Lect. 2: Light as EM Waves

(Cheng 7.6, 7.7)

Maxwell's Equations

$$\nabla \cdot \overline{D} = \rho$$

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

$$\nabla \cdot \overline{B} = 0$$

$$\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t}$$

$$\overline{D} = \varepsilon \overline{E}$$

$$\overline{B} = \mu \overline{H}$$

✓ Simplification

1) source free medium ➔ $\rho = 0, \ \overline{J} = 0$

2) uniform medium ➔ $\varepsilon, \mu \neq f(x, y, z)$

$$\nabla \times \left(\nabla \times \overline{E}\right) = -\nabla \times \left(\frac{\partial \overline{B}}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla \times \overline{B})$$

$$= -\mu \frac{\partial}{\partial t}(\nabla \times \overline{H}) = -\mu \frac{\partial^2 \overline{D}}{\partial t^2}$$

$$= -\mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2}$$

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$$\nabla \times \left(\nabla \times \overline{E}\right) = -\mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2}$$

$$\nabla \times \left(\nabla \times \overline{E}\right) = \nabla\left(\nabla \cdot \overline{E}\right) - \nabla^2 \overline{E}$$

$$\nabla \cdot \overline{E} = 0$$

$$\nabla \times \left(\nabla \times \overline{E}\right) = -\nabla^2 \overline{E}$$

$$\nabla^2 \overline{E} = \overline{x}\ \nabla^2 E_x + \overline{y}\ \nabla^2 E_y + \overline{z}\ \nabla^2 E_z$$ (Vector Laplacian)

$$\nabla^2 E_x = \frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2}$$ (Laplacian)

$$\Rightarrow \quad \nabla^2 \overline{E} = \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2}$$

EM Wave Equations

(source free, uniform medium)

→ light propagation

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Solutions for EM Wave Equations $\nabla^2 \overline{E} = \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2}$

Guess $\overline{E} = \overline{x} E_0 \cos(\omega t - kz)$

ω : (angular) frequency k : wave numbers

$$\nabla^2 \overline{E} = \overline{x}(-k^2) E_0 \cos(\omega t - kz)$$

$$\mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2} = \overline{x}\mu\varepsilon(-\omega^2) E_0 \cos(\omega t - kz)$$

Our guess is correct if $k^2 = \mu\varepsilon\omega^2$

Or $k = \omega\sqrt{\mu\varepsilon}$

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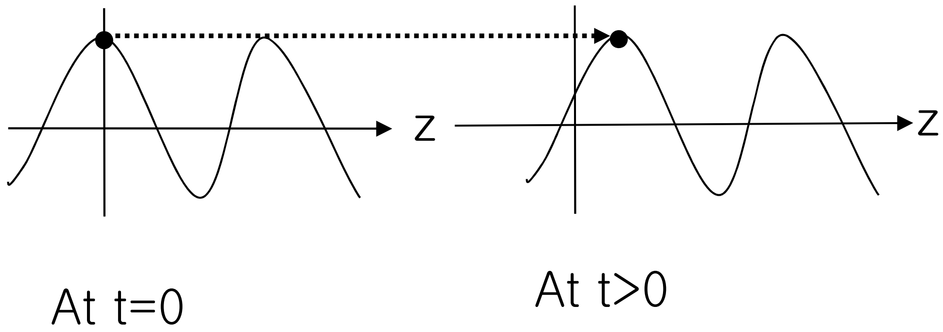
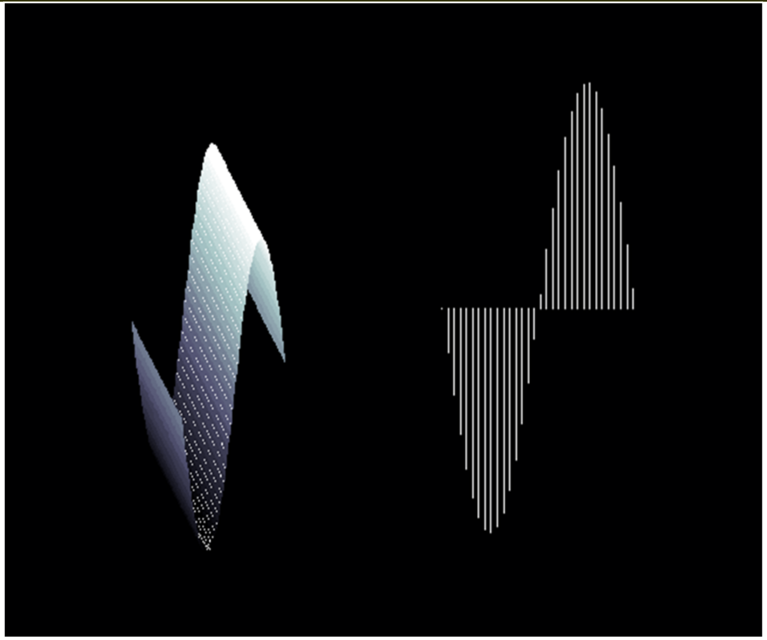
How does it look?

$$\overline{E} = \overline{x}E_0 \cos(\omega t - kz) \qquad k = \omega\sqrt{\mu\varepsilon}$$





- Moving to the right maintaining its shape ➔ Wave
- Periodic in space with periodicity $2\pi / k = \lambda \qquad k = 2\pi / \lambda$
- Periodic in time with periodicity $2\pi / \omega = T \qquad \omega = 2\pi / T$

$$k = \omega\sqrt{\mu\varepsilon} \qquad \frac{\lambda}{T} = \frac{1}{\sqrt{\mu\varepsilon}}$$

EM wave propagates with velocity of $\frac{1}{\sqrt{\mu\varepsilon}}$

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Solutions for EM Wave Equations $\nabla^2 \overline{E} = \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2}$

More generally, $\overline{E} = \overline{x} E_0 e^{j(\omega t - kz)}$

$$\nabla^2 \overline{E} = \overline{x}(-k^2) E_0 e^{j(\omega t - kz)}$$

$$\mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2} = \overline{x}\mu\varepsilon(-\omega^2) E_0 e^{j(\omega t - kz)}$$

$$\therefore k = \omega\sqrt{\mu\varepsilon}$$

➔ Plane-wave solutions

(exponential solutions, phasor notation, …)

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$$\nabla^2 \overline{E} = \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2} \qquad \overline{E} = \overline{x} E_0 e^{j(\omega t - kz)}$$

How about H-field? H-field must have the same ω, *k*

➔ direction and magnitude?

