Test #1

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Quantum Mechanics
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Prob. 1(25)

A beam of mono-energetic electrons having mass M and energy E strikes the surface of a metal at normal incidence at x=0. Assume V(x) = 0 for x<0 and the metal surface can be modeled with a potential barrier having $-V_0$, or $V(x) = -V_0 < 0$ for x>0. Use E = 1eV and V_0 = 8 eV for answering following questions.

(a)(10) What is the probability for these electrons to get transmitted into the metal? Give a numerical answer.

(b)(15) Where is the probability of finding electronics largest for x<0? Give your answer in terms of variables given in this problem and other fundamental constants.

Prob. 2(35)

Consider a particle with mass M confined in a quantum well where V(x) is infinitely large for x < 0, V(x) = 0 for 0<x<L, and V(x) = V_0 for x>L.

(a)(10) Determine the relationship between k and κ, where k is the wave number in the well and κ is the decaying coefficient of the wave function for x>L. Simplify you answer as much as possible.

(b)(10) Using the answer obtained in (b), determine the requirement so that there is only one eigen state in the well.

(c)(10) Sketch wave functions when there are two eigen states in the well. Your sketch should clear show different features of two wave functions such as peak locations and the amount of penetration into the barrier.

(d)(5) If the above system has the equal superposition of two eigen states at t=0, explain how the probability density of finding the particle evolves with time.

Prob. 3(40)

Consider a particle with mass M confined in a three-dimensional quantum well having infinitely large potential barriers where $V(x) = 0$ for $0<x<L$, $0<y<L$, and $0<z<L$, and infinitely large everywhere else.

(a)(10) Assuming $\Psi(x,y,z)=X(x)Y(y)Z(z)$, derive three 1-dimentional Schrödinger equations, each of which involves only $X(x)$, $Y(y)$ or $Z(z)$.

(b)(10) Solve each of three Schrödinger equations obtained in (a) using the proper boundary condition.

(c)(10) What is the ground state eigen energy? Your answer should be an expression involving M,L and other fundamental constants.

(d)(5) What is the second lowest eigen energy? How many distinctive eigen states does have this eigen energy have?

(e)(5) Assuming the eigen energy, E, is much larger than the ground state eigen energy, show that the rate of increase in the number of eigen states with respect to E or, $\frac{dN}{dE}$, is proportional to $\sqrt{E}$.