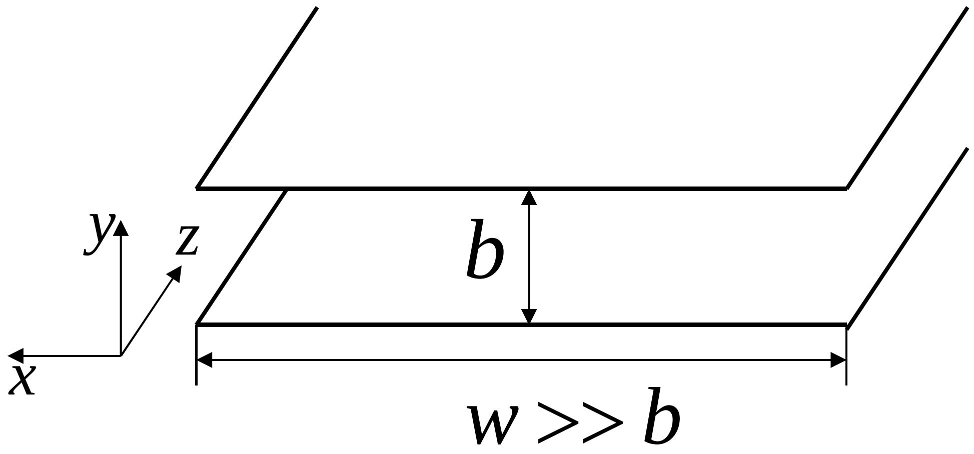
Lect. 13: Metallic Waveguides

Can EM waves propagate without diffraction?   Waveguides

Metallic Waveguide



Solve the E&M wave equation with above boundaries

$$\nabla^2 \overline{E} - \mu\varepsilon \frac{\partial^2}{\partial t^2}\overline{E} = 0$$

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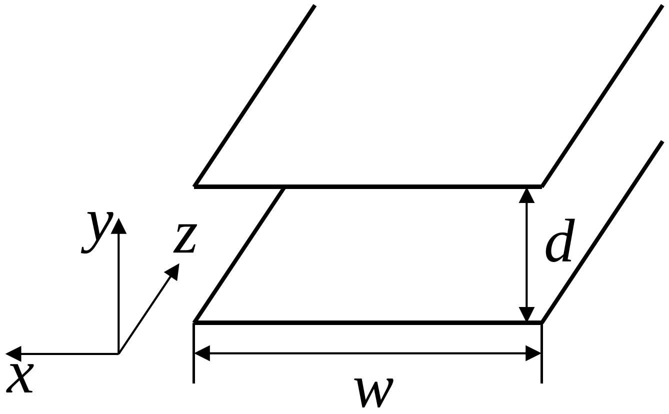

$$\nabla^2 \bar{E} - \mu\varepsilon \frac{\partial^2}{\partial t^2}\bar{E} = 0$$

Assume $\bar{E}(x, y, z, t) = \bar{E}(y) \cdot e^{-j\beta z} \cdot e^{j\omega t}$

$$\nabla^2 \bar{E} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \bar{E} = \left(\frac{d^2}{dy^2} - \beta^2 \right) \bar{E}(y)$$

$$\mu\varepsilon \frac{\partial^2}{\partial t^2}\bar{E} = -\omega^2 \mu\varepsilon \bar{E} = -k^2 \bar{E} \quad (k^2 = \omega^2 \mu\varepsilon)$$

$$\frac{d^2}{dy^2}\bar{E}(y) + (k^2 - \beta^2)\bar{E}(y) = 0$$

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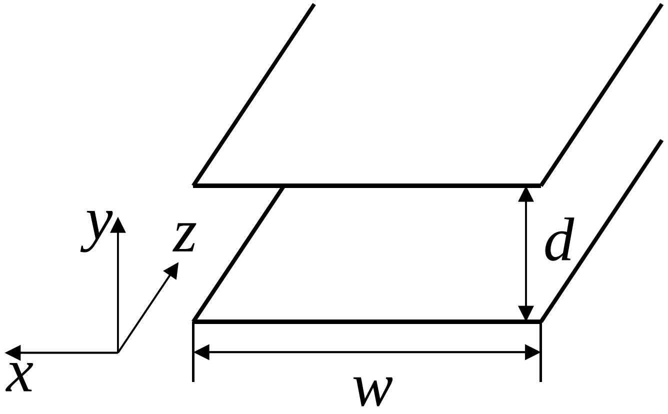

