Test #2

Dec. 12, 2016
Quantum Mechanics
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Prob. 1 (20)

A beam of mono-energetic electrons having mass M and energy E strikes the surface of a metal at normal incidence at x=0. Assume V(x) = 0 for x<0 and the metal surface can be modeled with a potential barrier having $-V_0$, or $V(x) = -V_0 < 0$ for x>0. Use E = 1eV and V_0 = 8 eV.

(a)(10) Determine the transmission probability for the electrons.

(b)(10) Determine the location (x<0) nearest to the interface, where the observed electron number is the largest.

Prob. 2 (25)

A certain quantum mechanical measurement is described by $\hat{A}|f\rangle = \mu|f\rangle$, where $\hat{A}$ is an operator, $|f\rangle$ is the non-zero normalized eigen vector, and μ is the non-degenerate eigen value. It is experimentally observed that performing two consecutive measurements brings the system back to the original state or $\hat{A}\hat{A}|f\rangle = |f\rangle$.

(a)(5) Determine the numerical value for μ.
(b)(5) What is the dimension of $\hat{A}$?
(c)(5) Determine $\hat{A}$ in the vector space spanned by the eigen vectors.
(d)(10) Determine $|f\rangle$.

Prob. 3(15)

The angular momentum of a quantum mechanical particle is measured and its magnitude square (L^2) is found $30\hbar^2$.

(a)(5) If the z-component of the angular momentum is subsequently measured, what are the possible measurement results?

(b)(5) Suppose measurement in (a) produces the largest value for L_z. Then, what is the minimum uncertainty for $\Delta L_x \Delta L_y$?

(c)(5) L^2 is once again measured after (a) and (b) measurements, what is (are) the possible measurement result(s)?

Prob. 4 (20)

A quantum system has two measurable quantities: c with associated operator $\hat{C}$, and d with associated operator $\hat{D}$. Operator $\hat{C}$ has two eigen vectors, $|\phi_1\rangle$ and $|\phi_2\rangle$, and operator $\hat{D}$ has two eigen vectors, $|\Psi_1\rangle$ and $|\Psi_2\rangle$. The relation between these eigen vectors is given as:

$$|\phi_1\rangle = \frac{3}{5}|\Psi_1\rangle + \frac{4}{5}|\Psi_2\rangle$$

$$|\phi_2\rangle = \frac{4}{5}|\Psi_1\rangle - \frac{3}{5}|\Psi_2\rangle$$

Suppose a measurement is done on c and the system is found in state $|\phi_1\rangle$. Then another measurement is made on d, and then c is again measured. Determine the probability that the system will be found in the state $|\phi_1\rangle$ on the second measurement of c.

Prob. 5 (20)

As we discussed in the class, cryptographic keys can be transmitted quantum mechanically without danger of eavesdropping. In this problem, the basic idea of this quantum mechanical key distribution is outlined.

A sender (Alice) randomly selects one out of two different pairs of orthorgonal bases (photon polarization) to encode 0 or 1 bit, and the receiver (Bob) also uses the randomly selected orthogonal basis to decode the received photon. The figure below shows the random bit sequence, polarization basis Alice uses, the resulting photon polarization, and Bob's random measuring basis.

Alice's random bit	0	1	1	0	1	0	0	1
Alice's random sending basis	+	+	×	+	×	×	×	+
Photon polarization Alice sends	↑	→	↘	↑	↘	↗	↗	→
Bob's random measuring basis	+	×	×	×	+	×	+	+

(a)(5) Determine the photon polarization Bob measures for each bit sent. Represent your answer in a sequence of arrows in the same manner as shown in the row for "Photon polarization Alice sends". If there are more than one possible photon polarization states, show all of them.

(b)(5) After the key transmission, Alice can tell Bob the bases she used in an open channel and, with this, Bob keeps only those received bits for which Alice and Bob used the same basis. Determine which bits are kept by Bob.

(c)(10) If an intruder (Eve) eavesdropps the key Alice sent to Bob, Alice and Bob can determine this by measuring the error rate in the randomly selected bits among those bits Bot keeps. Determine this error rate. Explain your answer.