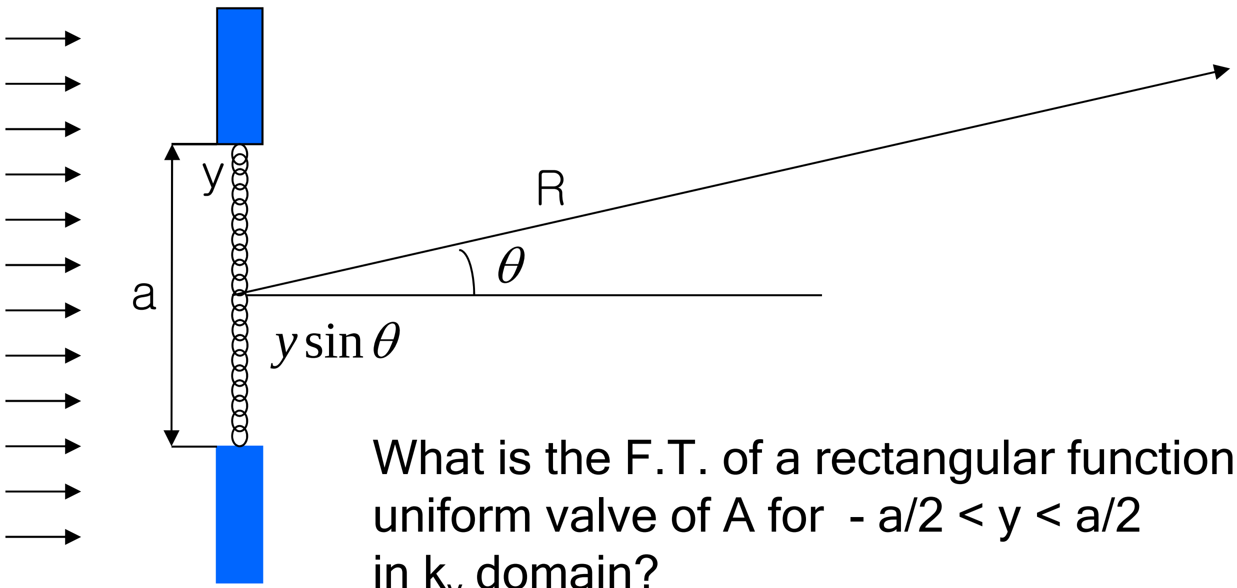

$$\frac{E(R,\theta)}{E(R,0)} = ?$$

What is the F.T. of a rectangular function having uniform valve of A for - a/2 < y < a/2 in k_y domain?

$$A\frac{\sin\left(k_y\frac{a}{2}\right)}{k_y\frac{a}{2}}$$

$$\frac{E(R,k_y)}{E(R,0)} = \frac{A\frac{\sin\left(k_y\frac{a}{2}\right)}{k_y\frac{a}{2}}}{A} = \frac{\sin\left(k_y\frac{a}{2}\right)}{k_y\frac{a}{2}}$$

(Same as the expression on p. 3)

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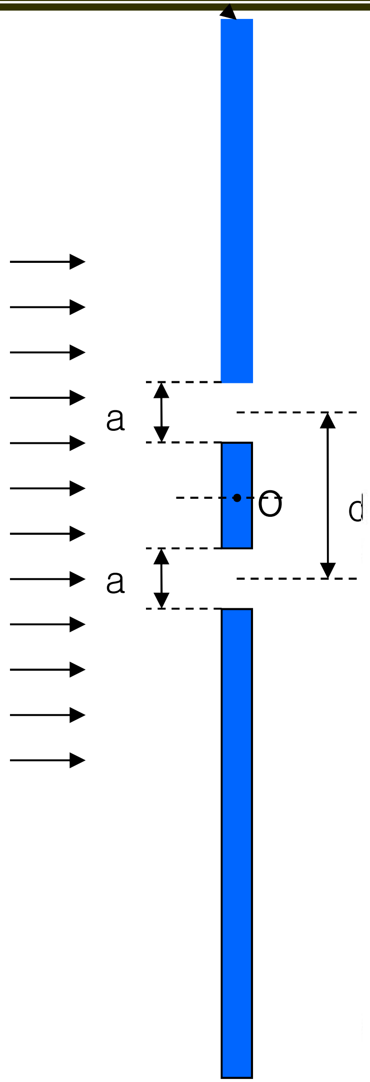

