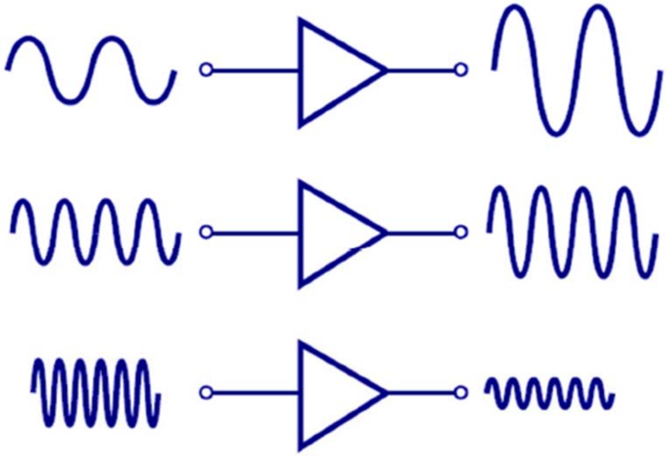
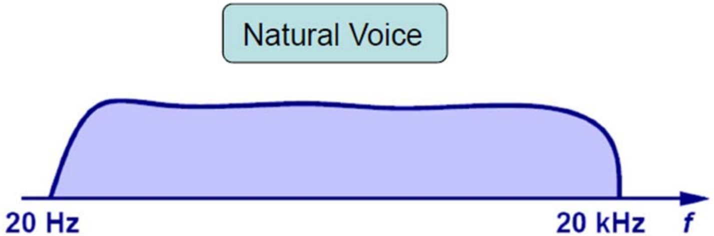
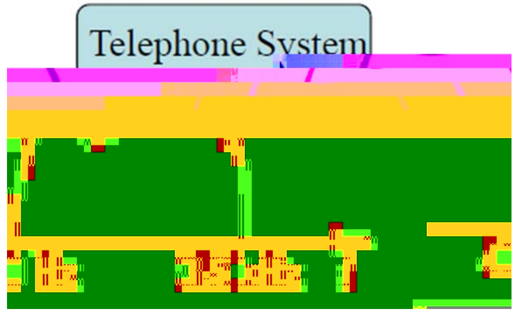
Lect. 10: Pole, Zero, Bode Plot (Razavi 11.1)

Frequency response is an important element of circuit/system characteristics





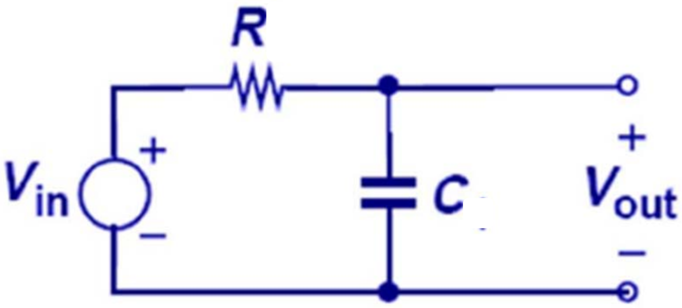


➔ What determines frequency responses of MOS amplifiers?

Lect. 10: Pole, Zero, Bode Plot

- How do we express frequency-domain characteristics of circuits?

➔Transfer functions in s-domain (Laplace Transform)

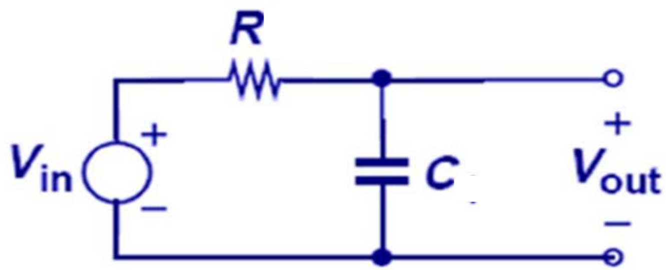


$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1/sC}{1/sC + R} = \frac{1}{1 + sRC}$$

