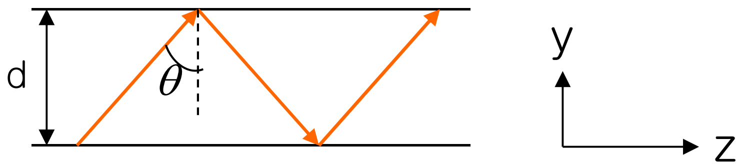
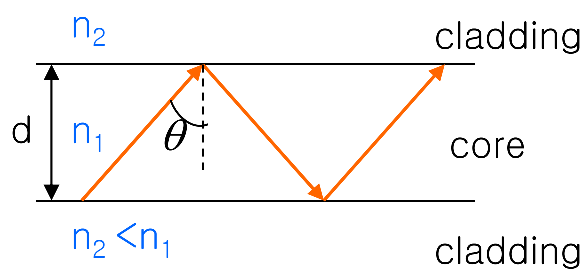
Lect. 14: Dielectric Waveguides (1)

Guidance condition in a waveguide

Metallic waveguide (TE, TM)



Dielectric waveguide (TIR)



$$e^{-jk_y d}(-1)e^{-jk_y d}(-1) = e^{-j2k_y d} = 1$$

$$\therefore 2k_y d = 2m\pi \text{ and } k_y = \frac{m\pi}{d}$$

$$\beta(=k_z) = \sqrt{(nk_0)^2 - \left(\frac{m\pi}{d}\right)^2}$$

$$\tan\theta = \frac{\beta}{k_y}$$

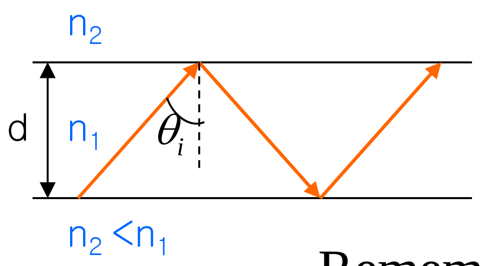
$$e^{-jk_y d}\ r_{\perp,//}\ e^{-jk_y d}\ r_{\perp,//} = 1$$

$$\text{Since } r_{\perp,//} = e^{j\phi_{\perp,//}},\quad e^{-j2k_y d} e^{j2\phi_{\perp,//}} = 1$$

$$\therefore 2k_y d - 2\phi_{\perp,//} = 2m\pi$$

$$\text{Or } k_y d - \phi_{\perp,//} = m\pi$$

Lect. 14: Dielectric Waveguides (1)



$$k_y d - \phi_{\perp,//} = m\pi$$

Remember from Lect. 7 ($n = \dfrac{n_2}{n_1}$)

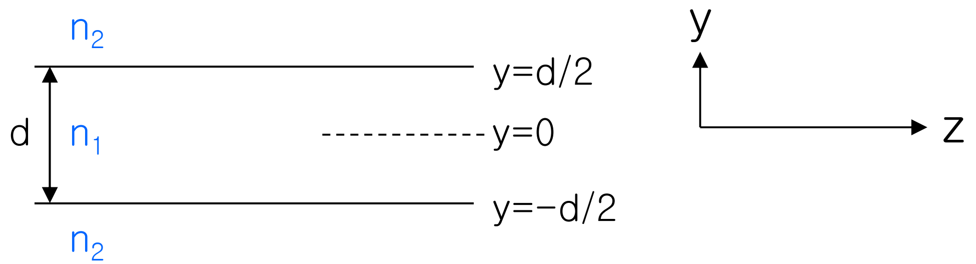
$$\phi_{\perp} : \tan\left(\frac{\phi_{\perp}}{2}\right) = \frac{(\sin^2\theta_i - n^2)^{1/2}}{\cos\theta_i} \qquad \text{(TE)}$$

$$\phi_{//} : \tan\left(\frac{\phi_{\parallel} + \pi}{2}\right) = \frac{(\sin^2\theta_i - n^2)^{1/2}}{n^2\cos\theta_i} \qquad \text{(TM)}$$

Numerically solve for $\phi_{\perp}$ and $\phi_{//}$ ➔ k_y for each mode

$$\beta = \sqrt{\omega^2\mu\varepsilon - k_y^{\,2}} = \sqrt{(n_1 k_0)^2 - k_y^{\,2}}$$

Lect. 14: Dielectric Waveguides (1)



What are field profiles for guided modes?

