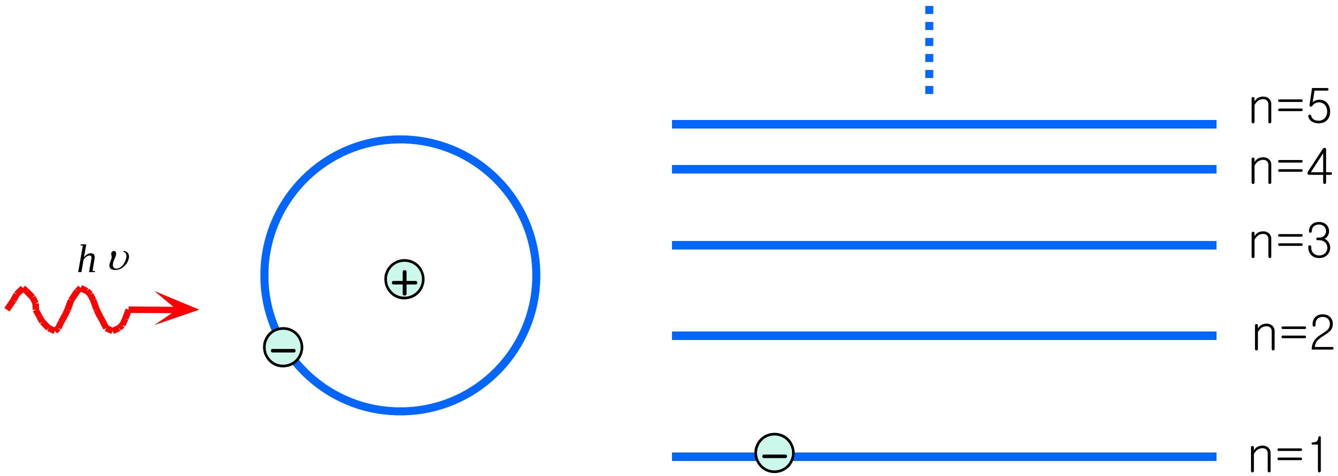
Lect. 20: Interaction between Light and Matter

What happens when photons interact with a matter
whose electron transition energies are compatible with photon energies?

Example: Electron energy levels in an hydrogen atom



$$E_n = -\frac{13.6}{n^2}\,[\text{eV}]$$

According to QM, energy levels inside an atom are quantized

What happens when $h\nu = E_n - E_m$?

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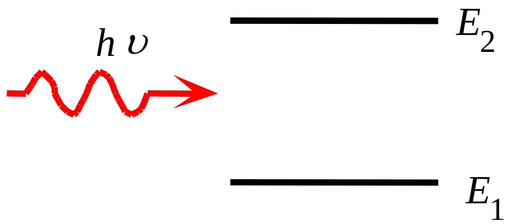
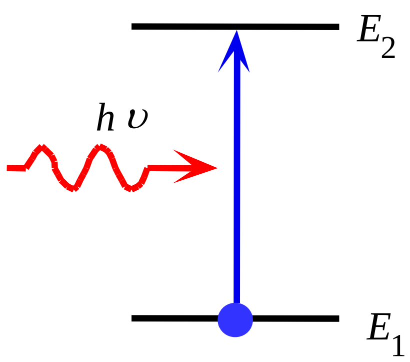
Consider for simplicity only two energy levels: ground state and first excited state