We are often interested in the sinusoidal steady-state response $s = j\omega$

$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

Lect. 10: Pole, Zero, Bode Plot



$$H(j\omega)=\frac{1}{1+j\omega RC}$$

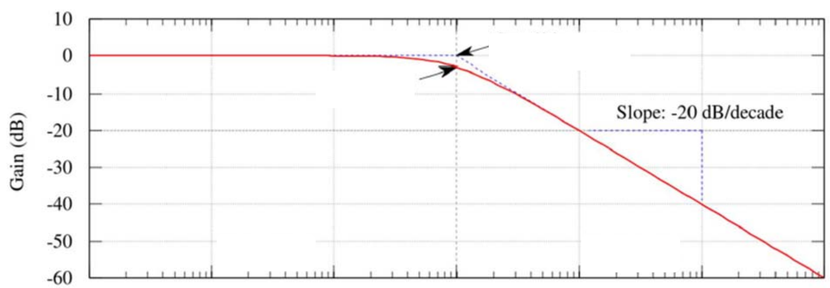
- How do magnitude and phase of $H(j\omega)$ change as frequency changes?

$$|H(j\omega)|=\frac{1}{\sqrt{1+(\omega RC)^2}}$$

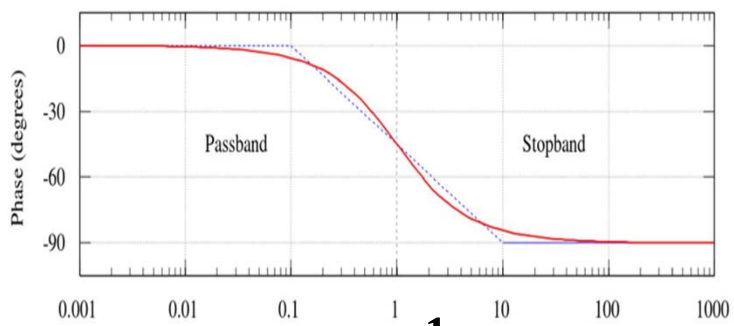
$$\angle H(j\omega)=-\tan^{-1}(\omega RC)$$

Magnitude: $20\log_{10}|H(j\omega)|$ **(dB) vs** $\log\omega$

Phase: $\angle H(j\omega)$ **vs** $\log\omega$

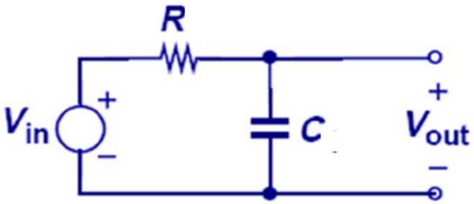


$$\omega=\frac{1}{RC}$$

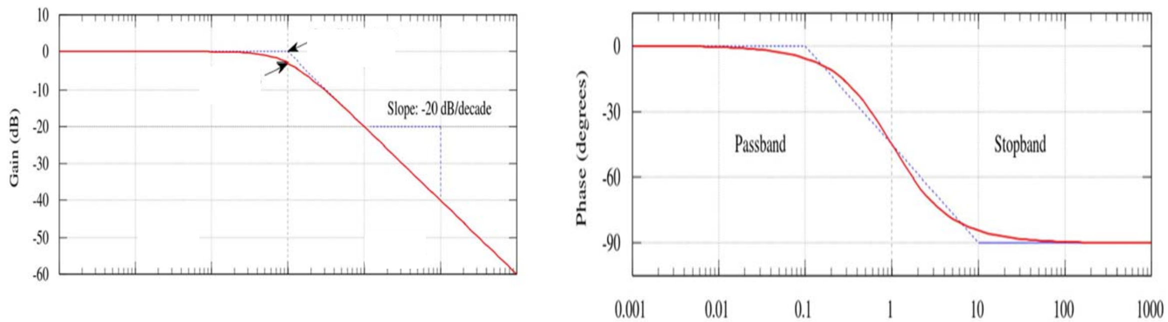


$$\omega=\frac{1}{RC}$$

Lect. 10: Pole, Zero, Bode Plot



Bode Plots



Named after Hendrik Bode (1905 – 1982),
an American engineer who specialized in control theory

