Lect. 3: Polarization

(8-2.3 in Cheng)

EM Wave Equations

(source free, uniform medium)

$$\nabla^2 \overline{E} = \mu\varepsilon \frac{\partial^2 \overline{E}}{\partial t^2}$$

Plane-wave solutions

$$\overline{E} = \overline{x} E_0 e^{j(\omega t - kz)} \qquad \overline{H} = \overline{y}\sqrt{\frac{\varepsilon}{\mu}} E_0 e^{j(\omega t - kz)}$$

When an EM wave is propagating into

Plane wave solutions

+z direction: $e^{j(\omega t - kz)}$

-z direction: $e^{j(\omega t + kz)}$

+y direction: $e^{j(\omega t - ky)}$

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EM wave propagating in any direction?

$$e^{j\omega t}e^{-jk_x x}e^{-jk_y y}e^{-jk_z z} \quad = e^{j(\omega t - \overline{k}\bullet\overline{R})}$$

$$\overline{R} = \overline{x}x + \overline{y}y + \overline{z}z$$ representing (x,y,z)

$$\overline{k} = \overline{x}k_x + \overline{y}k_y + \overline{z}k_z$$ wave vector

$$\left|\overline{k}\right| = \frac{2\pi}{\lambda}$$

$\angle\overline{k}$ direction of propagation

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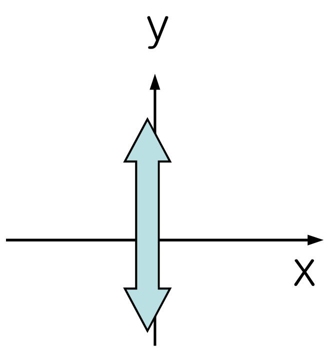
Polarization: Time varying characteristics of E-, H-field direction

Consider $$\overline{E} = \left(\overline{x}E_1 + \overline{y}E_2\right)e^{j\omega t}e^{jkz}$$

How does E-field direction change in time at a given location (z=0)?

$$\mathrm{Re}\left[\overline{E}\right] = \overline{x}E_1\cos\left(\omega t\right) + \overline{y}E_2\cos(\omega t)$$

1) If E_1=0 → $\mathrm{Re}\left[\overline{E}\right] = \overline{y}E_2\cos(\omega t)$



Linear polarization

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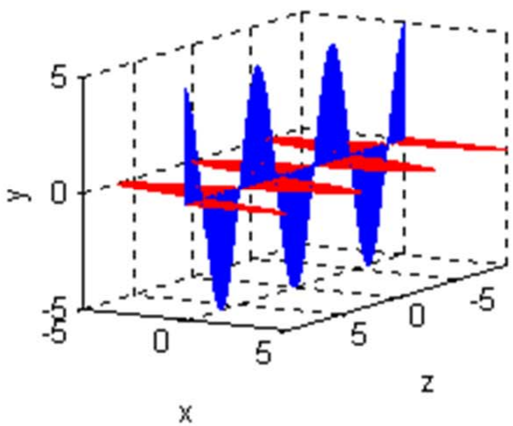
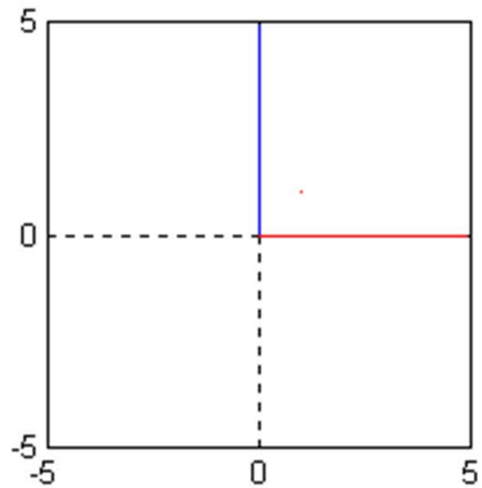
$$\overline{E} = \overline{y} E_2 e^{j\omega t} e^{jkz}$$