Maxwell's Equation in Source-free region

$$\nabla \cdot \overline{D} = 0$$

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

$$\nabla \cdot \overline{B} = 0$$

$$\nabla \times \overline{H} = \frac{\partial \overline{D}}{\partial t}$$

$$\nabla \times \overline{E} =$$

$$\overline{H} = \overline{y} \sqrt{\frac{\varepsilon}{\mu}} E_0 e^{j(\omega t - kz)}$$

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$$\nabla^2 \overline{E} = \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2} \qquad \overline{E} = \overline{x} E_0 e^{j(\omega t - kz)} \qquad \overline{H} = \overline{y} \sqrt{\frac{\varepsilon}{\mu}} E_0 e^{j(\omega t - kz)}$$

Direction of E, H fields?

Direction of propagation?

Speed of propagation?

$$\frac{E_0}{H_0} = \sqrt{\frac{\mu}{\varepsilon}} = \eta \;\; [\Omega]$$ (377Ω for vacuum)

Plot E(t,z) and H(t,z)?

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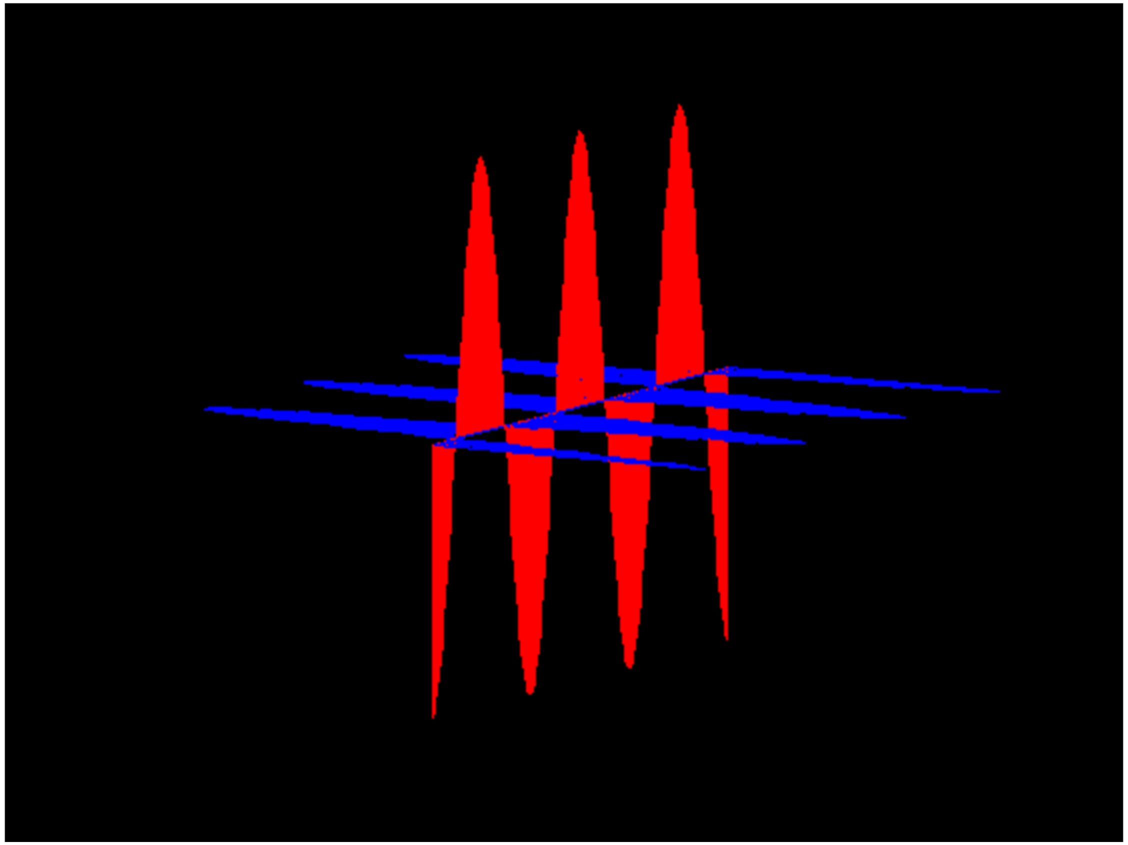
How does the plane-wave solution look like?



EM wave animation available at tera.yonsei.ac.kr
(Classes ➔ any 전자기학2 link ➔ Demonstration of EM waves)

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– Homework: Due on 9/6 in the class

A uniform plane wave propagating in a dielectric medium has the E-field given as

$$\overline{E}(t,z) = \overline{x}\,2\cos(10^8 t - z/\sqrt{3}) + \overline{y}\sin(10^8 t - z/\sqrt{3})$$

(a) What is the frequency of this EM-wave in Hz?
(b) What is the wavelength of this EM-wave in 1/m?
(c) What is the dielectric constant of the dielectric medium?
(d) What is the corresponding H-field?