Lect. 10: Pole, Zero, Bode Plot

In general, linear systems (electronic circuits included) are characterized by multiple poles and zeros

$$H(s) = A_0 \frac{\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)\cdots}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\cdots}$$

Zeros:

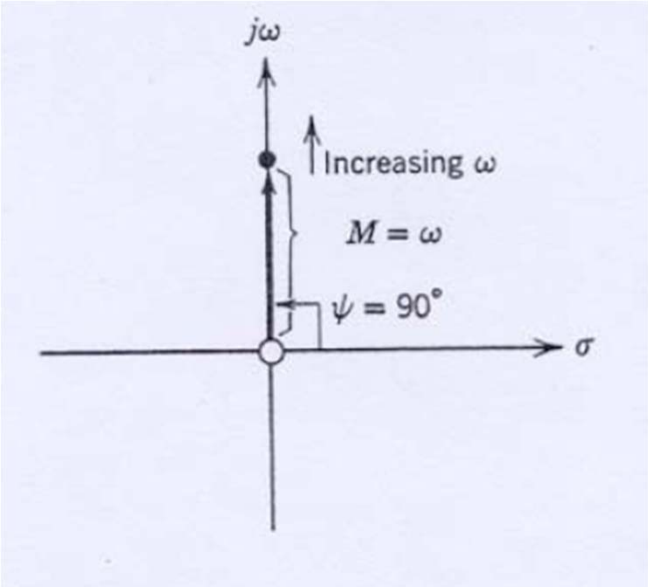
Poles:

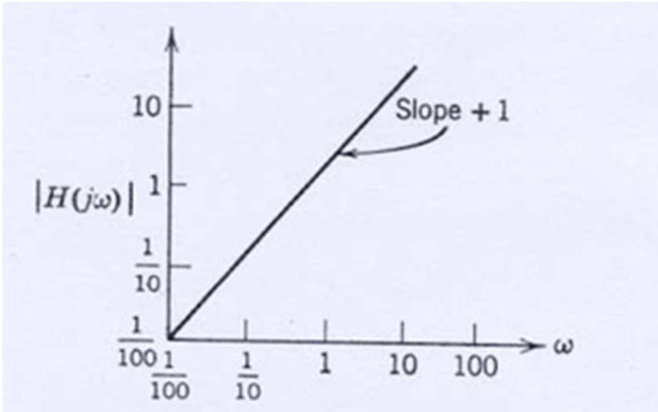
Bode plot?

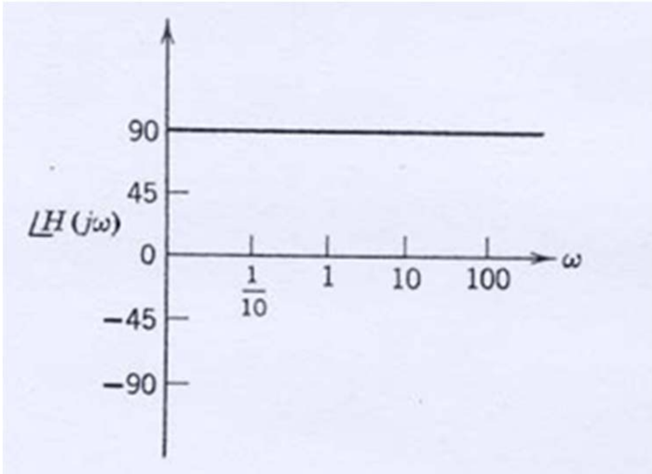
➔ Frequency response can be determined by pole, zero identification and 'addition' of each pole and zero response in log scale