Full analysis starting from wave equations:

$$\nabla^2 \overline{E}(y,z,t) = \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2}(y,z,t)$$

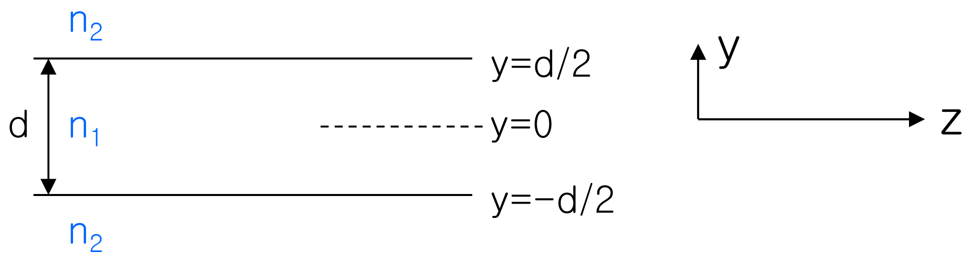
Assuming $\overline{E}(y,z,t) = \overline{E}(y) \cdot e^{-j\beta z} \cdot e^{j\omega t}$,

$$\frac{d^2}{dy^2}\overline{E}(y) + (k^2(y) - \beta^2)\overline{E}(y) = 0 \text{ where } k^2(y) = \omega^2 \mu\varepsilon(y)$$

$$k(y) = n_2 k_0 \text{ for } |y| > \frac{d}{2}; \text{ cladding}$$

$$k(y) = n_1 k_0 \text{ for } |y| < \frac{d}{2}; \text{ core}$$

Lect. 14: Dielectric Waveguides (1)



$$\frac{d^2}{dy^2}\bar{E}(y)+(k^2(y)-\beta^2)\bar{E}(y)=0,\ k^2(y)=\omega^2\mu\varepsilon(y)$$

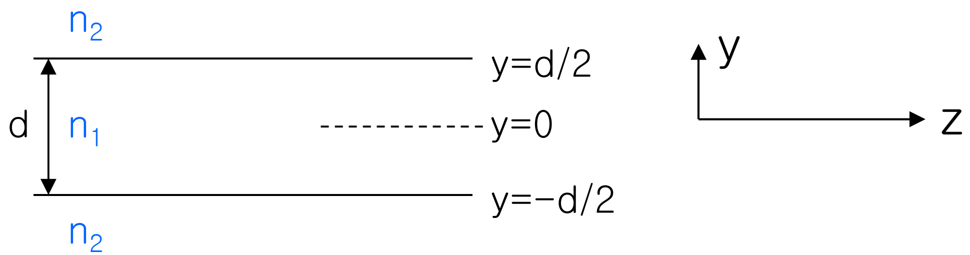
Consider TE Solution

$$\bar{E}(y)=\bar{x}\ E(y)$$

$$\frac{d^2E(y)}{dy^2}+(k^2(y)-\beta^2)E(y)=0$$

Solve for β and $E(y)$ (Eigen value equation)

Lect. 14: Dielectric Waveguides (1)



$$\frac{d^2E(y)}{dy^2}+(k^2(y)-\beta^2)E(y)=0$$

Sign of $(k^2(y)-\beta^2)$ determines the solution type

We know $n_1k_0>\beta>n_2k_0$

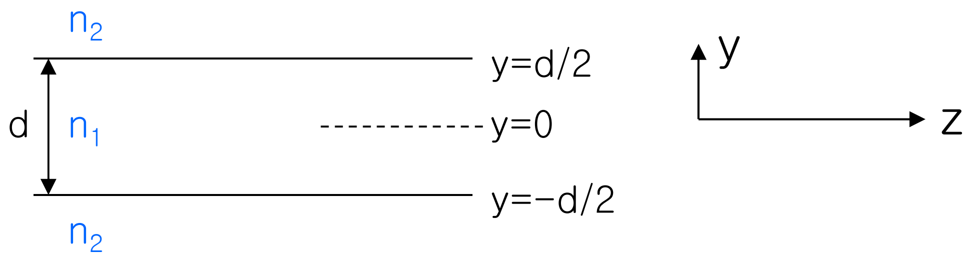
In core, $k^2(y)-\beta^2>0$ => $E(y)\sim\sin(k_y y)$ or $\cos(k_y y)$