$$\bar{E}(x,y,z,t)=\bar{E}(y)\cdot e^{-j\beta z}\cdot e^{j\omega t}$$

$$\frac{d^2}{dy^2}\bar{E}(y)+(k^2-\beta^2)\bar{E}(y)=0$$

$$(k^2=\omega^2\mu\varepsilon)$$

Simplest solution: $\bar{E}(y)=\bar{y}E_0$ with $\beta=k$

$\bar{E}(x,y,z,t)=\bar{y}E_0e^{-jkz}e^{j\omega t}$ (Plane wave between top and bottom plates)

Corresponding magnetic field? $\bar{H}(x,y,z,t)=-\bar{x}\dfrac{E_0}{\eta}e^{-jkz}e^{j\omega t}$

Boundary conditions?

➔ TEM waves

Used for co-axial cables

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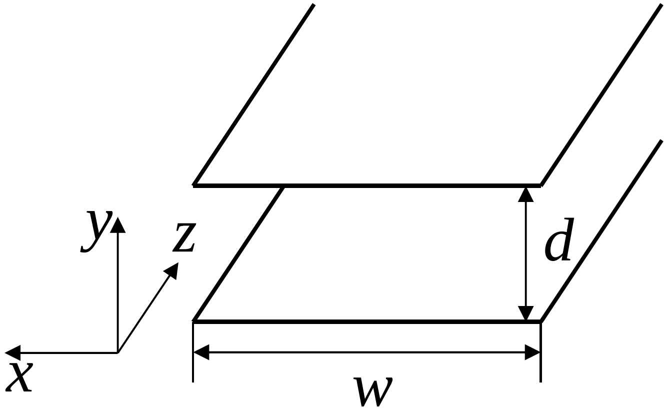

$$\bar{E}(x,y,z,t) = \bar{E}(y)\cdot e^{-j\beta z}\cdot e^{j\omega t}$$

$$\frac{d^2}{dy^2}\bar{E}(y) + (k^2-\beta^2)\bar{E}(y) = 0$$

Another type of solution: $\bar{E}(y) = \bar{x}E_0\sin(k_y y)$

$$\bar{E}(x,y,z,t) = \bar{x}E_0\sin(k_y y)e^{-j\beta z}e^{j\omega t} \quad (\beta^2 = k^2 - k_y{}^2)$$

Boundary conditions? $k_y = \dfrac{m\pi}{d}$ Quantization of k_y and β

➔ TE Solution

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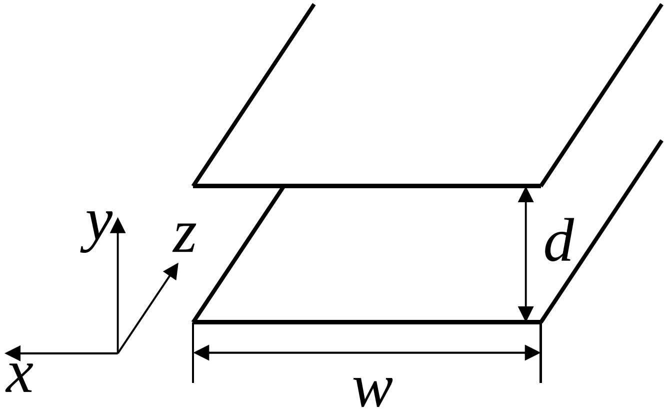


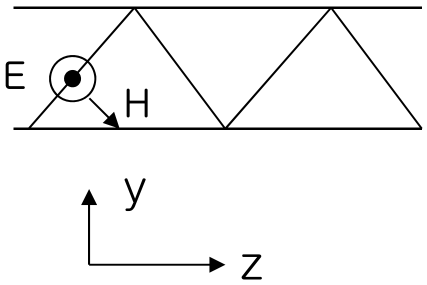
TE Solution

$$\bar{E}(y,z) = \bar{x}E_0 \sin(k_y y)e^{-j\beta z}$$

$$k_y = \frac{m\pi}{d} \qquad \beta^2 = k^2 - \left(\frac{m\pi}{d}\right)^2$$

Interpretation $\sin(k_y y)e^{-j\beta z} \sim (e^{jk_y y} - e^{-jk_y y})e^{-j\beta z}$