Lect. 10: Pole, Zero, Bode Plot

$H(s) = s$

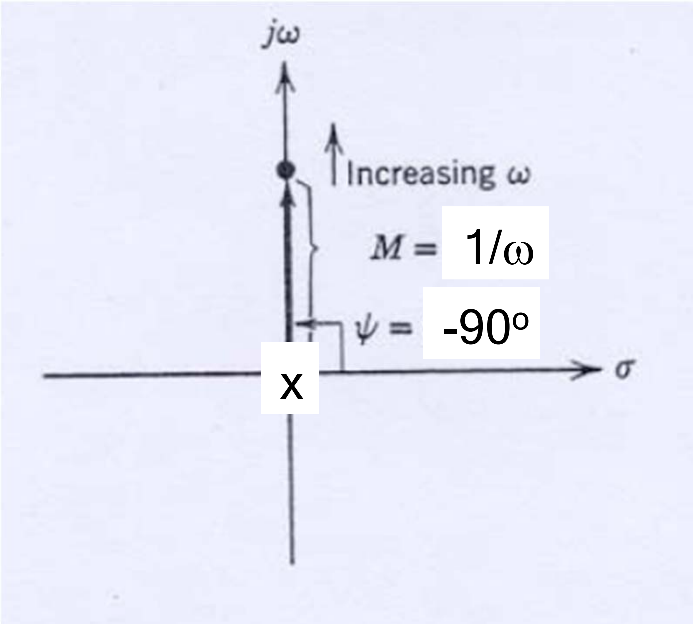


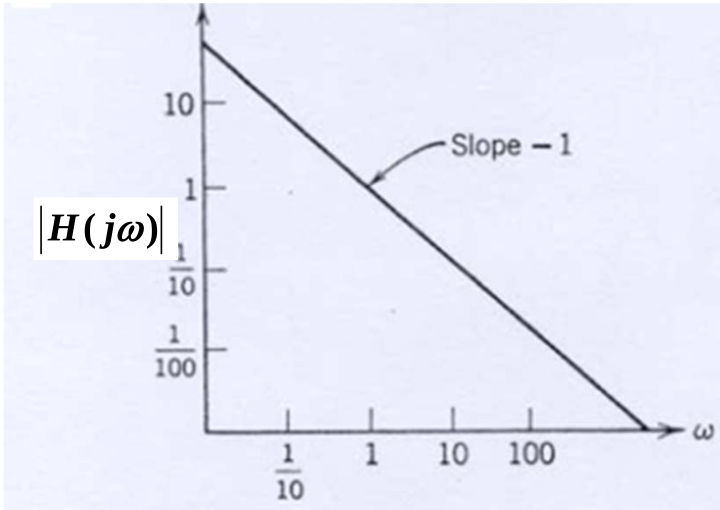


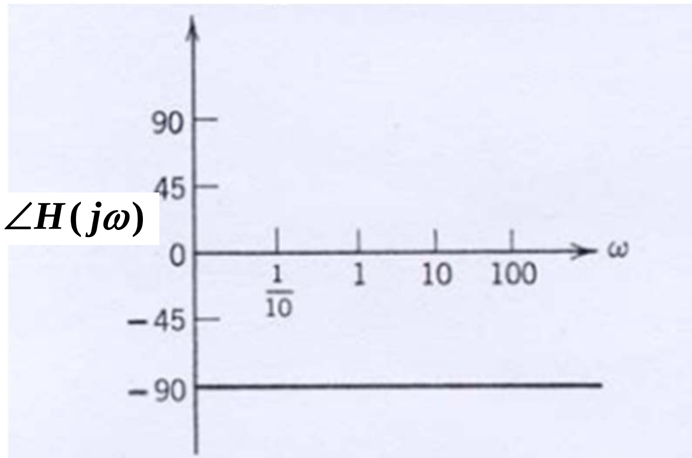


Lect. 10: Pole, Zero, Bode Plot

H(s) = 1/s

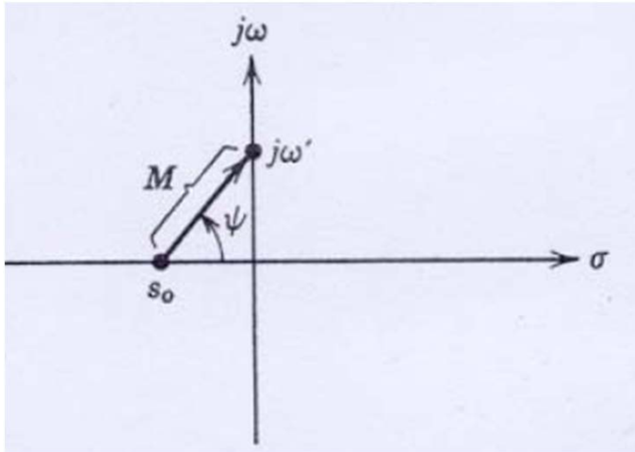


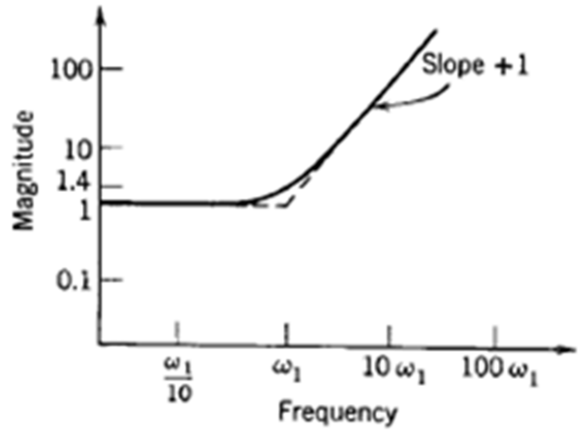




Lect. 10: Pole, Zero, Bode Plot

$$H(s) = 1 + \frac{s}{\omega_z}$$