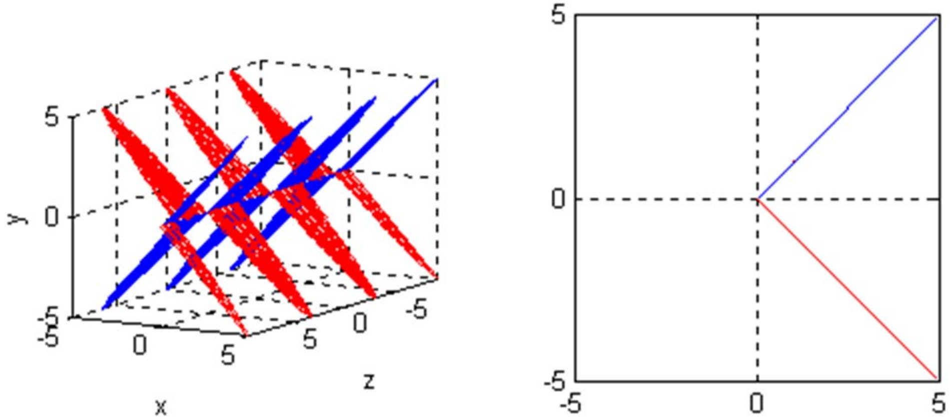




What if $E_1 = E_2$?

$$\overline{E} = \left(\overline{x}E_1 + \overline{y}E_1\right)e^{j\omega t}e^{jkz}$$

At z= 0, $\mathrm{Re}\left[\overline{E}\right] = \overline{x}E_1\cos\left(\omega t\right) + \overline{y}E_2\cos(\omega t)$



Linear Polarization

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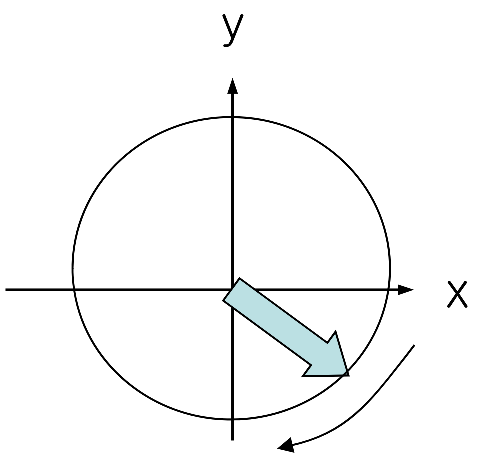
$$\overline{E} = \left(\overline{x}E_1 + \overline{y}E_2\right)e^{j\omega t}e^{jkz}$$

2) If $E_2 = j\,E_1$

$$\overline{E} = \left(\overline{x}E_1 e^{jkz} + \overline{y}\,jE_1 e^{jkz}\right)e^{j\omega t}$$

At z=0, $\mathrm{Re}(\overline{E}) = E_1\left\{\overline{x}\cos(\omega t) - \overline{y}\sin(\omega t)\right\}$



Circular polarization

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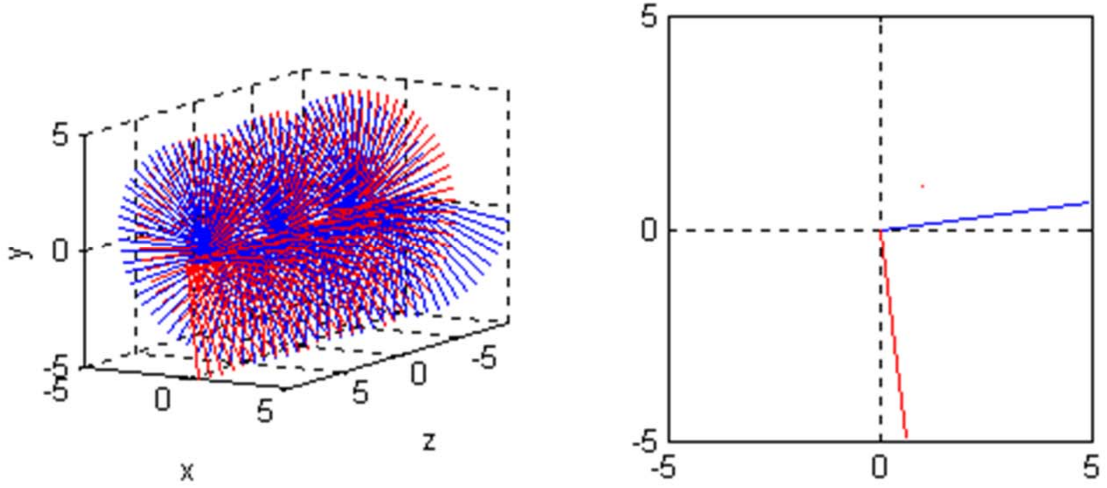
$$\overline{E} = \left(\overline{x} E_1 e^{jkz} + \overline{y} j E_1 e^{jkz} \right) e^{j\omega t}$$