➔ two plane waves propagating in different directions



(perpendicular polarization)

In y-direction, the phase should be the same after one round-trip

$$\exp\left(-j2k_y d\right) = 1 \quad \Rightarrow \quad k_y = \frac{2m\pi}{2d} = \frac{m\pi}{d}$$

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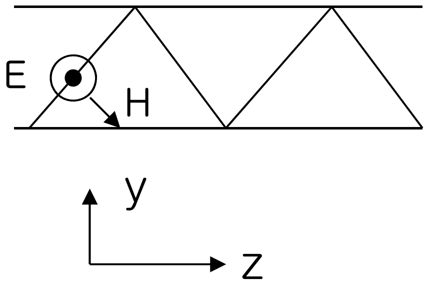


TE Solution

$$\bar{E}(y,z) = \bar{x}E_0 \sin(k_y y)e^{-j\beta z}$$

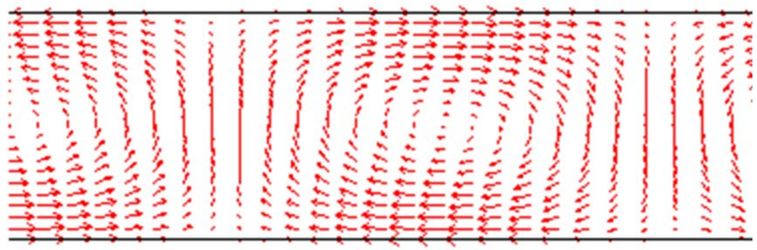
$$\bar{H}(y,z) = \left[\bar{y}H_1 \sin(k_y y) + \bar{z}H_2 \cos(k_y y)\right]e$$

$$k_y = \frac{m\pi}{d}$$

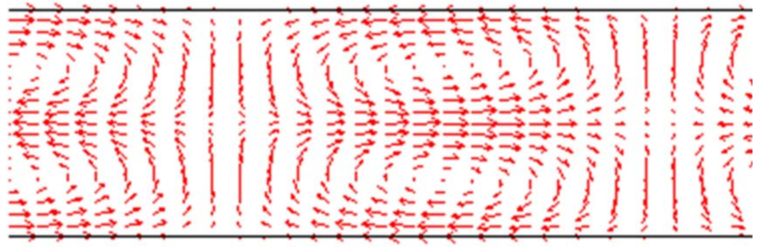
Each *m* gives different solution

➔ mode: TE_m

TE_1



TE_2



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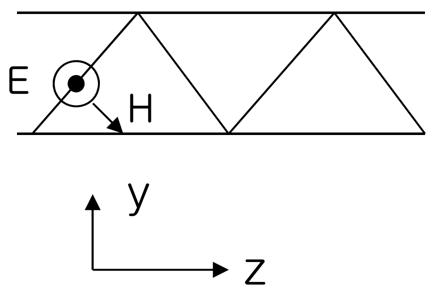

TE Solution

$$\bar{E}(y,z) = \bar{x}E_0 \sin(k_y y)e^{-j\beta z} \qquad k_y = \frac{m\pi}{d}$$

Each mode has its own propagation constant β

$$\beta = \sqrt{k^2 - k_y^{\,2}} = \sqrt{\omega^2 \mu\varepsilon - \frac{m^2\pi^2}{d^2}}$$

How many modes for a given waveguide at ω?