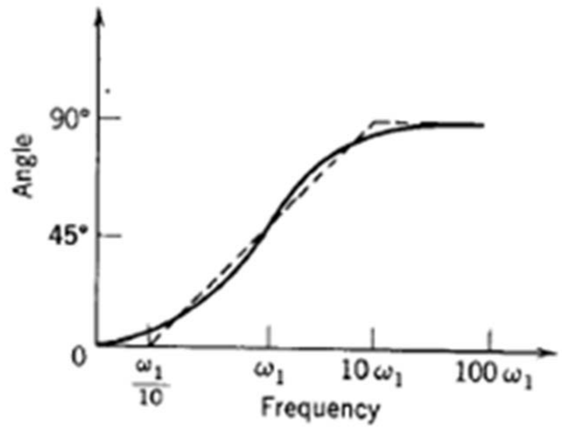




Magnitude

$\omega \ll \omega_z$: Does not change much with ω

$\omega \gg \omega_z$: Proportionally increase with ω,



Phase

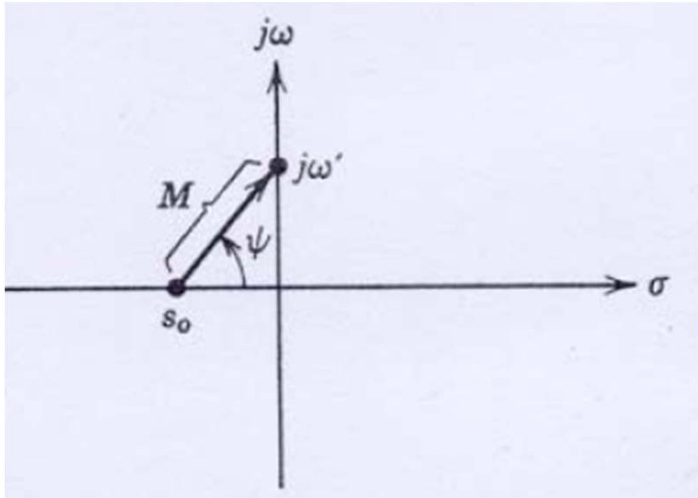
$\omega \ll \omega_z$: 0

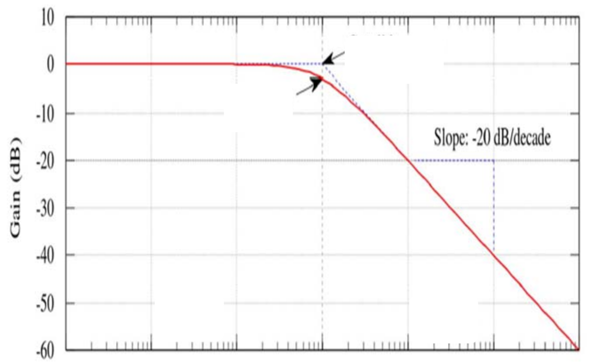
$\omega \gg \omega_z$: 90 deg

$$H(s) = -1 + \frac{s}{\omega_z}$$

Lect. 10: Pole, Zero, Bode Plot

