Review of classical mechanics

I. Energy & Force
II. Oscillations
III. Waves

Momentum & Kinetic Energy

- Momentum
  - Expressed as $\vec{p} = m\vec{v}$

- Kinetic energy
  - Energy associated with the motion
  - Expressed as $K.E. = \frac{1}{2}mv^2 = \frac{p^2}{2m}$, where $p^2 = \vec{p} \cdot \vec{p}$

- Potential energy
  - Energy due to position
  - Denoted by *V*(r) in quantum mechanics, in unit of Joules

- Total energy
  - Sum of kinetic energy and potential energy, "Hamiltonian"

$$H = \frac{p^2}{2m} + V(r)$$

Force

- Force & momentum
  - Newton's 2nd law: $F = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d\vec{p}}{dt}$

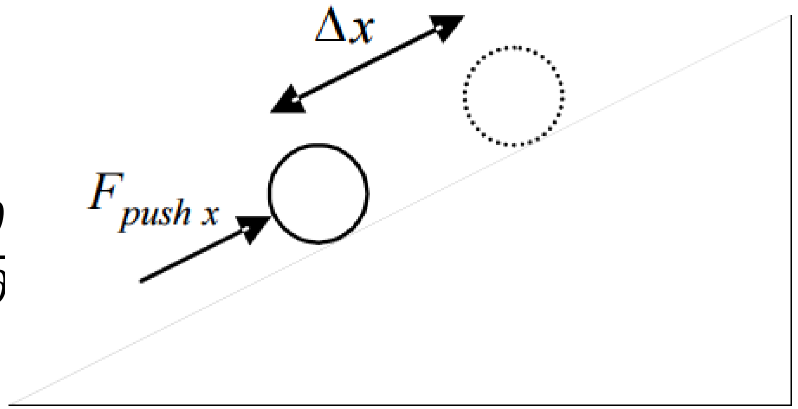
- Force & potential energy, *V*(r)
  - $\Delta V = F_{pushx}\Delta x$

    ➔$F_{pushx} = \frac{\Delta V}{\Delta x}$

    ➔$F_{pushx} = \frac{dV}{dx}$
  - Force due to potential energy difference is downhill

    ➔$F = -\frac{dV}{dx}$
  - Generalized relation

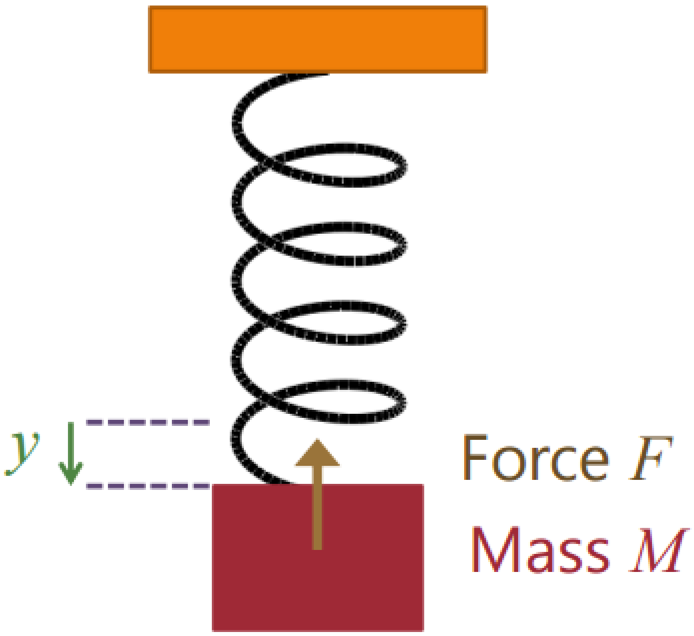
$$F = -\nabla V$$
$$= -[\frac{\partial V}{\partial x}i + \frac{\partial V}{\partial y}j + \frac{\partial}{\partial}$$



