2016-2
QUANTUM MECHANICS
FOR ELECTRICAL AND ELECTRONIC ENGINEERS

Wave Propagation

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Outline

Background

a. Plane waves and interference

b. Diffraction

c. Diffraction from periodic structures

Summary

Background

In the previous lecture…

1-D wave equation

$$\frac{\partial^2 y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = 0$$

c: wave velocity

For monochromatic wave (single angular frequency ω)

$$\phi(z,t) = Z(z)T(t)$$

$$\frac{\partial^2 \phi(z,t)}{\partial t^2} = -\omega^2 \phi(z,t)$$

ω:angular frequency

$$\frac{d^2 Z(z)}{dz^2} + k^2 Z(z) = 0 \qquad k^2 = \frac{\omega^2}{c^2}$$

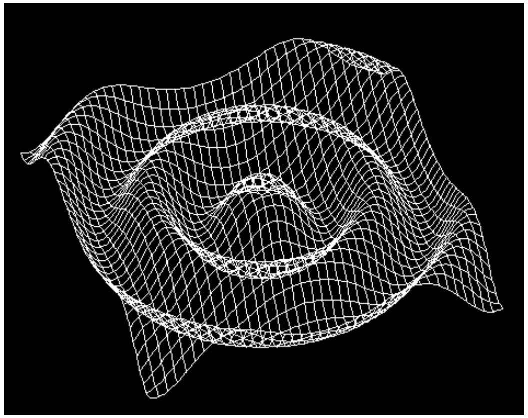
: Helmholtz wave equation

a. Plane waves and interference

Wave equation in 3 dimensions

$$\nabla^2 \phi(x,y,z,t) - \frac{1}{c^2}\frac{\partial^2 \phi(x,y,z,t)}{\partial t^2} = 0$$

where $\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$



For monochromatic wave (single angular frequency ω)

Verifying plane wave solutions of the form when k= ω /c

$$\phi(x,y,z,t) = \exp[i(\mathbf{k} \bullet \mathbf{r} - \omega t)]$$

where $\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$ $\quad \mathbf{k} = k_x\hat{\mathbf{x}} + k_y\hat{\mathbf{y}} + k_z\hat{\mathbf{z}}$

a. Plane waves and interference

For monochromatic wave (single angular frequency ω)

Verifying plane wave solutions of the form when k= ω /c

$$\nabla \exp[i(\mathbf{k}\bullet\mathbf{r} - \omega t)] = \left(\hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}\right)\exp\left\{i\left[k_x x + k_y y + k_z z - \omega t\right]\right\}$$

$$= i\left(k_x\hat{\mathbf{x}} + k_y\hat{\mathbf{y}} + k_z\hat{\mathbf{z}}\right)\exp\left\{i\left[k_x x + k_y y + k_z z - \omega t\right]\right\}$$

$$= i\mathbf{k}\exp[i(\mathbf{k}\bullet\mathbf{r} - \omega t)]$$

$$\nabla^2 \exp[i(\mathbf{k}\bullet\mathbf{r} - \omega t)] = \nabla\bullet\nabla \exp[i(\mathbf{k}\bullet\mathbf{r} - \omega t)]$$

$$= \nabla\bullet\left(i\vec{k}\exp[i(\mathbf{k}\bullet\mathbf{r} - \omega t)]\right)$$

$$= i\left(k_x\frac{\partial}{\partial x} + k_y\frac{\partial}{\partial x} + k_z\frac{\partial}{\partial x}\right)\exp[i(\mathbf{k}\bullet\mathbf{r} - \omega t)]$$

$$= -(k_x^2 + k_y^2 + k_z^2)\exp[i(\mathbf{k}\bullet\mathbf{r} - \omega t)]$$

$$= -k^2\exp[i(\mathbf{k}\bullet\mathbf{r} - \omega t)]$$

a. Plane waves and interference

For monochromatic wave (single angular frequency ω)