$$H(s) = \frac{1}{1 + \frac{s}{\omega_p}}$$





Magnitude

$\omega << \omega_p$: Does not change much with ω

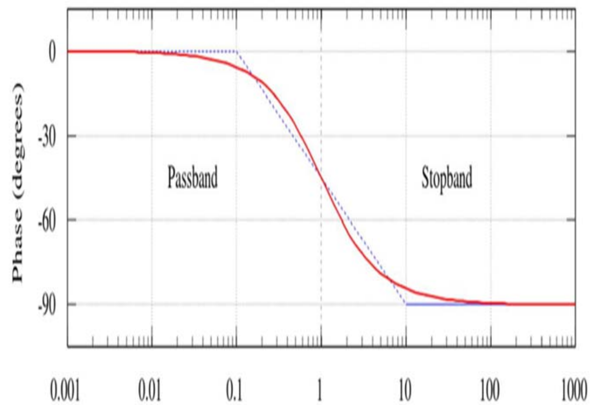
$\omega >> \omega_p$: Proportionally decrease with ω,

Phase

$\omega << \omega_p$: 0

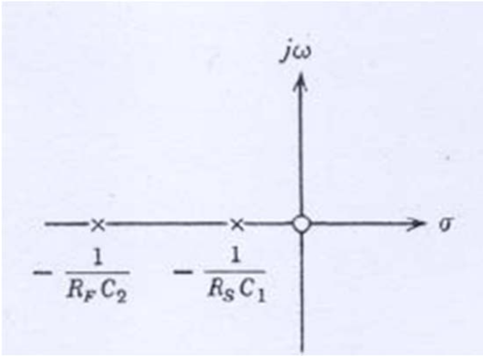
$\omega >> \omega_p$: - 90 deg

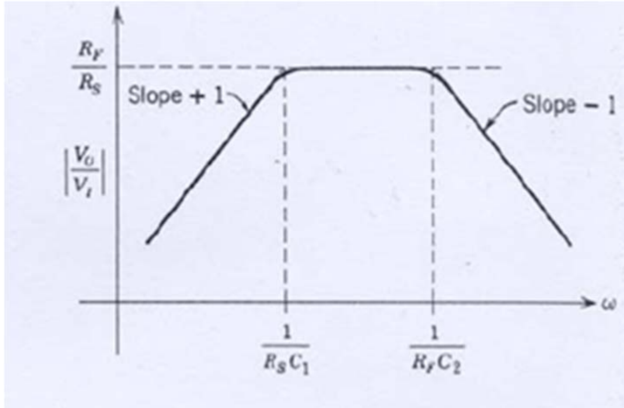
$$H(s) = \frac{1}{1 - \frac{s}{\omega_p}}$$