Assume $h\nu = E_2 - E_1$

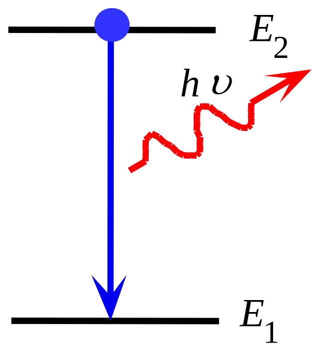


➔ Three interaction processes are possible

Absorption

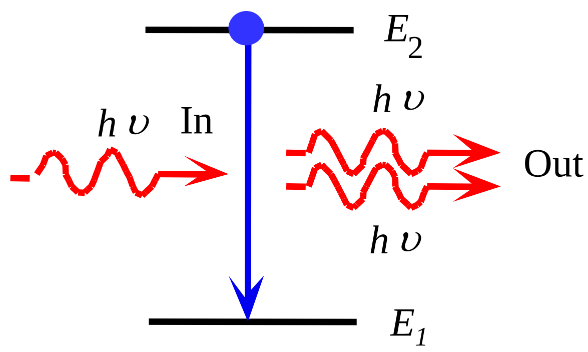


Spontaneous Emission



output photons are "random" except energy

Stimulated Emission



output photons are "identical" to input photons: amplification

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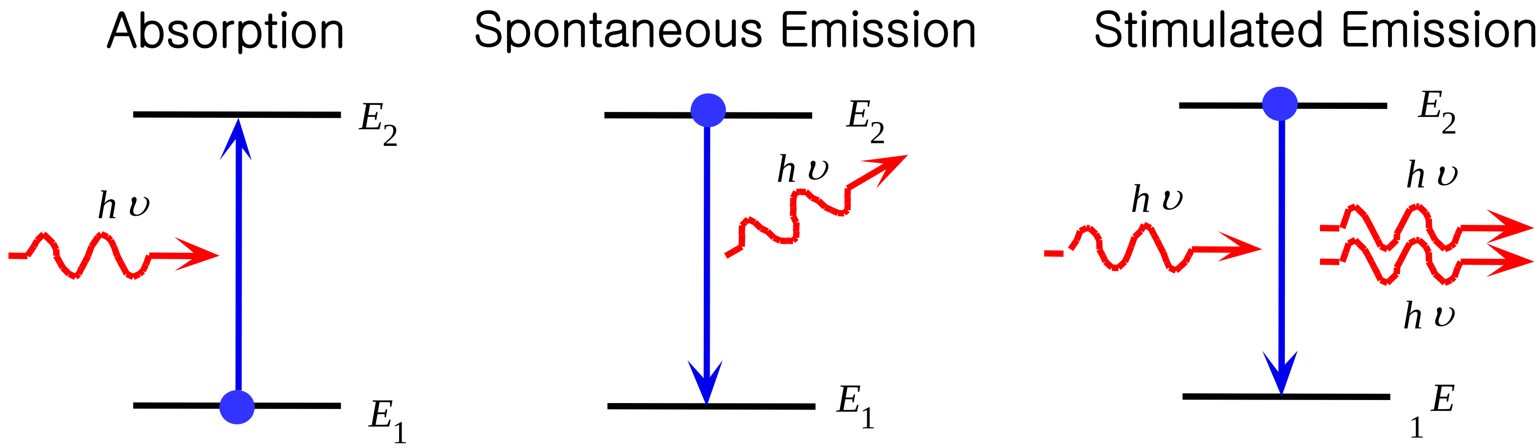
Determine the rate for each process: How many per unit volume per second



$$R_{12} = B_{12} \cdot N_1 \cdot \rho \qquad R_{sp} = A_{21} \cdot N_2 \qquad R_{21} = B_{21} \cdot N_2 \cdot \rho$$

ρ : photon density (spectral photon energy density)

$N_{1,2}$: electron density at $E_{1,2}$

B_{12}, B_{sp}, B_{21} : constants

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What happens at equilibrium? No net change of N_1, N_2, ρ

$$R_{12} = R_{sp} + R_{21}$$

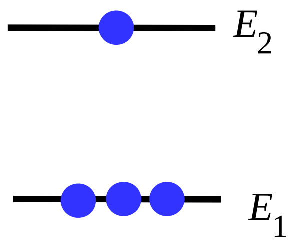
$$B_{12} \cdot N_1 \cdot \rho = A_{21} \cdot N_2 + B_{21} \cdot N_2 \cdot \rho$$

$$\rho = \frac{A_{21}/B_{12}}{\dfrac{N_1}{N_2} - \dfrac{B_{21}}{B_{12}}} \qquad \therefore \rho(E_2 - E_1) = \frac{A_{21}/B_{12}}{e^{\left(\frac{E_2 - E_1}{kT}\right)} - \dfrac{B_{21}}{B_{12}}}$$

