Lect. 10: Pole, Zero, Bode Plot  (Razavi 11.1)

Frequency response is an important element of circuit/system characteristics

What determines frequency responses of MOS amplifiers?
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- How do we express frequency-domain characteristics of circuits?

⇒ Transfer functions in s-domain (Laplace Transform)

![Circuit Diagram]

\[ H(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{1/sC}{1/sC + R} = \frac{1}{1 + sRC} \]

We are often interested in the sinusoidal steady-state response \( s = j\omega \)

\[ H(j\omega) = \frac{1}{1 + j\omega RC} \]
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- How do magnitude and phase of $H(j\omega)$ change as frequency changes?

$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

**Magnitude:**

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

**Phase:**

$$\angle H(j\omega) = -\tan^{-1}(\omega RC)$$

**Magnitude:** $20\log_{10}|H(j\omega)|$ (dB) vs log$\omega$

**Phase:** $\angle H(j\omega)$ vs log$\omega$
Bode Plots

Named after Hendrik Bode (1905 - 1982), an American engineer who specialized in control theory.
In general, linear systems (electronic circuits included) are characterized by multiple poles and zeros

\[ H(s) = A_0 \frac{\left(1 + \frac{s}{\omega_{z1}}\right)\left(1 + \frac{s}{\omega_{z2}}\right)\ldots}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)\ldots} \]

Zeros:

\[
\begin{align*}
\text{Zeros:} & \quad s = -\omega_{z1}, -\omega_{z2}, \ldots \\
\text{Poles:} & \quad s = -\omega_{p1}, -\omega_{p2}, \ldots
\end{align*}
\]

Bode plot?

\[ \Rightarrow \text{Frequency response can be determined by pole, zero identification and 'addition' of each pole and zero response in log scale} \]
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\[ H(s) = s \]
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\[ H(s) = \frac{1}{s} \]

\[ M = \frac{1}{\omega} \]

\[ \psi = -90^\circ \]

\[ |H(j\omega)| \]

\[ \angle H(j\omega) \]
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\[ H(s) = 1 + \frac{s}{\omega_z} \]

**Magnitude**

- \( \omega \ll \omega_z \): Does not change much with \( \omega \)
- \( \omega \gg \omega_z \): Proportionally increase with \( \omega \),

**Phase**

- \( \omega \ll \omega_z \): 0
- \( \omega \gg \omega_z \): 90 deg

\[ H(s) = -1 + \frac{s}{\omega_z} \]
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\[ H(s) = \frac{1}{1 + \frac{s}{\omega_p}} \]

**Magnitude**

- \( \omega << \omega_p \): Does not change much with \( \omega \)
- \( \omega >> \omega_p \): Proportionally decrease with \( \omega \)

**Phase**

- \( \omega << \omega_p \): 0
- \( \omega >> \omega_p \): - 90 deg
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\[ H(s) = \frac{s}{(s + \frac{1}{R_S C_1})(s + \frac{1}{R_F C_2})}, \quad R_S C_1 > R_F C_2 \]

\[ \log |H(s)| = \log |s| + \log \left(1 + \frac{1}{s + \frac{1}{R_S C_1}}\right) + \log \left(1 + \frac{1}{s + \frac{1}{R_F C_2}}\right) \]

\[ \angle H(s) = \angle s + \angle \left(\frac{1}{s + \frac{1}{R_S C_1}}\right) + \angle \left(\frac{1}{s + \frac{1}{R_F C_2}}\right) \]
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- Bode Plot for MOS circuit
  (Ignoring MOS frequency response, $\lambda = 0$)
- Homework:
Determine magnitude and phase Bode plots for small-signal voltage gain \((V_{out}/V_{in})\).
Ignore the frequency response of \(M_1\). Assume \(\lambda = 0\), the input pole frequency is lower than the output pole and zero frequencies, and all pole zero frequencies are well separated.