Verifying plane wave solutions of the form when k= ω /c

Since $\frac{\partial^2}{\partial t^2}\exp[i(\mathbf{k}\bullet\mathbf{r}-\omega t)] = -\omega^2\exp[i(\mathbf{k}\bullet\mathbf{r}-\omega t)]$,

with $\nabla^2\exp[i(\mathbf{k}\bullet\mathbf{r}-\omega t)] = -k^2\exp[i(\mathbf{k}\bullet\mathbf{r}-\omega t)]$

$$\nabla^2\phi(x,y,z,t) - \frac{1}{c^2}\frac{\partial^2\phi(x,y,z,t)}{\partial t^2} = 0$$

$$-k^2\exp[i(\mathbf{k}\bullet\mathbf{r}-\omega t)] + \frac{\omega^2}{c^2}\exp[i(\mathbf{k}\bullet\mathbf{r}-\omega t)] = 0$$

choosing $k = \omega / c$,

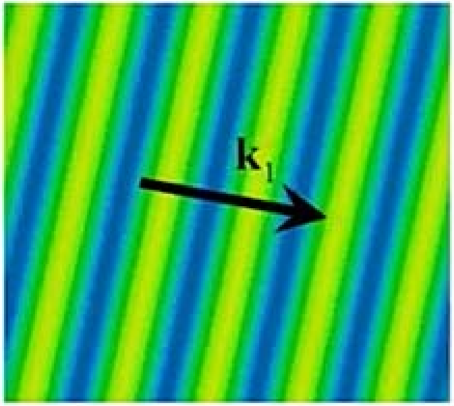
$\phi(x,y,z,t) = \exp[i(\mathbf{k}\bullet\mathbf{r}) - \omega t]$ is a solution for any vector direction $\mathbf{k}$

provided $|\mathbf{k}| = \frac{\omega}{c}$

a. Plane waves and interference

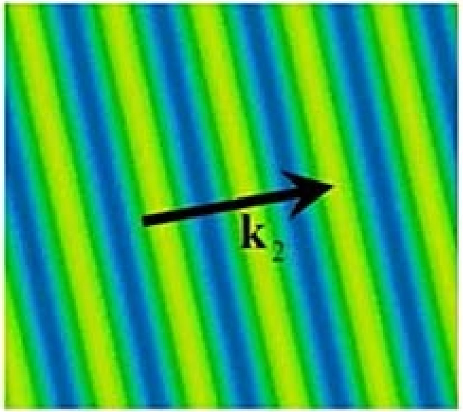
Wave interference

Linearity of wave equation



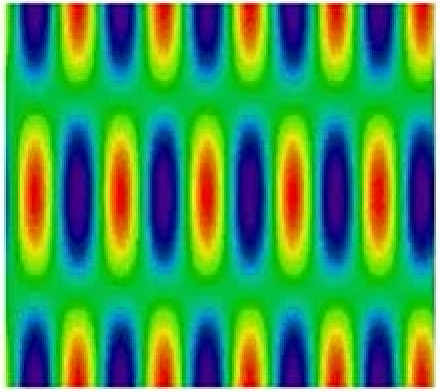
A plane wave solution $\mathbf{k_1}$

+



A plane wave solution $\mathbf{k_2}$

=



A plane wave solution $\mathbf{k_1+k_2}$

Wave interference between 2 waves is also a solution of wave equation because of **Linearity of wave equation**

b. Diffraction

What is Diffraction?

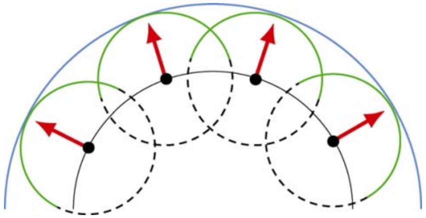
Diffraction refers to various phenomena which occur when a wave encounters an **obstacle** or a **slit**.

Interference is a phenomenon in which two waves **superpose** to form a resultant wave of greater, lower, or the same amplitude.

Huygens' principle

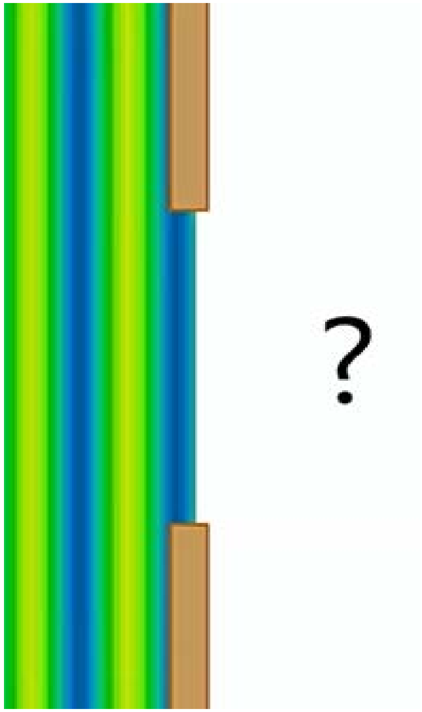
Christiaan Huygens (1629-1695)