Right-Handed Circular Polarization

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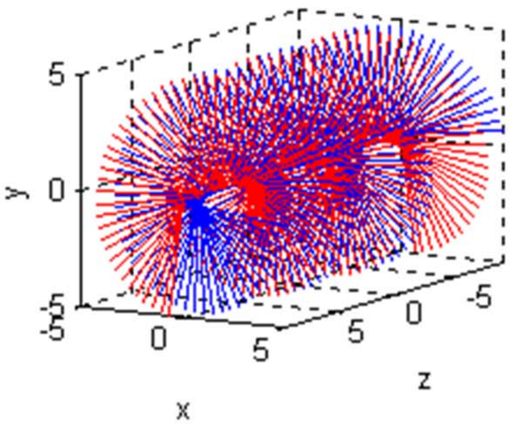
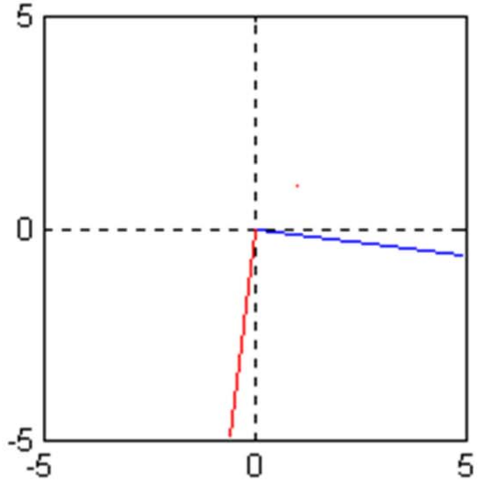
$$\overline{E} = \left(\overline{x}E_1 e^{jkz} - \overline{y}\, jE_1 e^{jkz}\right) e^{j\omega t}$$





Left-Handed Circular Polarization

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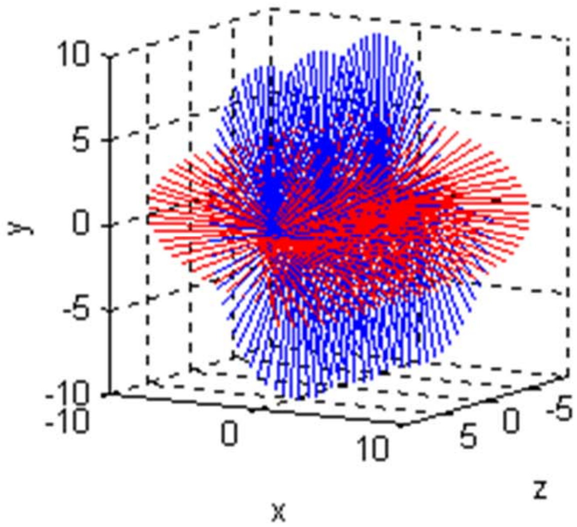
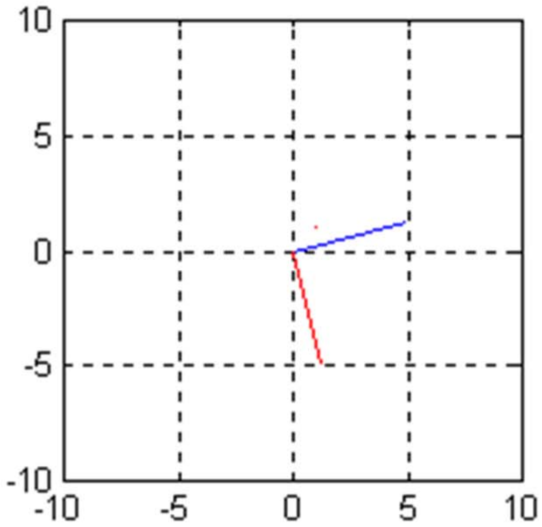
3) If $E_1 < E_2$

$$\overline{E} = \left(\overline{x}E_1 e^{jkz} + \overline{y}\,jE_2 e^{jkz}\right)e^{j\omega t}$$