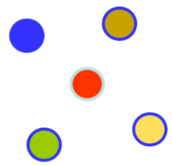
From another branch of physics (statistical mechanics),

electron distribution at equilibrium

$$\frac{N_2}{N_1} = \exp\left(-\frac{E_2 - E_1}{kT}\right)$$



From Lect. 19,
(Planck law for black-body radiation)



$$\rho(h\upsilon) = \frac{8\pi h\upsilon^3}{c^3\left[\exp\left(\dfrac{h\upsilon}{kT}\right) - 1\right]}$$

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Compare

$$\rho(E_2 - E_1) = \frac{A_{21}/B_{12}}{e^{\left(\frac{E_2 - E_1}{kT}\right)} - \frac{B_{21}}{B_{12}}}$$

and

$$\rho(h\upsilon) = \frac{8\pi h\upsilon^3}{c^3\left[e^{\left(\frac{h\upsilon}{kT}\right)} - 1\right]}$$

For hν = $E_1 - E_2$, two expressions should be identical.

$$\frac{B_{21}}{B_{12}} = 1 \qquad \frac{A_{21}}{B_{12}} = \frac{8\pi h\upsilon^3}{c^3}$$

➔ Einstein's A, B constants

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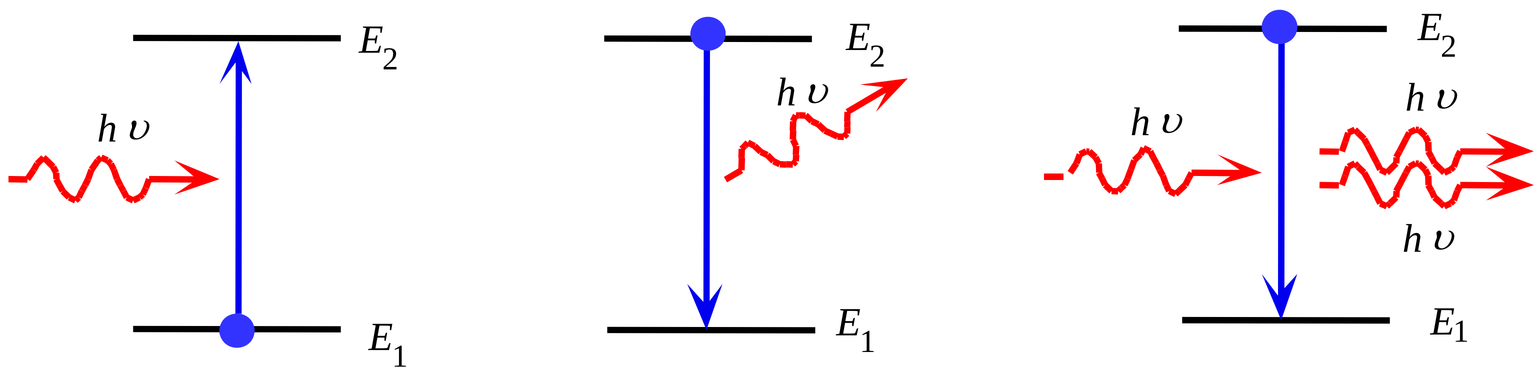
Absorption | Spontaneous Emission | Stimulated Emission



$$R_{12} = B_{12} \cdot N_1 \cdot \rho \qquad R_{sp} = A_{21} \cdot N_2 \qquad R_{21} = B_{21} \cdot N_2 \cdot \rho$$

$$\frac{B_{21}}{B_{12}} = 1$$

Interpretations:

–Absorption and simulated emission have the same coefficeint
➔ The only difference is N_1 and N_2

$$\frac{A_{21}}{B_{12}} = \frac{8\pi h \upsilon^3}{c^3}$$

–Spontaneous emission and stimulated emission are intrinsically related
➔ Spontaneous emission is simulated emission due to *vacuum fluctuation* (QM interpretation of EM waves)

