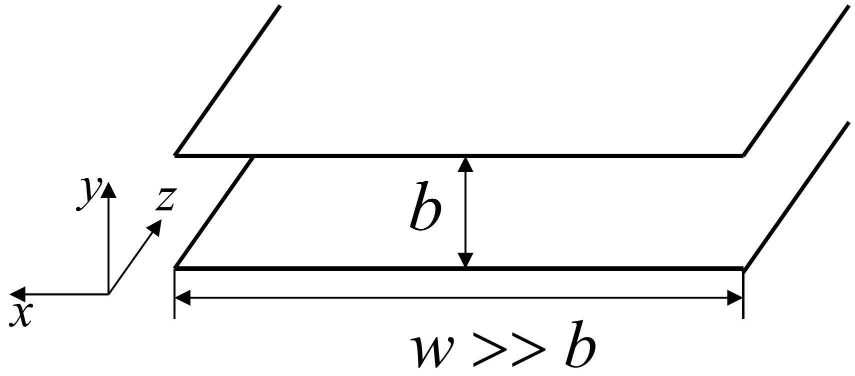
Lect. 16 Rectangular Metallic Waveguides

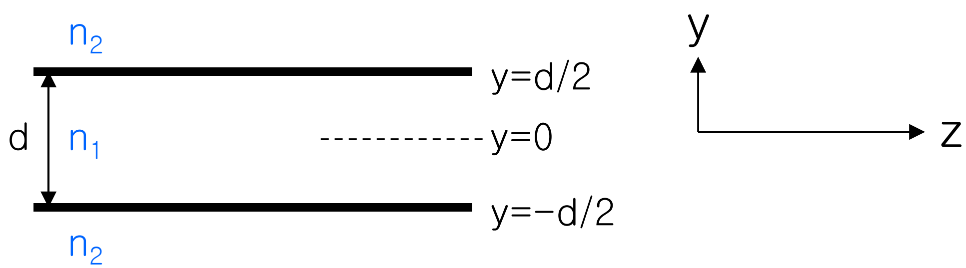


- TEM mode

  E-field: Only E_y
  H-field: Only H_x
  $E_z=0, H_z=0$

- TE mode

  E-field: Only E_x
  H-field: H_y and H_z
  $E_z=0, H_z\neq0$



- TM mode

  E-field: E_y and E_z
  H-field: Only H_x
  $E_z\neq0, H_z=0$

- k_y is quantized for TE and TM

3-D Rectangular waveguide?

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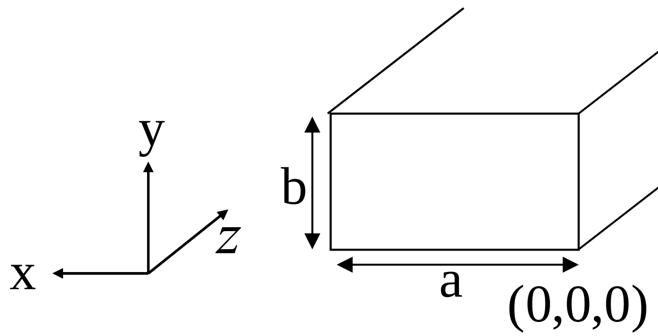
Rectangular metallic waveguide



$$\nabla^2 \bar{E} - \mu\varepsilon \frac{\partial^2}{\partial t^2}\bar{E} = 0$$

$$\nabla^2 \bar{H} - \mu\varepsilon \frac{\partial^2}{\partial t^2}\bar{H} = 0$$

$$\bar{E}(x,y,z,t) = \bar{E}(x,y)\cdot e^{-j\beta z}\cdot e^{j\omega t}$$

$$\bar{H}(x,y,z,t) = \bar{H}(x,y)\cdot e^{-j\beta z}\cdot e^{j\omega t}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\bar{E}(x,y) + (k^2 - \beta^2)\bar{E}(x,y) = 0$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\bar{H}(x,y) + (k^2 - \beta^2)\bar{H}(x,y) = 0$$

Solve for E_z (TM)

Solve for H_z (TE)