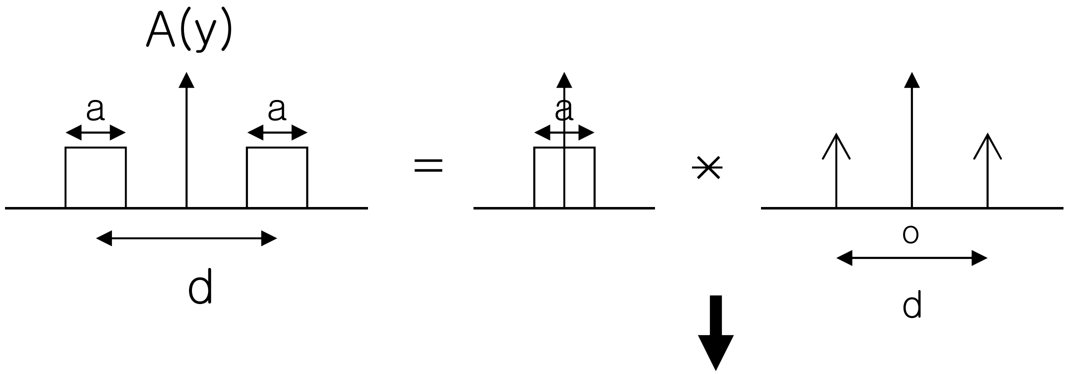

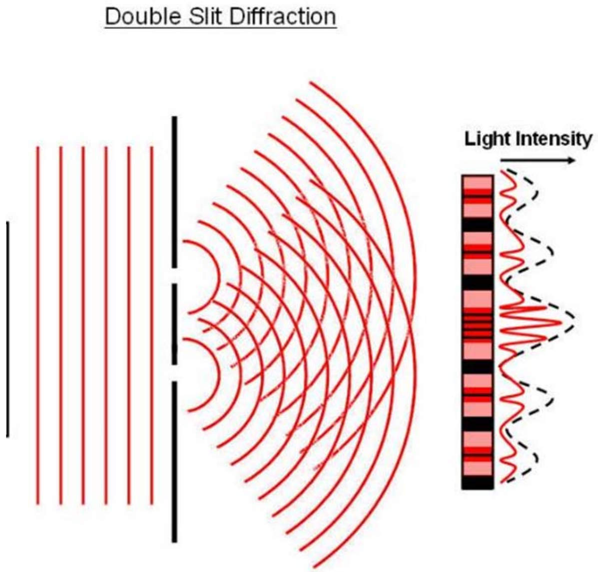

$$\frac{\sin(k_y \frac{a}{2})}{k_y \frac{a}{2}} \quad \text{X} \quad e^{jk_y \frac{d}{2}} + e^{-jk_y \frac{d}{2}} = 2\cos(k_y \frac{d}{2})$$

$$\frac{E(k_y)}{E(0)} = 2\cos(k_y \frac{d}{2}) \frac{\sin(\frac{k_y a}{2})}{\frac{k_y a}{2}}$$