$$k_y{}^2=k(y)^2-\beta^2,\quad k_y=\sqrt{(n_1k_0)^2-\beta^2}$$

In cladding, $k^2(y)-\beta^2<0$ => $E(y)\sim\exp(\alpha y)$ or $\exp(-\alpha y)$

$$\alpha^2=\beta^2-k(y)^2,\qquad \alpha=\sqrt{\beta^2-(n_2k_0)^2}$$

Lect. 14: Dielectric Waveguides (1)



Solutions

$$y > \frac{d}{2} : E(y) = A\exp(\alpha y) + B\exp(-\alpha y)$$

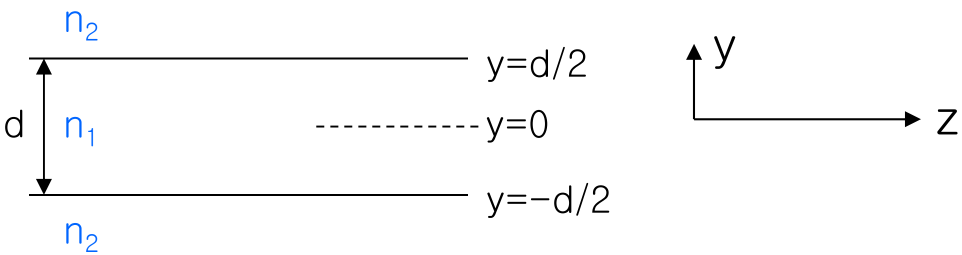
$$|y| < \frac{d}{2} : E(y) = C\sin(k_y y) + D\cos(k_y y)$$

$$y < -\frac{d}{2} : E(y) = E\exp(\alpha y) + F\exp(-\alpha y)$$

A=0 and F=0

For easier analysis, divide the solutions into even and odd solutions

Lect. 14: Dielectric Waveguides (1)



Even Solutions

$$y > \frac{d}{2} : E(y) = B\exp(-\alpha y)$$

$$|y| < \frac{d}{2} : E(y) = D\cos(k_y y)$$

$$y < -\frac{d}{2} : E(y) = B\exp(\alpha y)$$

$$(E = B)$$

Odd Solutions

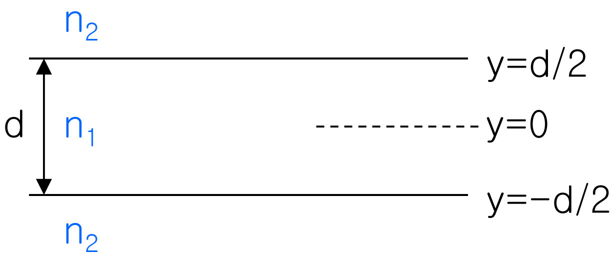
$$y > \frac{d}{2} : E(y) = B\exp(-\alpha y)$$

$$|y| < \frac{d}{2} : E(y) = D\sin(k_y y)$$

$$y < -\frac{d}{2} : E(y) = -B\exp(\alpha y)$$

$$(E = -B)$$

Lect. 14: Dielectric Waveguides (1)



Even Solutions

$$y > \frac{d}{2} : E(y) = B\exp(-\alpha y)$$

$$|y| < \frac{d}{2} : E(y) = D\cos(k_y y)$$

$$y < -\frac{d}{2} : E(y) = B\exp(\alpha y)$$

$(E = B)$

Apply boundary conditions:

$E(y)$ is continuous at $y = \pm\frac{d}{2}$

H_{tan} should be continuous at $y = \pm\frac{d}{2}$

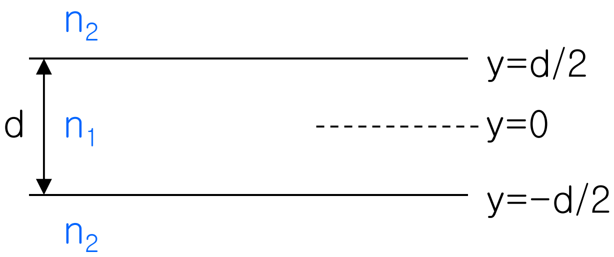
For $\overline{E}\left(y, z\right) = \overline{x}E(y)e^{-j\beta z}$

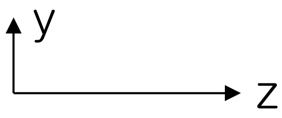
$\overline{H}$ has $\overline{y}$ and $\overline{z}$ components, where $H_z \sim \frac{dE(y)}{dy}$

H_z should be continuous across the boundary

$\therefore E(y)$ and $\frac{dE(y)}{dy}$ are continuous at $y = \pm\frac{d}{2}$

Lect. 14: Dielectric Waveguides (1)





Even Solutions

$$y > \frac{d}{2} : E(y) = B\exp(-\alpha y)$$

$$|y| < \frac{d}{2} : E(y) = D\cos(k_y y)$$

$$y < -\frac{d}{2} : E(y) = B\exp(\alpha y)$$

$$(E = B)$$

Boundary conditions:

$E(y)$ and $\dfrac{dE(y)}{dy}$ are continuous at $y = \pm\dfrac{d}{2}$

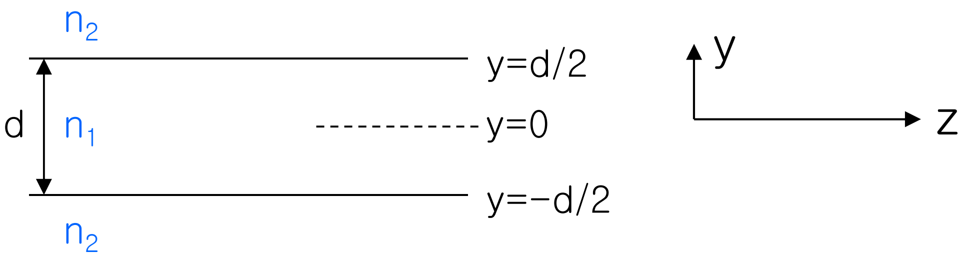
At $y = \dfrac{d}{2}$,

$$B\exp(-\alpha\frac{d}{2}) = D\cos(k_y\frac{d}{2}) \quad \text{-------- (1)}$$

$$-\alpha B\exp(-\alpha\frac{d}{2}) = -k_y D\sin(k_y\frac{d}{2}) \quad \text{------ (2)}$$

$$\frac{(2)}{(1)}: \ \alpha = k_y\tan(k_y\frac{d}{2})$$

Lect. 14: Dielectric Waveguides (1)



Odd Solutions

$$y > \frac{d}{2} : E(y) = B\exp(-\alpha y)$$

$$|y| < \frac{d}{2} : E(y) = D\sin(k_y y)$$

$$y < -\frac{d}{2} : E(y) = -B\exp(\alpha y)$$

$(E = -B)$

Apply boundary conditions.

$E(y)$ and $\dfrac{dE(y)}{dy}$ are continuous at $y = \pm\dfrac{d}{2}$

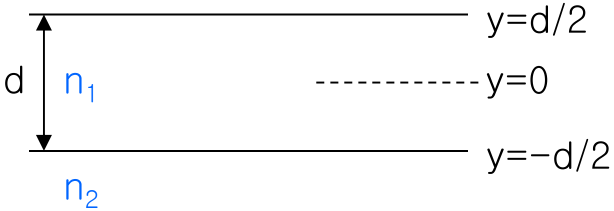
At $y = \dfrac{d}{2}$,

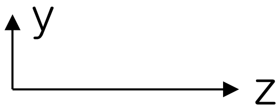
$$B\exp(-\alpha\frac{d}{2}) = D\sin(k_y\frac{d}{2}) \quad \text{------- (1)}$$

$$-\alpha B\exp(-\alpha\frac{d}{2}) = k_y D\cos(k_y\frac{d}{2}) \quad \text{----- (2)}$$

$$\frac{(2)}{(1)}: \ \alpha = -k_y\cot(k_y\frac{d}{2}) = k_y\tan(k_y\frac{d}{2} - \frac{\pi}{2})$$

Lect. 14: Dielectric Waveguides (1)





Even: $\alpha = k_y \tan(k_y \frac{d}{2})$

Odd: $\alpha = k_y \tan(k_y \frac{d}{2} - \frac{\pi}{2})$

Determine k_y and α that satisfy above conditions.