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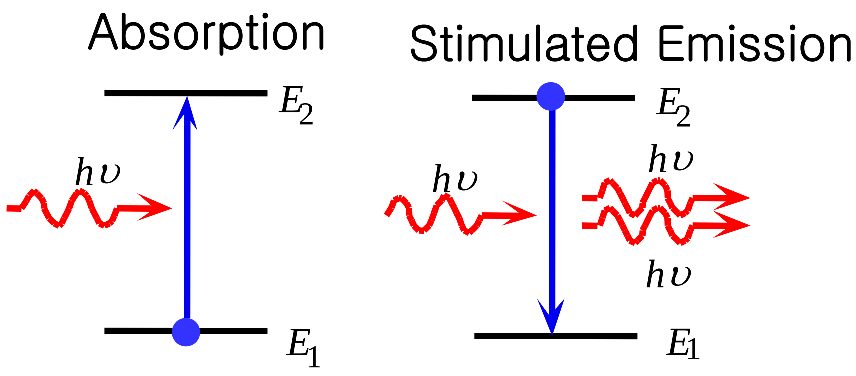
Which process is dominant at equilibrium?

Stimulated emission vs. absorption



$\mathrm{R}_{12} = \mathrm{B}_{12} \cdot \mathrm{N}_1 \cdot \rho$ $\mathrm{R}_{21} = \mathrm{B}_{21} \cdot \mathrm{N}_2 \cdot \rho$

$$\frac{R_{21}}{R_{12}} = \frac{B_{21} N_2 \rho}{B_{12} N_1 \rho} = \frac{N_2}{N_1} = \exp\left(-\frac{E_2 - E_1}{kT}\right)$$

$kT \sim 26meV$ (T=300K)

For example, $\lambda = 1.55\mu m$ $E_{photon} = h\nu \simeq \frac{1.24}{\lambda[\mu m]} eV = \frac{1.24}{1.55} eV = 0.8eV$

$$\frac{R_{21}}{R_{12}} = \exp\left(-\frac{0.8eV}{0.026eV}\right) \sim 4.3 \times 10^{-14}$$

Almost all incident photons are absorbed at equilibrium

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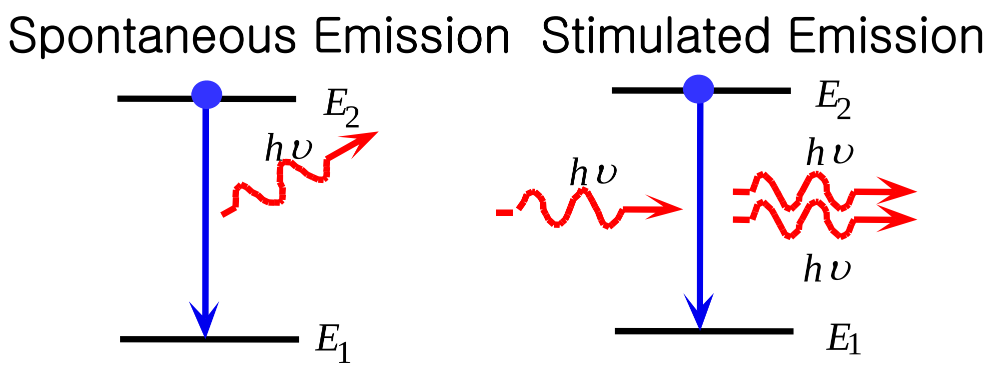
Which process is dominant at equilibrium?

Stimulated emission vs. spontaneous emission



$$R_{sp} = A_{21} \cdot N_2 \qquad R_{21} = B_{21} \cdot N_2 \cdot \rho$$

$$\frac{R_{21}}{R_{sp}} = \frac{B_{21} N_2 \rho}{A_{21} N_2} = \frac{c^3}{8\pi h\upsilon^3} \frac{8\pi h\upsilon^3}{c^3\left[\exp\left(\frac{h\upsilon}{kT}\right)-1\right]} = \frac{1}{\exp\left(\frac{E_2 - E_1}{kT}\right)-1} = \frac{1}{\exp\left(\frac{0.8eV}{0.026eV}\right)-1} << 1$$