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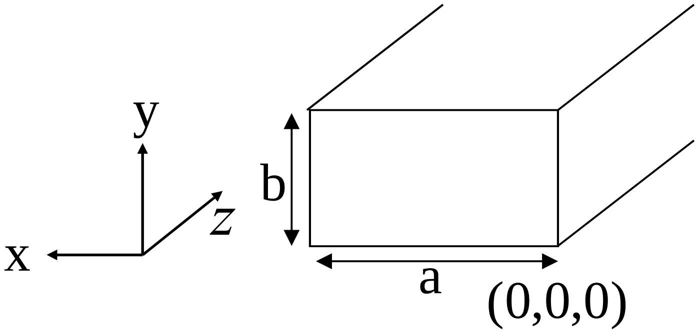
Rectangular Metallic Waveguide: TE modes



$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)H_z(x,y)+(k^2-\beta^2)H_z(x,y)=0$$

$H_z(x,y)=X(x)\cdot Y(y)$ Separation of variables

$$\frac{\partial^2}{\partial x^2}(XY)+\frac{\partial^2}{\partial y^2}(XY)+(k^2-\beta^2)(XY)=0 \qquad \frac{1}{X}\frac{\partial^2}{\partial x^2}X+\frac{1}{Y}\frac{\partial^2}{\partial x^2}Y+(k^2-\beta^2)=0$$

$$\frac{1}{X}\frac{\partial^2}{\partial x^2}X+\frac{1}{Y}\frac{\partial^2}{\partial x^2}Y+(k_x{}^2+k_y{}^2)=0 \qquad \left(k^2=k_x{}^2+k_y{}^2+\beta^2\right)$$

$$\frac{d^2}{dx^2}X+k_x{}^2X=0 \qquad \frac{d^2}{dx^2}Y+k_y{}^2Y=0$$

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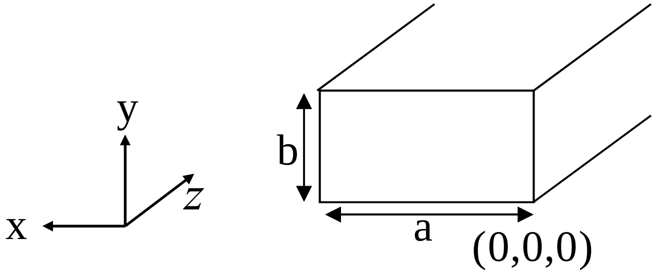

$$H_z(x,y) = X(x)Y(y),$$

$$\frac{d^2}{dx^2}X + k_x^{\,2}X = 0$$

$$X = A\sin(k_x x) + B\cos(k_x x)$$

$$\left.\partial X/\partial x\right|_{x=0} = 0 \quad \Rightarrow \; A = 0$$

$$\left.\partial X/\partial x\right|_{x=a} = 0 \quad \Rightarrow \; k_x a = m\pi, \quad k_x = \frac{m\pi}{a}$$

$$X = B\cos(k_x x)$$

$$\therefore H_z(x,y) = H_0 \cos(k_x x)\cos(k_y y)$$

$$\frac{d^2}{dx^2}Y + k_y^{\,2}Y = 0$$

$$Y = C\sin(k_y y) + D\cos(k_y y)$$

$$\left.\partial Y/\partial y\right|_{y=0} = 0 \quad \Rightarrow \; C = 0$$

$$\left.\partial Y/\partial y\right|_{y=b} = 0 \quad \Rightarrow k_y b = n\pi, \; k_y = \frac{n\pi}{b}$$

$$Y = D\cos(k_y y)$$

$$k_x = \frac{m\pi}{a}, \; k_y = \frac{n\pi}{b}, \qquad \beta^2 = k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2$$