For easier solutions, do following normalization:

Let $X = k_y \frac{d}{2}$, $Y = \alpha \frac{d}{2}$

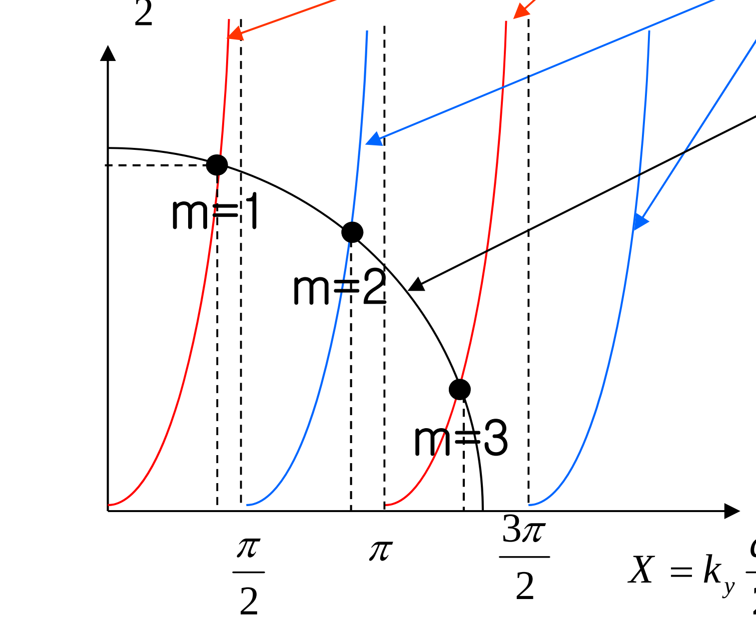
Then, $Y = X \tan X$ for even (1) $\qquad Y = X \tan(X - \frac{\pi}{2})$ for odd (2)

$$X^2 + Y^2 = \left(\frac{d}{2}\right)^2 (k_y{}^2 + \alpha^2) = \left(\frac{d}{2}\right)^2 [(n_1 k_0)^2 - \beta^2] + [\beta^2 - (n_2 k_0)^2] = \left(\frac{d}{2}\right)^2 k_0{}^2 (n_1{}^2 - n_2{}^2)$$

$X^2 + Y^2 = r^2$ (3)

Plot (1), (2), (3) on X-Y plane!

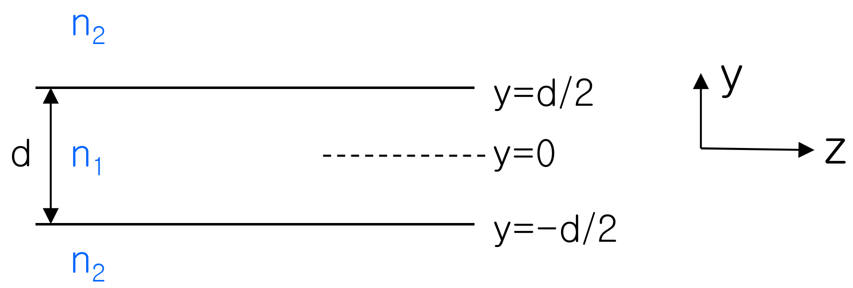
Lect. 14: Dielectric Waveguides (1)



$Y = X \tan X$: even mode

$Y = X \tan(X - \pi/2)$: odd mode

$$X^2 + Y^2 = \left(\frac{d}{2}\right)^2 k_0^2 (n_1^2 - n_2^2) = r^2$$



For m=1

k_y and α can be determined from m=1 point where two curves intersect

$$\beta = \sqrt{(n_1 k_0)^2 - k_y^2}$$

$$E(y) = D\cos(k_y y) \text{ for } |y| < \frac{d}{2}$$

$$= B\exp(-\alpha y) \text{ for } y > \frac{d}{2}$$

$$= B\exp(\alpha y) \text{ for } y < -\frac{d}{2}$$

$E(y,z) = E(y) \cdot e^{-j\beta z}$ determined!