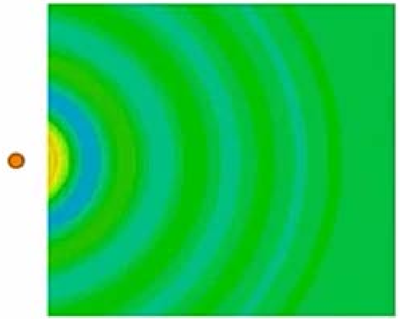
Every unobstructed point on a wavefront will act a source of secondary spherical waves. The new wavefront is the surface tangent to all the secondary spherical waves.



b. Diffraction

Waves from an aperture





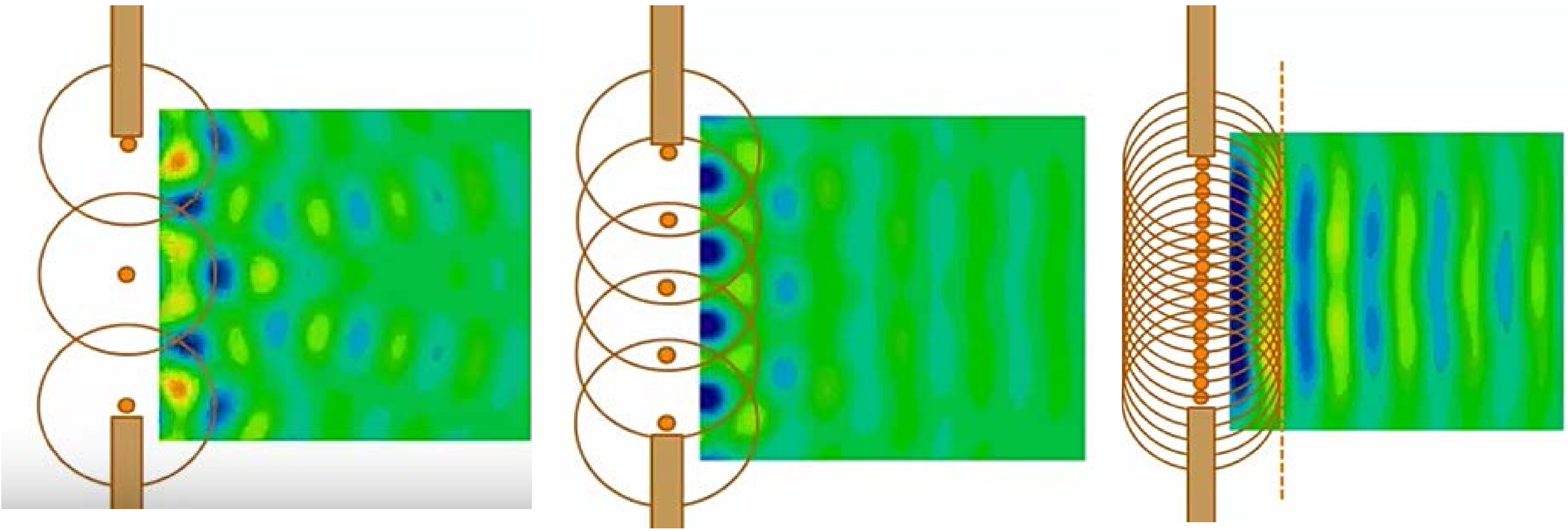
Waves from a point source

-Circular waves propagating from a point source

By putting huygens' sources between the aperture, wave from an aperture can be modeled.