Both k_x and k_y are quantized: TE_{mn} mode

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From $H_z(x,y) = H_0 \cos(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)$, it can be determined for $\mathrm{TE_{mn}}$ mode (p. 552 in Cheng)

$$E_x(x,y) = \frac{j\omega\mu}{h^2}(\frac{n\pi}{b})H_0 \cos(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y) \qquad \left[h^2 = (\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2\right]$$

$$E_y(x,y) = -\frac{j\omega\mu}{h^2}(\frac{m\pi}{a})H_0 \sin(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)$$

$$H_x(x,y) = \frac{j\beta}{h^2}(\frac{m\pi}{a})H_0 \sin(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)$$

$$H_y(x,y) = \frac{j\beta}{h^2}(\frac{n\pi}{b})H_0 \cos(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)$$

$$\beta = \sqrt{k^2 - k_x{}^2 - k_y{}^2} = \sqrt{\omega^2\mu\varepsilon - (\frac{m\pi}{a})^2 - (\frac{n\pi}{b})^2} \qquad f_c = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2}$$

Dominant TE mode?

(Smallest f_c, or largest β)

$\mathrm{TE_{01}}$ or $\mathrm{TE_{10}}$ mode

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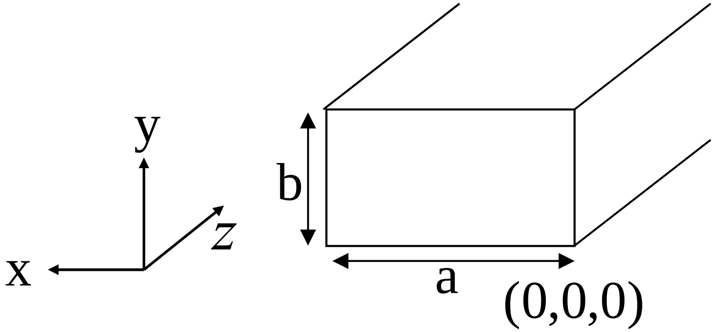
TM mode



$$\nabla^2 \overline{E} - \mu\varepsilon \frac{\partial^2}{\partial t^2}\overline{E} = 0$$

$$\overline{E}(x,y,z,t) = \overline{E}(x,y)\cdot e^{-j\beta z}\cdot e^{j\omega t}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\overline{E}(x,y) + (k^2 - \beta^2)\overline{E}(x,y) = 0$$

Solve for E_z

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$$E_z(x,y) = E_0 \sin(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)$$ TM$_{mn}$ Mode

$$E_x(x,y) = -\frac{j\beta}{h^2}(\frac{m\pi}{a})E_0 \cos(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)$$ $\left[h^2 = (\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2\right]$

$$E_y(x,y) = -\frac{j\beta}{h^2}(\frac{n\pi}{b})E_0 \sin(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)$$

$$H_x(x,y) = \frac{j\omega\varepsilon}{h^2}(\frac{n\pi}{b})E_0 \sin(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)$$

$$H_y(x,y) = -\frac{j\omega\varepsilon}{h^2}(\frac{m\pi}{a})E_0 \cos(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)$$

$$\beta = \sqrt{k^2 - k_x{}^2 - k_y{}^2} = \sqrt{\omega^2\mu\varepsilon - (\frac{m\pi}{a})^2 - (\frac{n\pi}{b})^2}$$ $$f_c = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2}$$

Dominant TM mode? TM$_{11}$ Mode

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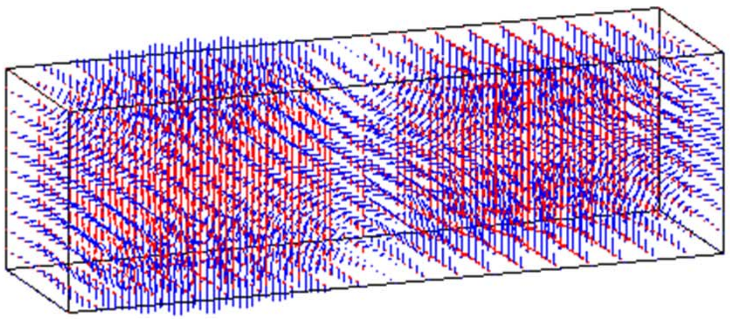
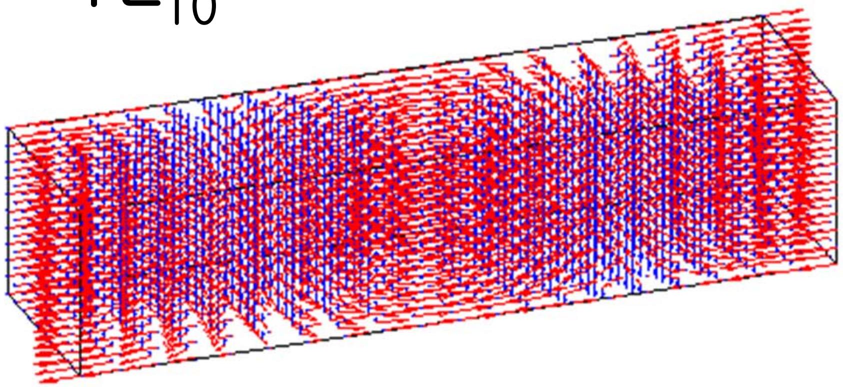
TM_{11}