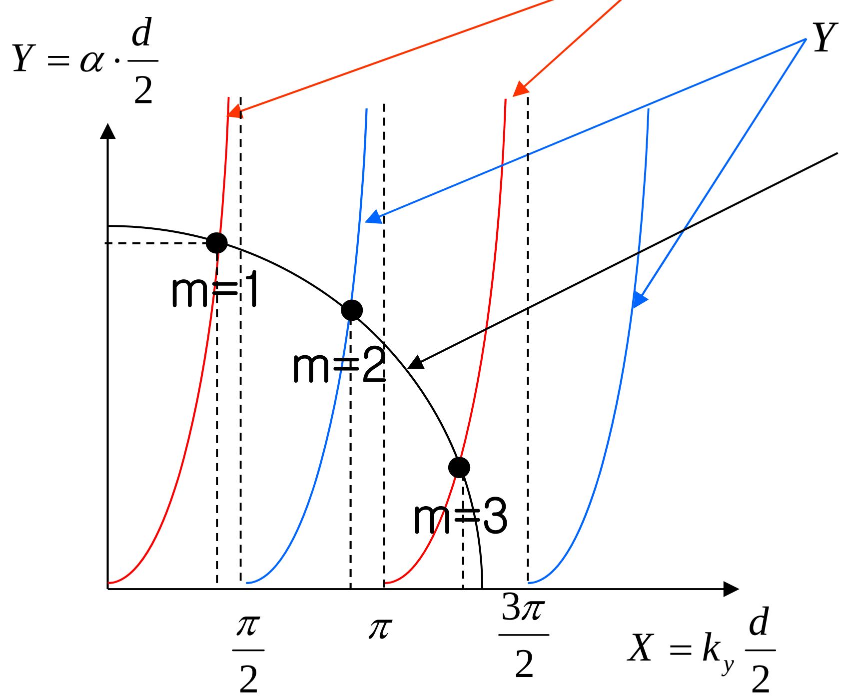
$B, D = ?$

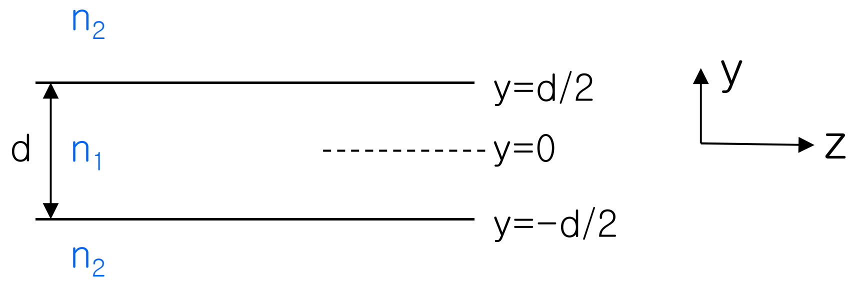
Lect. 14: Dielectric Waveguides (1)

$Y = X \tan X$: even mode

$Y = X \tan(X - \pi/2)$: odd mode

$$X^2 + Y^2 = \left(\frac{d}{2}\right)^2 k_0^2 (n_1^2 - n_2^2) = r^2$$





Observations:

- Even, odd, even, odd ··· as *r* increases
- With larger r, more modes
- There is at least one even TE mode

Lect. 14: Dielectric Waveguides (1)

Homework:

A symmetric three-layer waveguide is shown below. Consider only TE mode for this problem.

(a) Determine how many modes the waveguide can support for λ=1.0μm.

(b) Sketch the electric field intensity in the waveguide for each mode.

(c) When λ is increased, the number of modes the waveguide can support may change. What is the largest wavelength for which the mode number remains the same as what was obtained in (a)?