Mass on Spring: Oscillation

- Simple harmonic oscillator
  - Restoring force
    : $F = -Ky$, K is spring constant
  - $F = Ma = M\frac{d^2y}{dt^2}$
  - $\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$
    ω is angular frequency of oscillation
  - Solution for equation is

$$y \propto \sin(\omega t)$$



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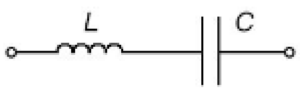
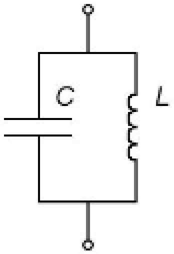
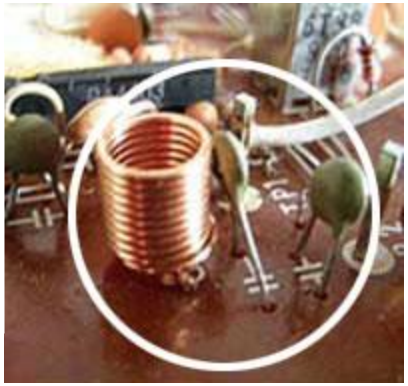
Harmonic Oscillators

LC Oscillators



$$f = \frac{1}{2\pi\sqrt{LC}}$$

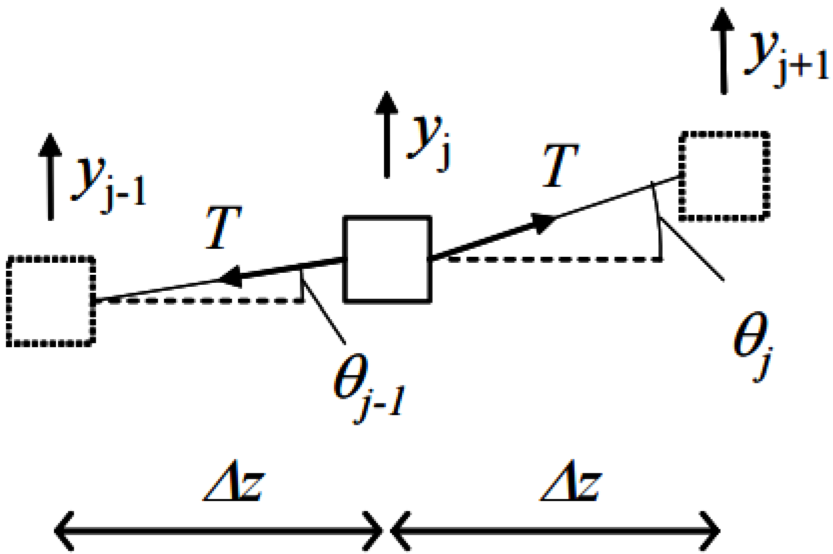




“Helmholtz” resonator in acoustics

Classical Wave Equation: Standing Waves

- Force at mass j
  - $F_j = T(sin\theta_j - sin\theta_{j\text{-}1})$
    $\cong T\left(\frac{y_{j+1}-y_j}{\Delta z} - \frac{y_j - y_{j-1}}{\Delta z}\right)$
    $= T\Delta z \frac{y_{j+1}-2y_j+y_{j-1}}{\Delta z^2}$
    $= T\Delta z \frac{\partial^2 y}{\partial z^2}$
  - Think $m = \rho\Delta z$
    Then, $F = ma = \rho\Delta z \frac{\partial^2 y}{\partial t^2}$
  - $T\Delta z \frac{\partial^2 y}{\partial z^2} = \rho\Delta z \frac{\partial^2 y}{\partial t^2}$
    ➔ $\frac{\partial^2 y}{\partial z^2} = \frac{\rho}{T}\frac{\partial^2 y}{\partial t^2}$
    ➔ $\frac{\partial^2 y}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = 0$, where velocity $v = \sqrt{\frac{T}{\rho}}$ (Wave equation)
  - Solution for wave equation: $f(z - ct)$ or $g(z + ct)$