Right-Handed Elliptical Polarization

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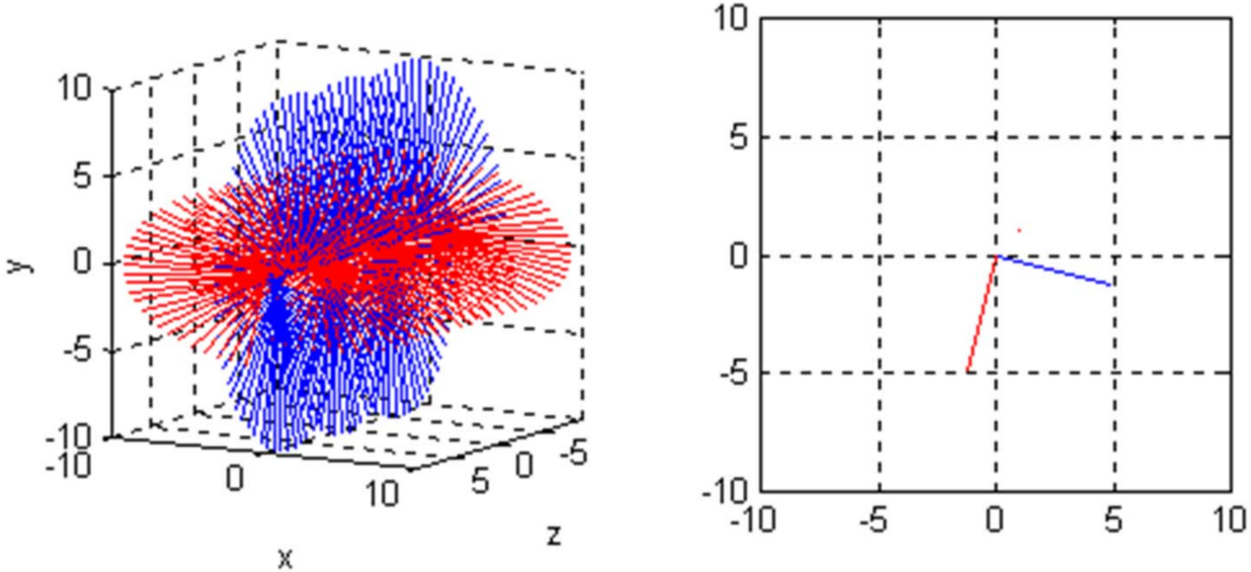
$$\overline{E} = \left(\overline{x}E_1 - \overline{y}\,jE_2\right)e^{j\omega t}e^{jkz}$$



Left-Handed Elliptical Polarization

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In general, for $\overline{E} = \left(\overline{x}E_1 + \overline{y}E_2\right)e^{j\omega t}e^{jkz}$

$$E_1 = \left|E_1\right|\exp(j\theta_1),\ E_2 = \left|E_2\right|\exp(j\theta_2)$$

If $\theta_1 = \theta_2$, linear

else if $\left(\left|E_1\right| = \left|E_2\right| \text{ and } \theta_2 - \theta_1 = \pm\pi/2\right)$, circular

otherwise elliptical

Lect. 3: Polarization

- Homework: Due on 9/11

(1) A plane EM wave in a vacuum has its E−field given as $\overline{E} = (\overline{x} + \overline{y} + \overline{z})\exp(-j\overline{k} \cdot \overline{R})$.

Determine the constraint that exists among three components of $\overline{k}$.

(2) Determine the polarization of the following EM wave.

$$\overline{E}(t, z) = \overline{x}\,2\cos(10^8 t - z/\sqrt{3}) + \overline{y}\sin(10^8 t - z/\sqrt{3})$$