b. Diffraction

Waves from a point source/an aperture

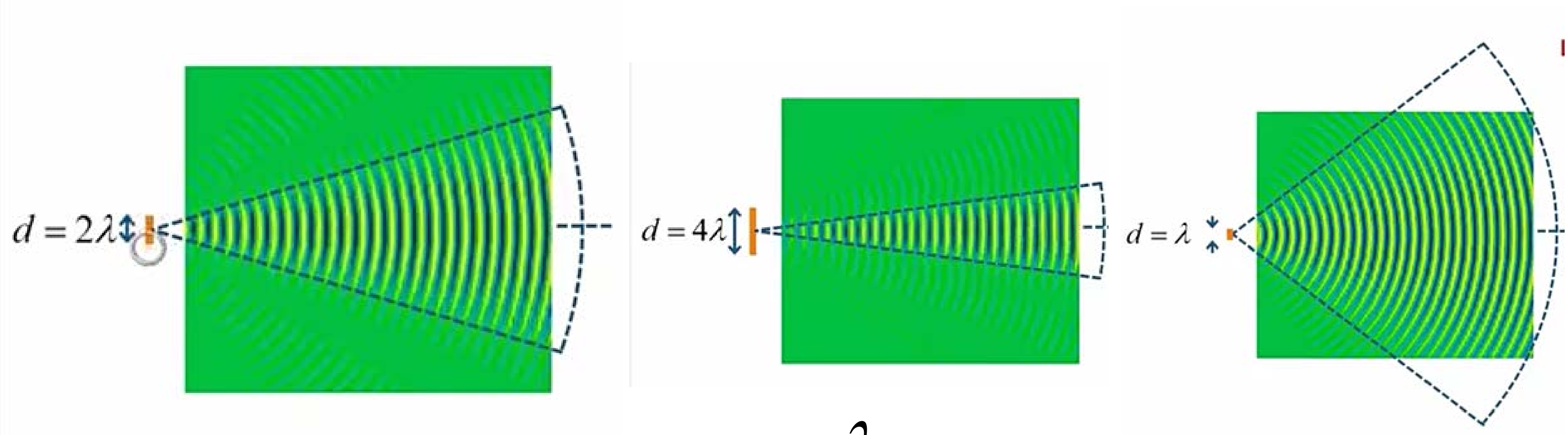


As the number of point sources increase, we have converged on a good description in this model!

b. Diffraction

Diffraction angle

If we look far away from a set of sources,



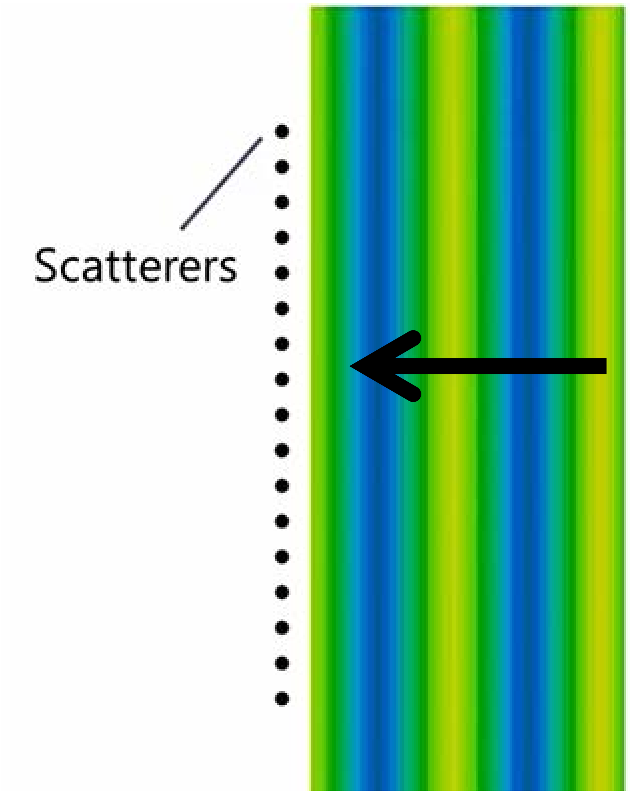
$$\theta \sim \frac{\lambda}{d}$$

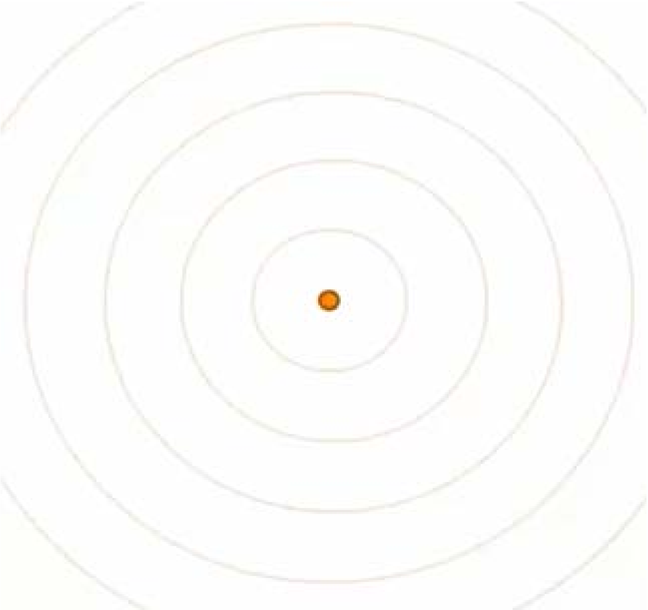
High frequency speakers : very directional
Low frequency speakers : not very directional

참고 사이트
http://yjh-phys.tistory.com/1459

c. Diffraction from periodic structures

What does the back-scattered light look like?