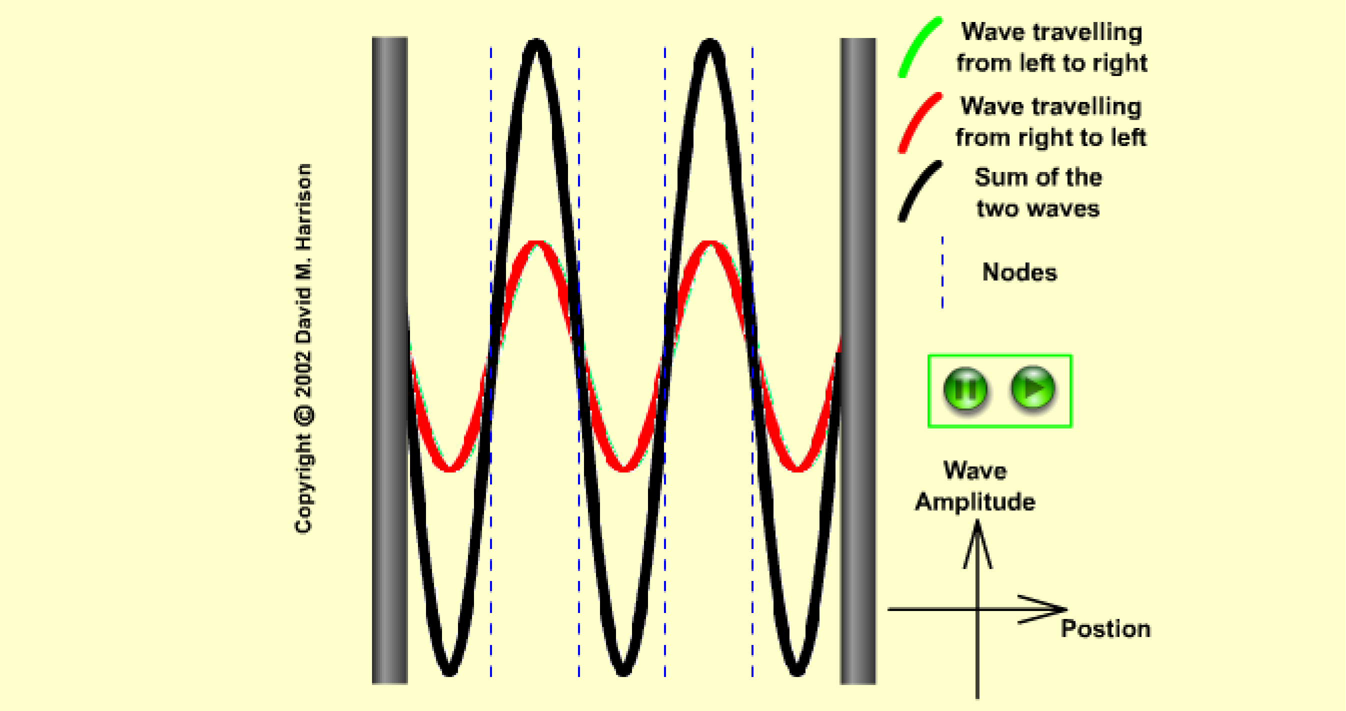


Helmholtz Wave Equation

- Considering oscillation at single angular frequency ω (Monochromatic wave)
  - Solution for wave equation is combination of
    : $\exp(i\omega t)$, $\exp(-i\omega t)$, $\cos(\omega t)$, $\sin(\omega t)$
  - Solution $\phi(z,t) \equiv Z(z) \cdot T(t) \Longrightarrow \frac{\partial^2 \phi}{\partial t^2} = -\omega^2 \phi$
  - Helmholtz wave equation
    (Only spatial part remaining from wave equation)

$$\frac{d^2 Z(z)}{dz^2} + k^2 Z(z) = 0, where\ k^2 = \frac{\omega^2}{c^2}$$

Standing Waves



Standing wave: sum of forward wave and backward wave

$$\phi(z,t) = \sin(kz - \omega t) + \sin(kz + \omega t)$$
$$= 2\cos(\omega t)\sin(kz)\,, where\ k = \frac{\omega}{c} = \frac{2\pi L}{L}$$

Conclusion

- Expression of energy, force in quantum mechanics

- Simple harmonic oscillation
  - $\frac{d^2y}{dt^2} = -\frac{K}{M}y = -\omega^2 y$

- Wave equation
  - $\frac{\partial^2 y}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = 0$
  - $\frac{d^2Z(z)}{dz^2} + k^2Z(z) = 0, where\ k^2 = \frac{\omega^2}{c^2}$(Helmholtz wave equation)