TE_{10}



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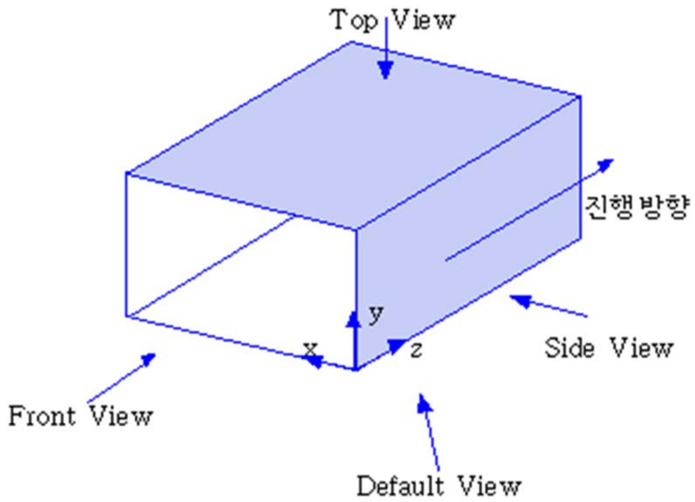
Wave propagation for TE_{10} and TM_{11} modes in rectangular waveguide



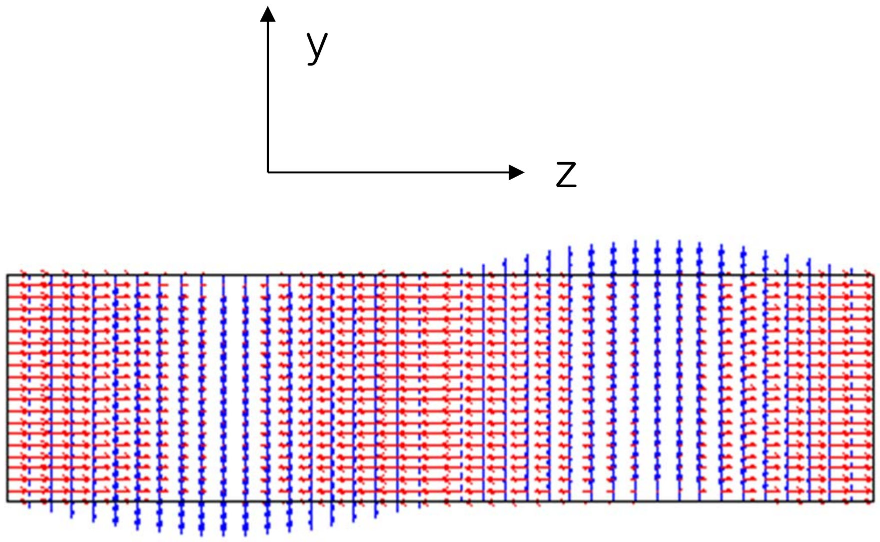
Lect. 16 Rectang

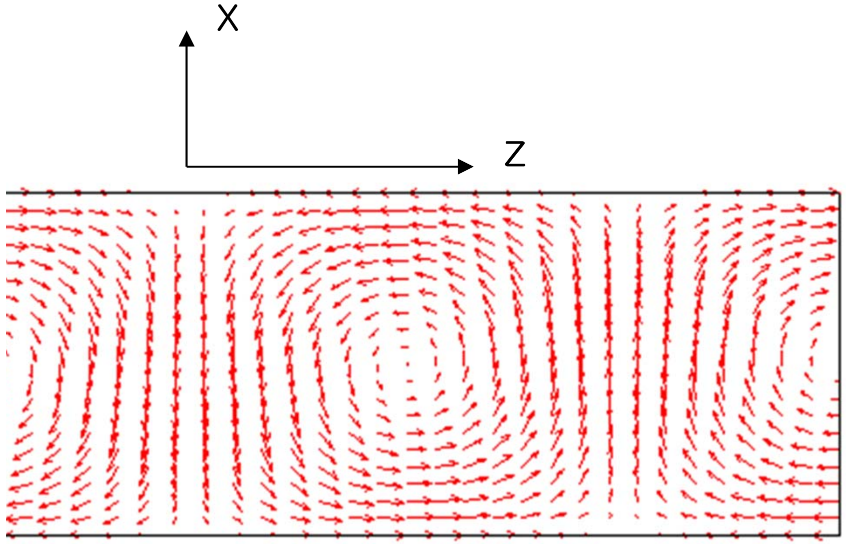
TE$_{10}$

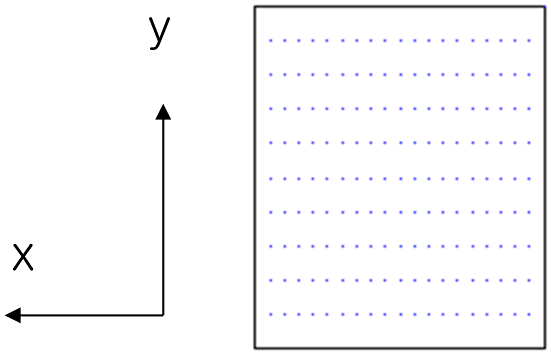
$$E_y(x,y,t) = \frac{\omega\mu}{h^2}(\frac{\pi}{a})H_0 \sin(\frac{\pi}{a}x)\sin(\omega t - \beta z)$$