Virtually all photon emission at equilibrium is due to spontaneous emission

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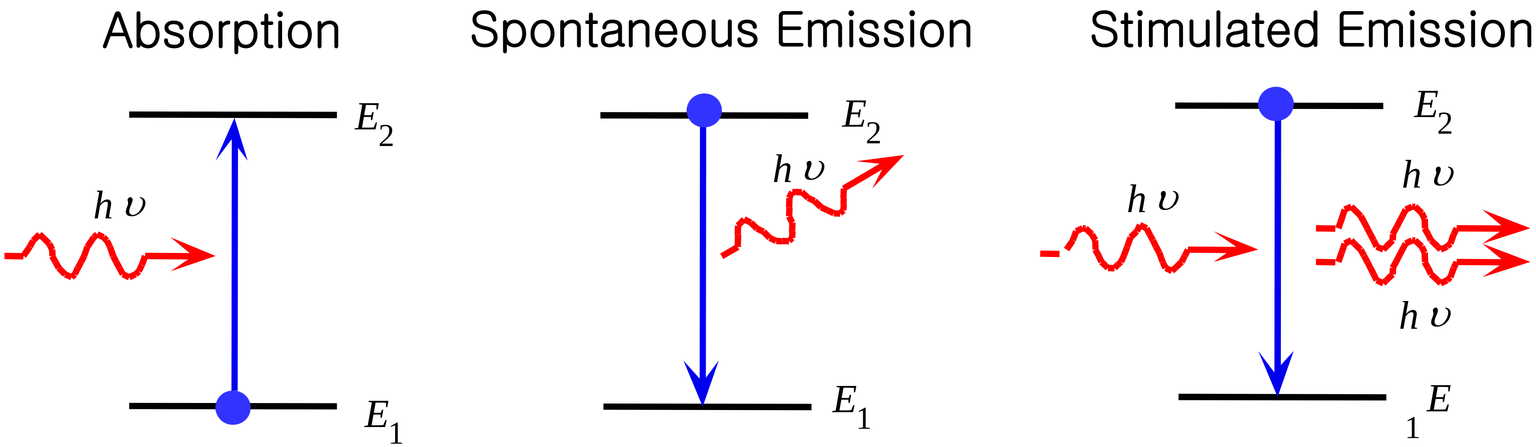
How can we induce stimulated emission?



$$R_{12} = B_{12} \cdot N_1 \cdot \rho \qquad R_{sp} = A_{21} \cdot N_2 \qquad R_{21} = B_{21} \cdot N_2 \cdot \rho$$

Make N_2 larger than N_1: Break equilibrium by pumping carriers into E_2

$N_2 = N_1$: transparent

$N_2 > N_1$: population inversion

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Homework:

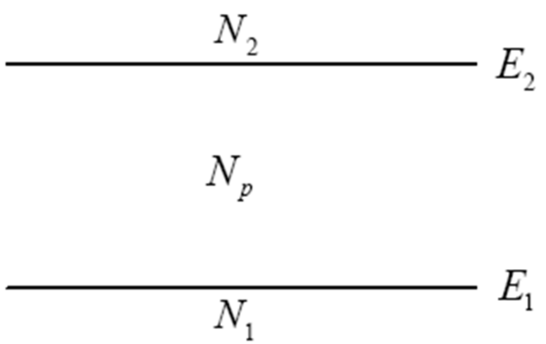
A material with two energy levels and photons are at the equilibrium state as shown below. The photon energy, E_p, is equal to $E_2 - E_1 = 100meV$. Use $kT = 25meV$.

(a) What is the expression for the stimulated emission rate?

(b) Determine the numerical value of N_1/N_2, the ratio between electron densities at E_2 and E_1.

(c) What is the percentage of photons that are due to stimulated emission?

(d) Electron are excited from E_1 to E_2 by optical pumping. If the total density of electrons in the material is N, what should be N_2 in order to reach the transparency condition?



$$E_2 - E_1 = E_p = 100meV$$

$$kT = 25meV$$