For given *m*, there is ω for which β=0

$$\omega_c = \frac{m\pi}{d\sqrt{\mu\varepsilon}} \quad \text{or} \quad f_c = \frac{m}{2d\sqrt{\mu\varepsilon}},$$

➔ cut-off frequency

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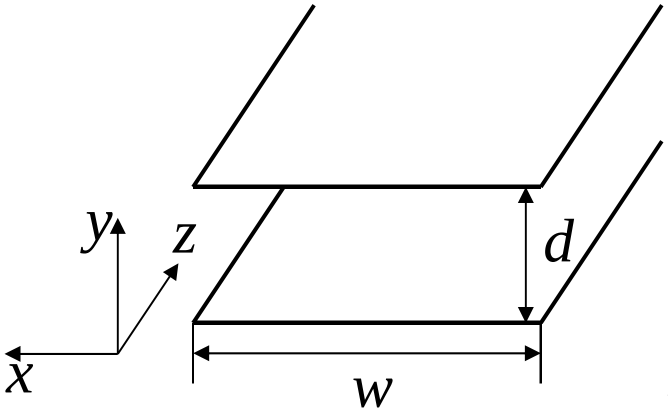

Wave equation for magnetic field

$$\frac{d^2}{dy^2}\bar{H}(y) + (k^2 - \beta^2)\bar{H}(y) = 0$$

$$\bar{H}(y) = \bar{x}H_0 \cos(k_y y) \quad (k^2 = k_y^2 + \beta^2)$$

$$\bar{H}(y,z) = \bar{x}H_0 \cos(k_y y) e^{-j\beta z}$$

Boundary conditions? $k_y = \frac{m\pi}{d}$ Quantization in k_y and β

Corresponding electric field?

$$\bar{E}(y,z) = \bar{y}E_1 \cos(k_y y) e^{-j\beta z} + \bar{z}E_2 \sin(k_y y) e^{-j\beta z}$$

➔ TM Solution

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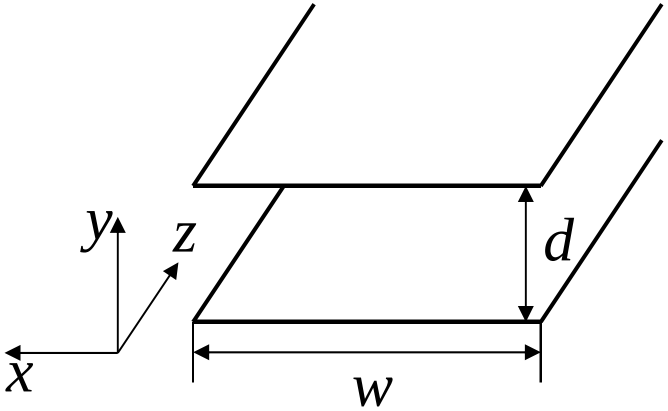
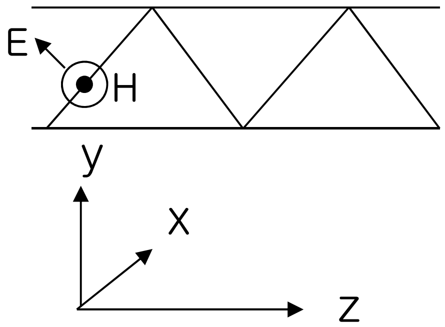


TM Solution

$$\bar{H}(y,z) = \bar{x}H_0 \cos(k_y y)e^{-j\beta z} \quad k_y = \frac{m\pi}{d}$$

Interpretation $\cos(k_y y)e^{-j\beta z} \sim (e^{jk_y y} + e^{-jk_y y})e^{-j\beta z}$

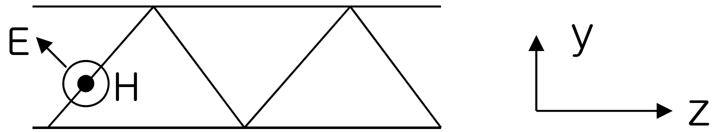
➔ two plane waves propagation in different directions