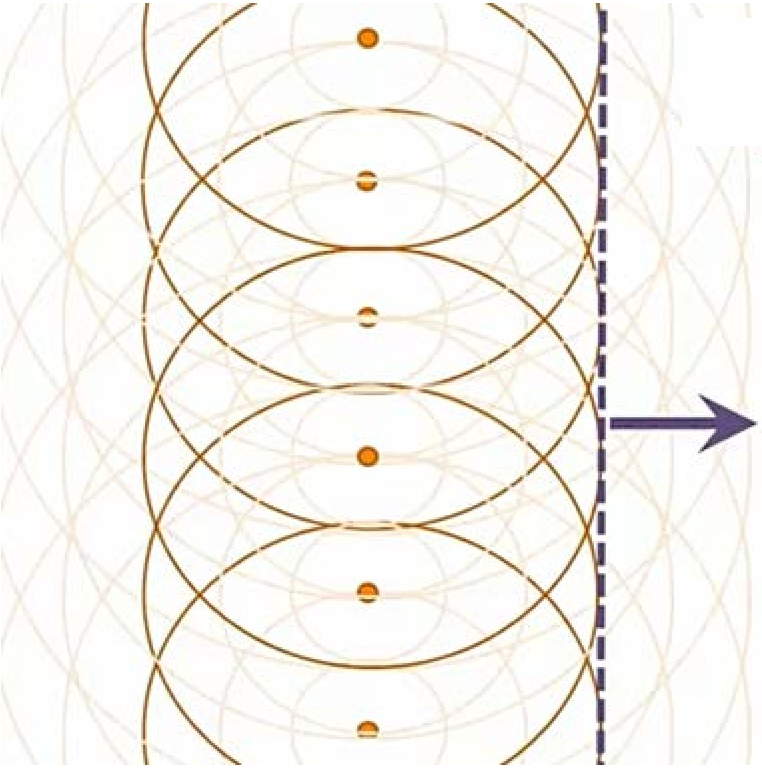


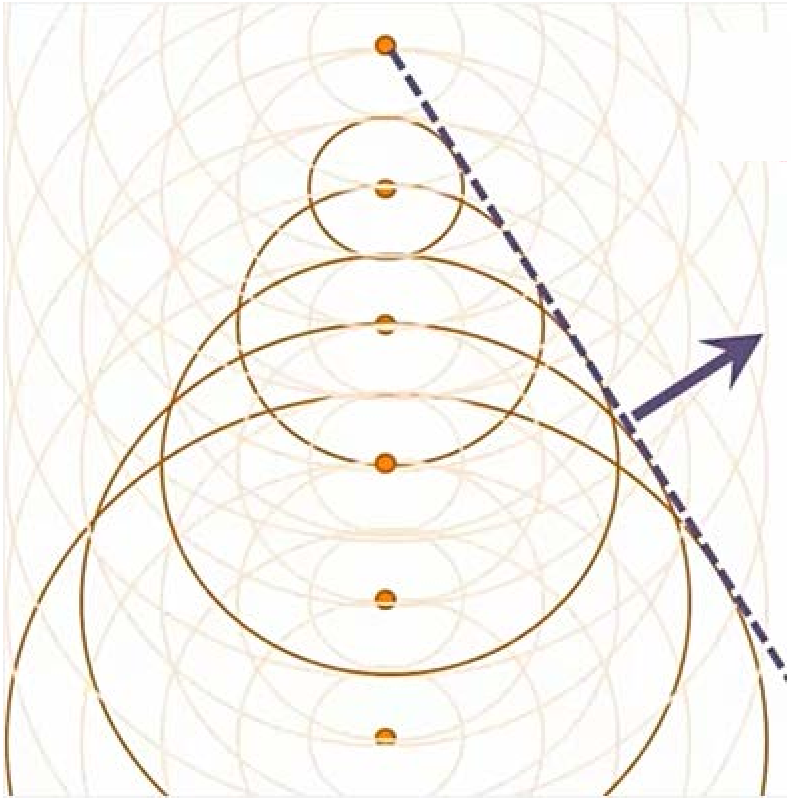


Concentric circle

c. Diffraction from periodic structures

What does the back-scattered look like?