$$H_x(x,y,t) = -\frac{\beta}{h^2}(\frac{\pi}{a})H_0 \sin(\frac{\pi}{a}x)\sin(\omega t - \beta z)$$

$$H_z(x,y,t) = H_0 \cos(\frac{\pi}{a}x)\cos(\omega t - \beta z)$$







Lect. 16 Rectang

TM_{11}

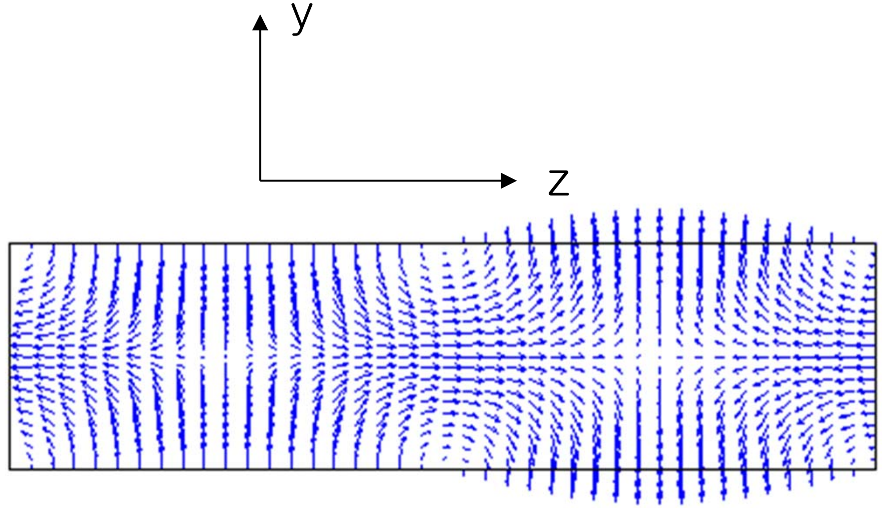
$$E_x(x,y,t) = \frac{\beta}{h^2}(\frac{\pi}{a})E_0 \cos(\frac{\pi}{a}x)\sin(\frac{\pi}{b}y)\sin(\omega t - \beta z)$$