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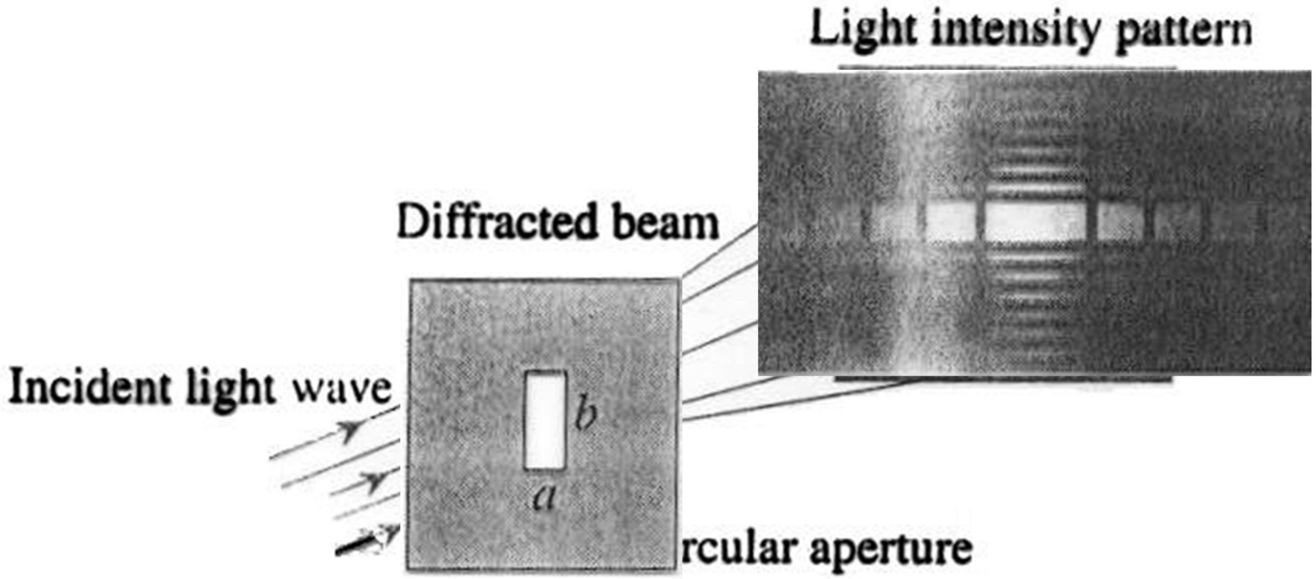

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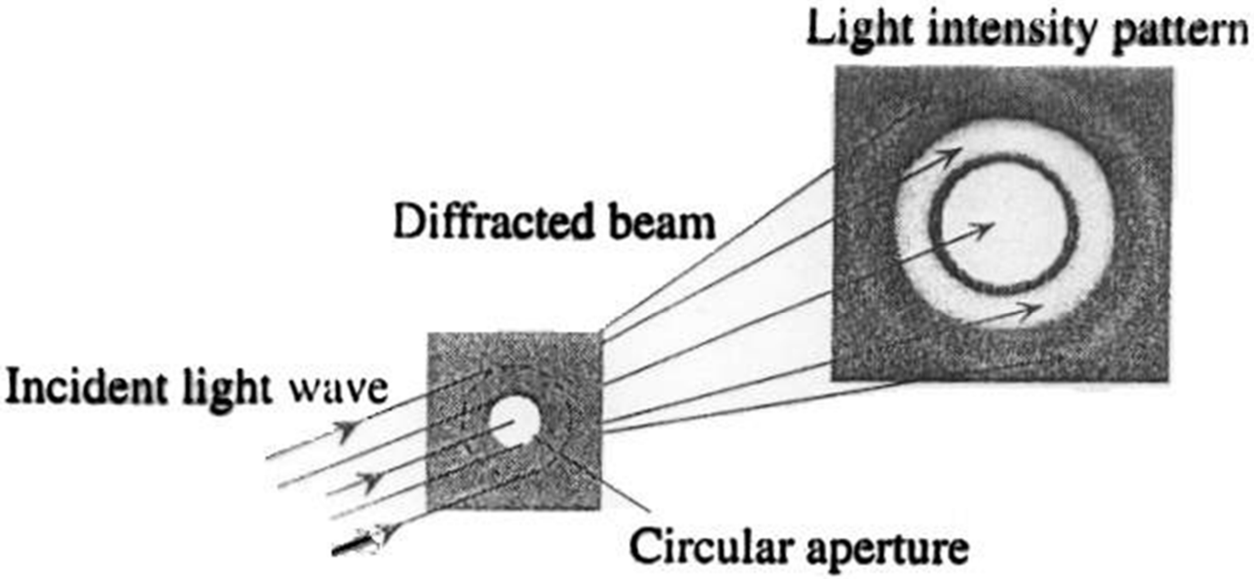

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Homework

Two point light sources are located as shown below. We are interested in the far-field pattern produced by the interference of these point sources. For each of cases given below, sketch the magnitude of E(Θ)/E(0). For the sketch, use sin(Θ)/λ as the x-axis. On the sketch, clearly indicate the locations of the max. and min. magnitudes.

(a) Two source are located near origin and E-fields from two sources are in-phase when they are produced at the source.

(b) Two source are located near origin and E-fields from two sources are out-of-phase when they are produced at the source.

(c) Same as in (a) but the location of sources are shifted by d.