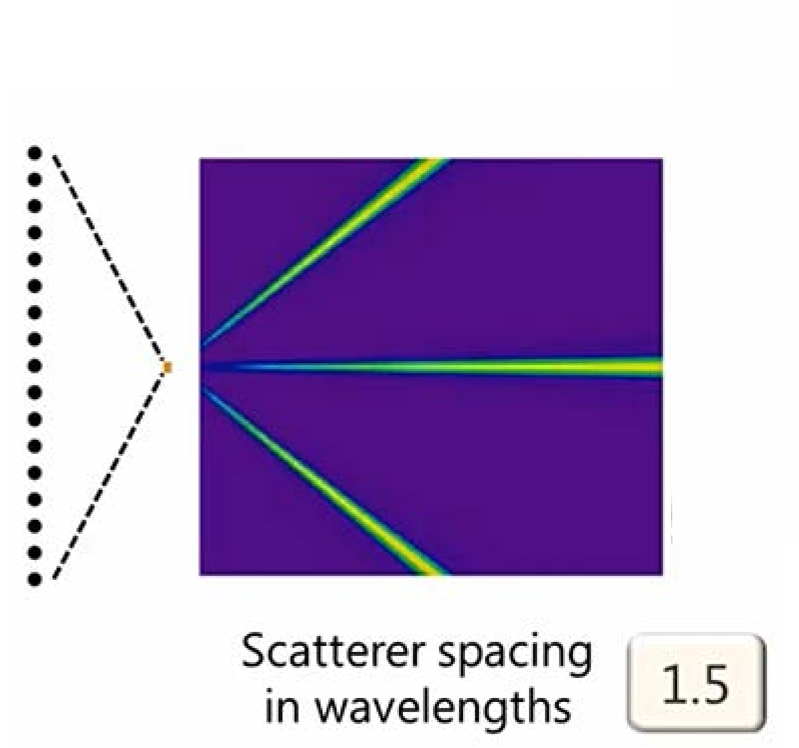
Zeroth order diffraction

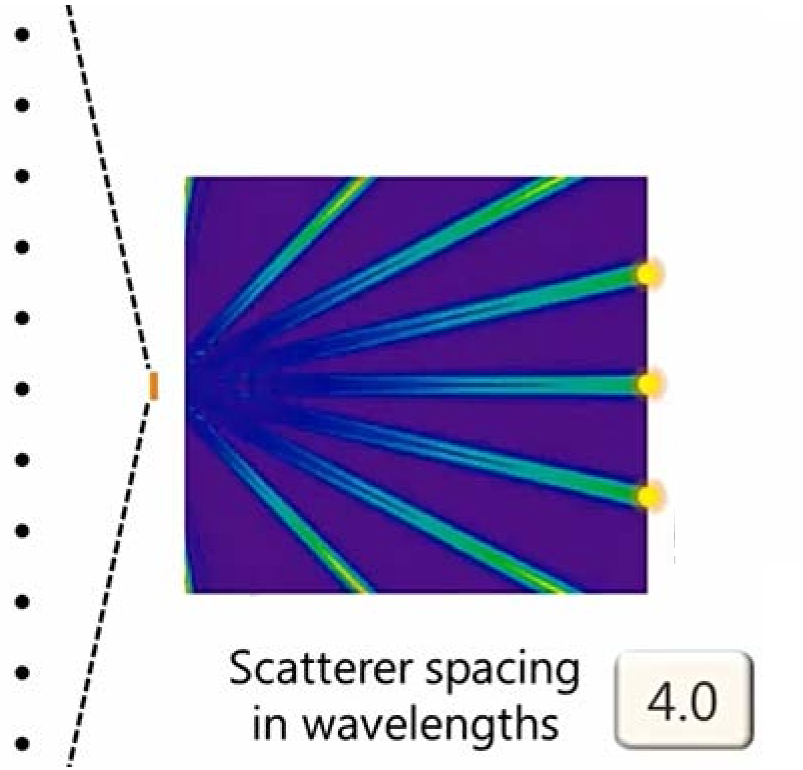
All of the sources add up in phase to generate a phase going back

An upward or downward direction is also possible

c. Diffraction from periodic structures

What does the back-scattered look like?