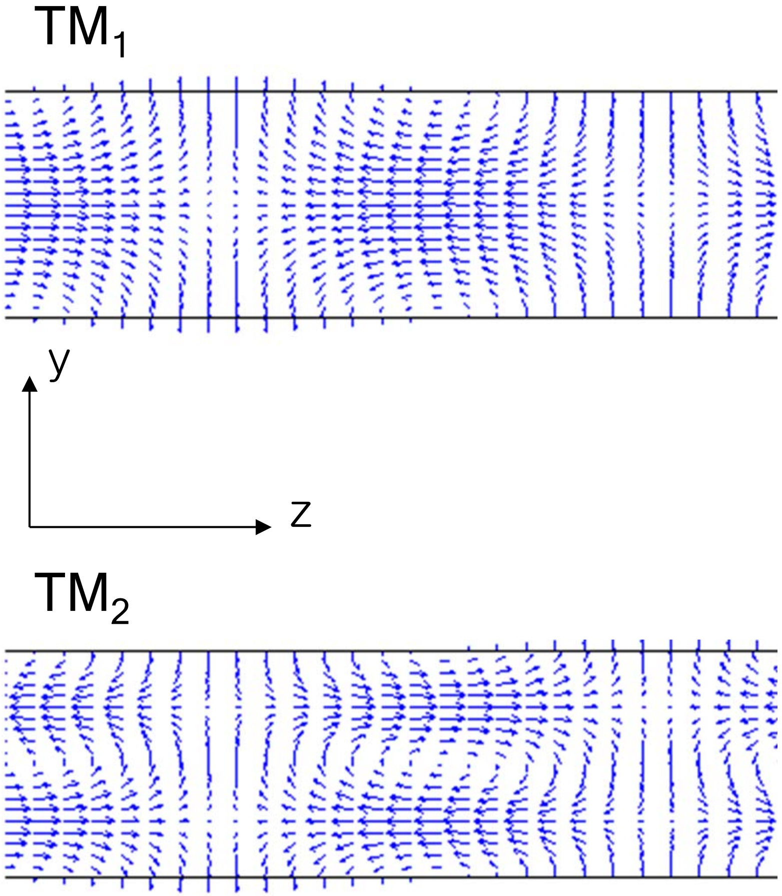


(parallel polarization)

In y-direction, same phase after one round-trip

$$\exp\left(-j2k_y d\right) = 1 \quad \Rightarrow \quad k_y = \frac{2m\pi}{2d} = \frac{m\pi}{d}$$





TM Solution

$$\bar{H}(y,z) = \bar{x}H_0 \cos(k_y y)e^{-j\beta z}$$

$$\bar{E}(y,z) = \left[\bar{y}E_1 \cos(k_y y) + \bar{z}E_2 \sin(k_y y)\right]e^{-j\beta z}$$

$$k_y = \frac{m\pi}{d}$$

Each *m* gives different solution

➔ mode: TM$_m$

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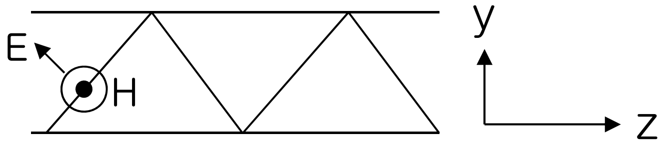
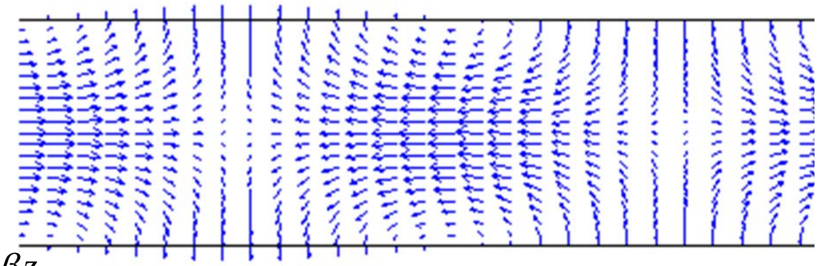
TM$_1$

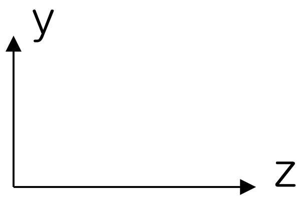


$$\bar{H}(y,z) = \bar{x}H_0 \cos(k_y y)e^{-j\beta z}$$

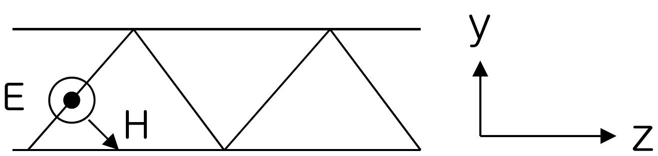
$$\bar{E}(y,z) = \left[\bar{y}E_1 \cos(k_y y) + \bar{z}E_2 \sin(k_y y)\right]e^{-j\beta z}$$