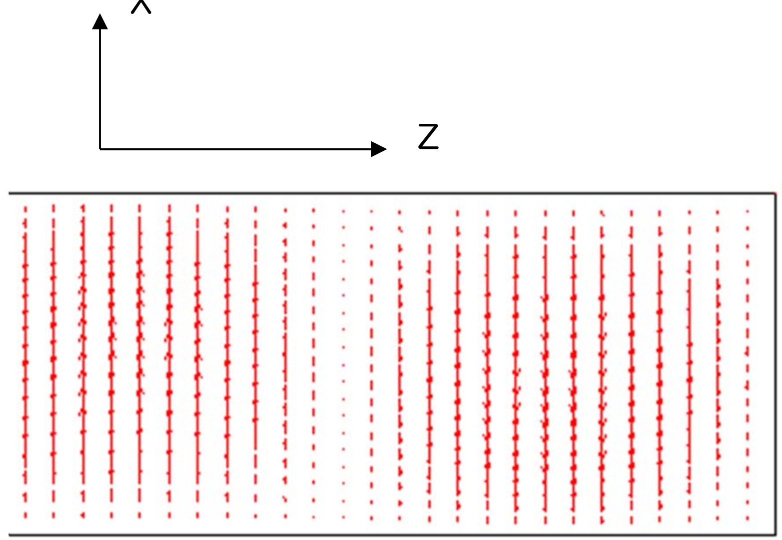
$$E_y(x,y,t) = \frac{\beta}{h^2}(\frac{\pi}{b})E_0 \sin(\frac{\pi}{a}x)\cos(\frac{\pi}{b}y)\sin(\omega t - \beta z)$$

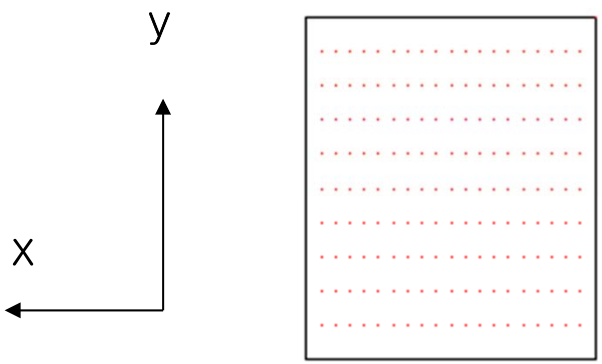
$$E_z(x,y,t) = E_0 \sin(\frac{\pi}{a}x)\sin(\frac{\pi}{b}y)\cos(\omega t - \beta z)$$

$$H_x(x,y,t) = -\frac{\omega\varepsilon}{h^2}(\frac{\pi}{b})E_0 \sin(\frac{\pi}{a}x)\cos(\frac{\pi}{b}y)\sin(\omega t - \beta z)$$

$$H_y(x,y,t) = \frac{\omega\varepsilon}{h^2}(\frac{\pi}{a})E_0 \cos(\frac{\pi}{a}x)\sin(\frac{\pi}{b}y)\sin(\omega t - \beta z)$$







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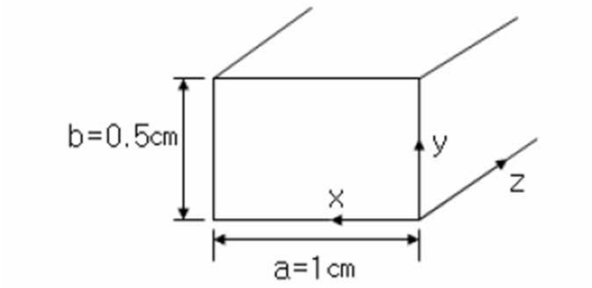
Consider an empty rectangular waveguide with dimensions 1 cm x 0.5 cm shown below.
The waves are propagating in z-direction.



(a,b) Identify the waveguide modes that have the field profiles shown below. In the figure, electric fields are represented by solid lines and magnetic fields by dashed lines.