Lect. 10: Pole, Zero, Bode Plot

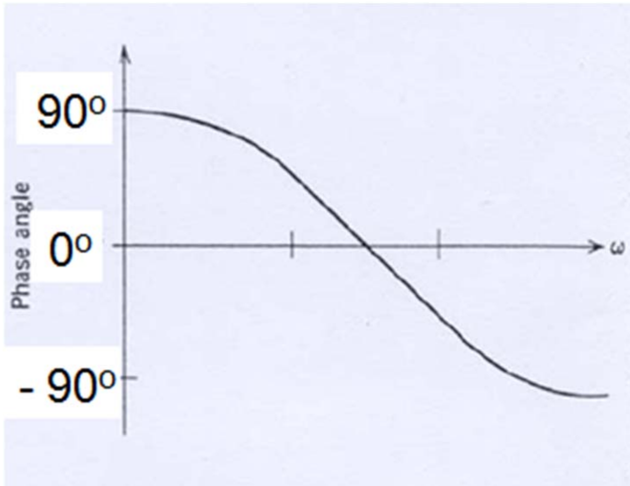
$$H(s)=\frac{s}{(s+\frac{1}{R_SC_1})(s+\frac{1}{R_FC_2})},\ R_SC_1>R_FC_2$$





$$\log|H(s)|=\log|s|+\log\left|\frac{1}{(s+\frac{1}{R_SC_1})}\right|+\log\left|\frac{1}{(s+\frac{1}{R_FC_2})}\right|$$

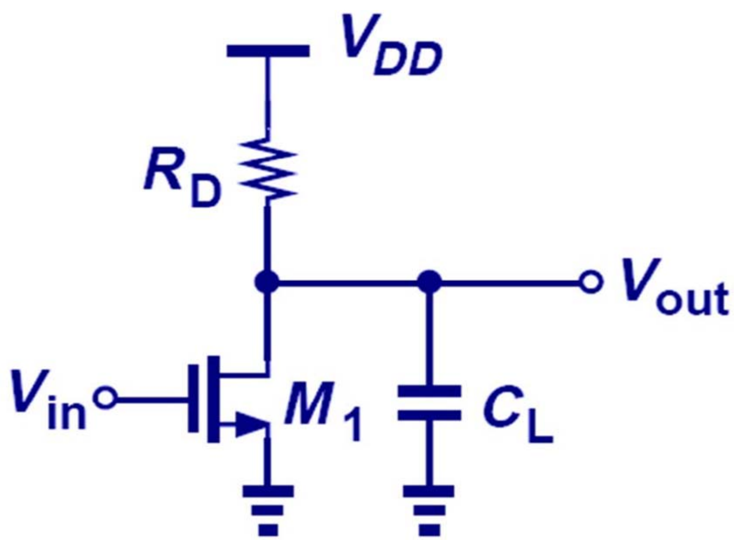
$$\angle H(s)=\angle s+\angle\frac{1}{(s+\frac{1}{R_SC_1})}+\angle\frac{1}{(s+\frac{1}{R_FC_2})}$$

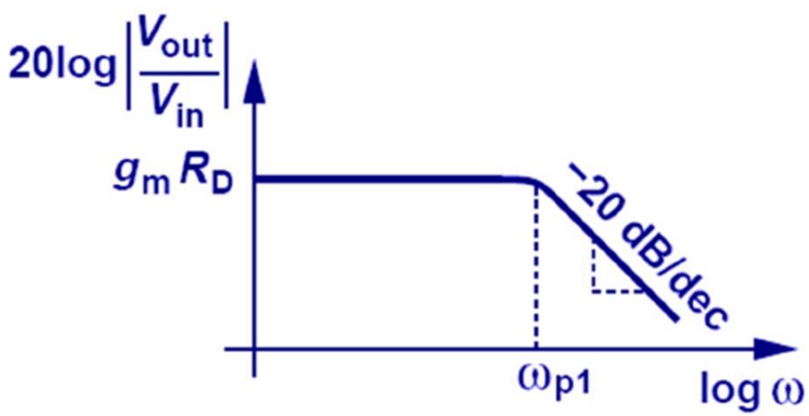


W.-Y. Choi

Lect. 10: Pole, Zero, Bode Plot

- Bode Plot for MOS circuit
(Ignoring MOS frequency response, $\lambda = 0$)





Lect. 10: Pole, Zero, Bode Plot

- Homework:

Determine magnitude and phase Bode plots for small-signal voltage gain (V_{out}/V_{in}). Ignore the frequency response of M_1. Assume $\lambda = 0$, the input pole frequency is lower than the output pole and zero frequencies, and all pole zero frequencies are well separated.