TM$_1$





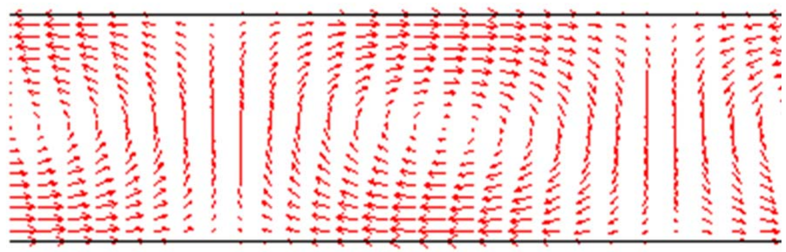
TE$_1$



$$\bar{E}(y,z) = \bar{x}E_0 \sin(k_y y)e^{-j\beta z}$$

$$\bar{H}(y,z) = \left[\bar{y}H_1 \sin(k_y y) + \bar{z}H_2 \cos(k_y y)\right]e^{-j\beta z}$$

TE$_1$



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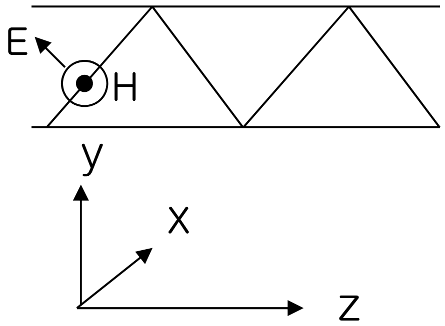


TM Solution

$$\bar{H}(y,z) = \bar{x}H_0 \cos(k_y y)e^{-j\beta z} \quad k_y = \frac{m\pi}{d}$$

Each mode has its own propagation constant β

$$\beta = \sqrt{k^2 - k_y^2} = \sqrt{\omega^2 \mu\varepsilon - \frac{m^2\pi^2}{d^2}}$$

For a given waveguide, there is a finite number of guided modes at ω

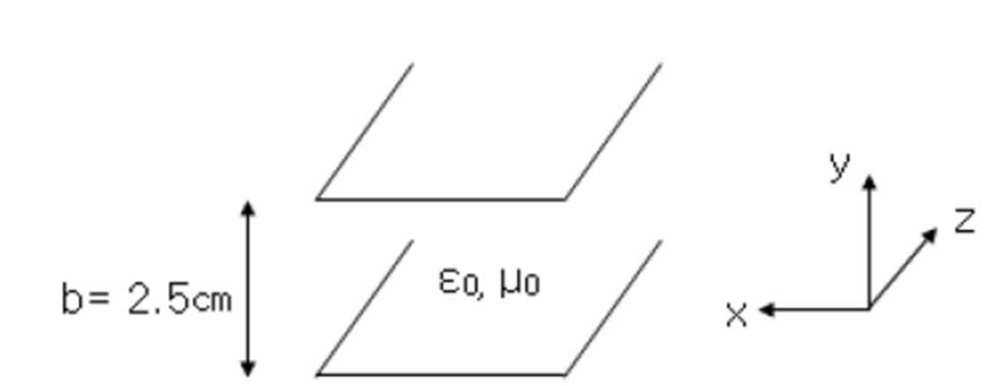
For given *m*, there is ω for which β=0

$$\omega_c = \frac{m\pi}{d\sqrt{\mu\varepsilon}} \quad \text{or} \quad f_c = \frac{m}{2d\sqrt{\mu\varepsilon}},$$

➔ cut-off frequency

Homework

Consider a lossless parallel–plate waveguide shown below.



(a) How many TE or TM modes are there that can propagate in the waveguide if the EM wave frequency is 30 GHz?

(b) Find the expression for E(x,y,z) of TE_2 mode.