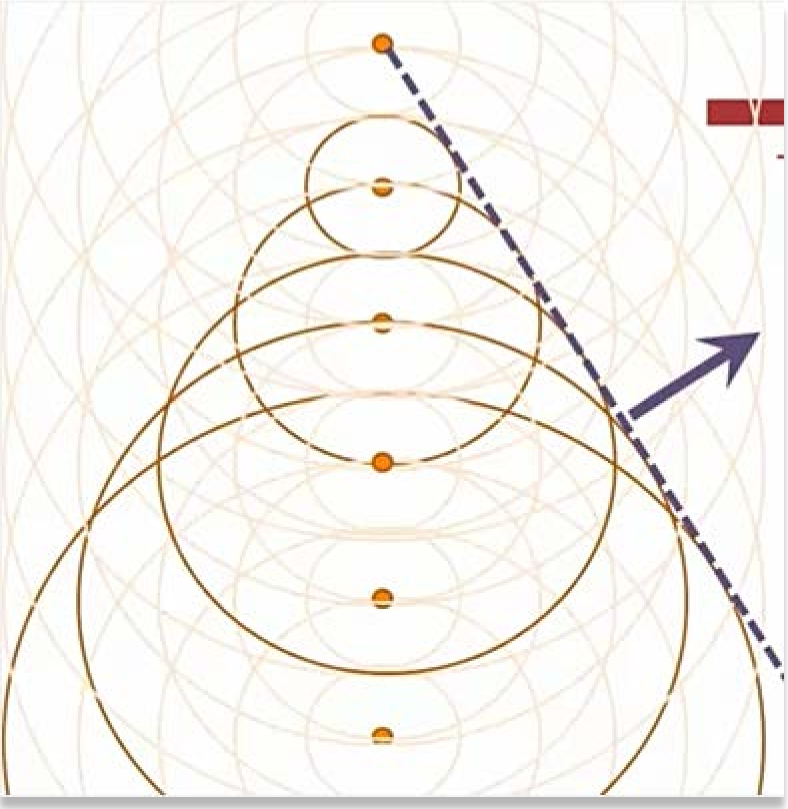




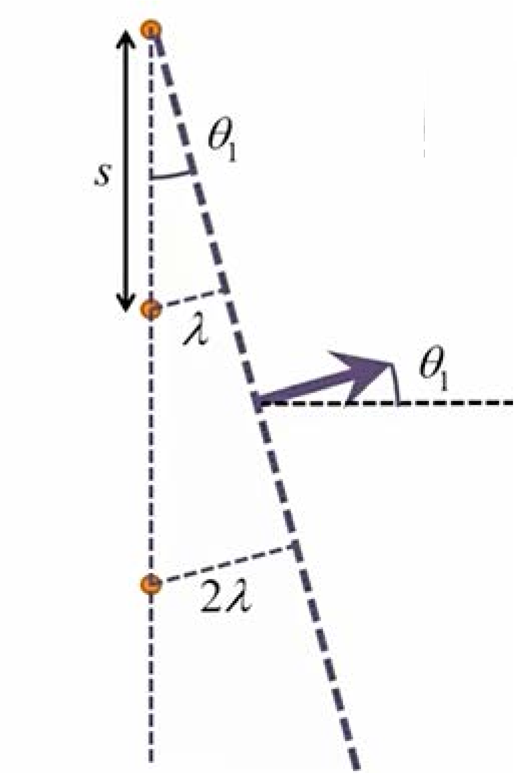
Larger scatterers seperation gives beam closer in angle

c. Diffraction from periodic structures

Calculate the angle Θ of diffracted waves



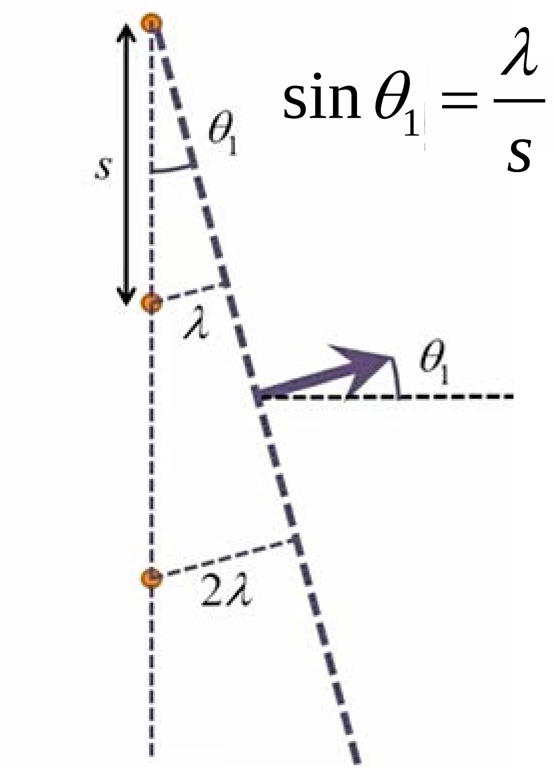
$$\sin\theta_1 = \frac{\lambda}{s}$$



Larger scatterers seperation gives beam closer in angle

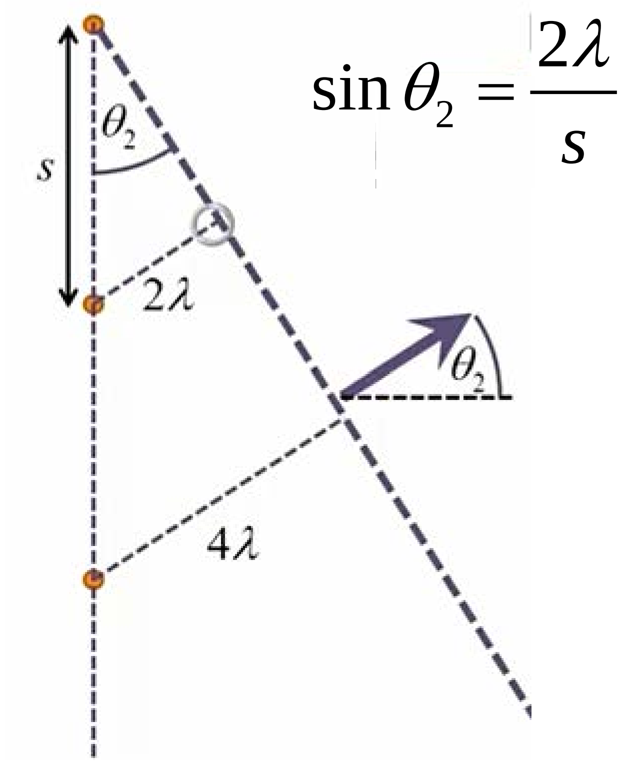
c. Diffraction from periodic structures

Calculate the angle Θ of diffracted waves



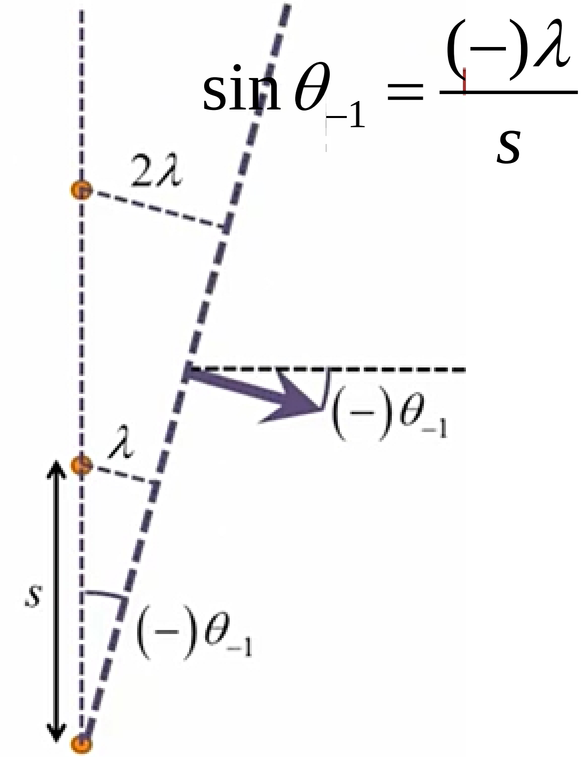
$$\sin\theta_1 = \frac{\lambda}{s}$$

First order diffraction



$$\sin\theta_2 = \frac{2\lambda}{s}$$

Second order diffraction



$$\sin\theta_{-1} = \frac{(-)\lambda}{s}$$

(negative)
First order diffraction

Summary

Wave equation in 3 dimensions

$$\nabla^2\phi(x,y,z,t) - \frac{1}{c^2}\frac{\partial^2\phi(x,y,z,t)}{\partial t^2} = 0$$

Plane wave solution for monochromatic wave (single angular frequency ω)

$$\phi(x,y,z,t) = \exp[i(\mathbf{k}\bullet\mathbf{r} - \omega t)]$$

Diffraction angle (by an aperture)

$$\theta \sim \frac{\lambda}{d}$$

Angle of diffracted waves by a periodic structure

$$\sin\theta_n = \frac{n\lambda}{s}$$