Lect. 4: Light Propagation in Media

Characteristics of media: $\begin{cases} \varepsilon : \text{permittivity} \\ \mu : \text{permeability} \end{cases}$ $\begin{cases} \overline{D} = \varepsilon \overline{E} \\ \overline{B} = \mu \overline{H} \end{cases}$

- Assume $\mu = \mu_0$ in this course

- With different ε, how do plane EM waves change?

For example, $\overline{E} = \overline{x} E_0 e^{j(\omega t - kz)}$ in dielectric material

$k = \omega\sqrt{\mu_0 \varepsilon_r \varepsilon_0} = nk_0$ ($k_0 = \omega\sqrt{\mu_0 \varepsilon_0}$, $n = \sqrt{\varepsilon_r}$ refractive index)

=> changes in λ phase velosity ($v = \dfrac{\omega}{k}$)

Lect. 4: Light Propagation in Media

Medium with conductivity $\sigma \neq 0$ $\quad \overline{J} = \sigma \cdot \overline{E} \neq 0$

Maxwell's Eq.

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

$$\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t}$$

$$\nabla \bullet \overline{D} = \rho$$

$$\nabla \bullet \overline{B} = 0$$

Assuming time-harmonic solutions,

$$(E, H \sim e^{j\omega t})$$

$$\nabla \times \overline{E} = -j\omega \overline{B}$$

$$\nabla \times \overline{H} = \overline{J} + j\omega \overline{D}$$

$$\nabla \bullet \overline{D} = \rho$$

$$\nabla \bullet \overline{B} = 0$$

t ➔ ω

Maxwell's Eq. in frequency domain

Lect. 4: Light Propagation in Media

Medium with conductivity $\sigma \neq 0$ $\quad \overline{J} = \sigma \cdot \overline{E} \neq 0$

$$\nabla \times \overline{E} = -j\omega\overline{B}$$

$$\nabla \times \overline{H} = \overline{J} + j\omega\overline{D}$$

$$\nabla \bullet \overline{D} = \rho$$

$$\nabla \bullet \overline{B} = 0$$

$$\nabla \times \overline{H} = \sigma\overline{E} + j\omega\varepsilon\overline{E}$$

$$= j\omega(\varepsilon - j\frac{\sigma}{\omega})\overline{E}$$

$$= j\omega\varepsilon_c\overline{E}$$

$$\varepsilon_c \equiv \varepsilon - j\frac{\sigma}{\omega}$$

➔ Lossy medium has complex ε

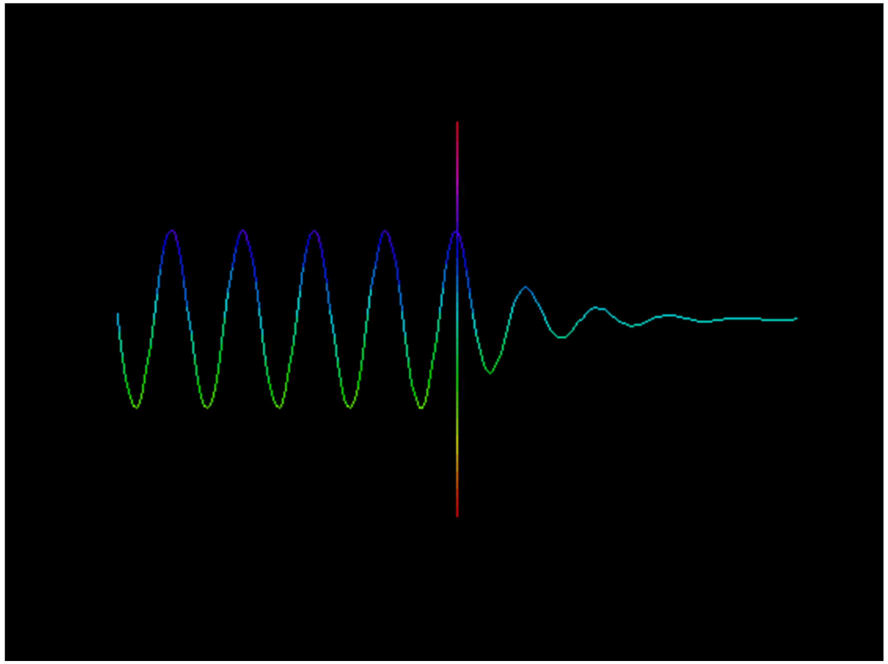
Lect. 4: Light Propagation in Media

With complex ε

$k = \omega\sqrt{\mu\varepsilon}$ $\quad$ k is also complex $\quad$ $k = \beta - j\alpha$

Consider $\quad \overline{E} = \overline{x}E_0 e^{-jkz}$ $\quad$ ($e^{j\omega t}$ is often emitted for time-harmonic solutions)

$$= \overline{x}E_0 e^{-j(\beta - j\alpha)z} = \overline{x}E_0 e^{-j\beta z} e^{-\alpha z}$$



EM waves get attenuated in conductive medium!

➔ lossy medium

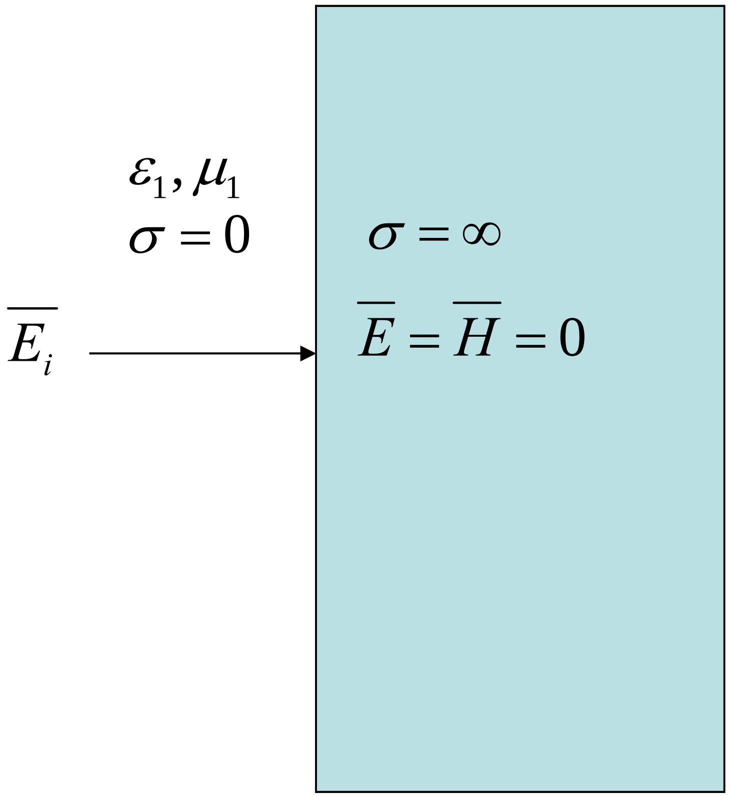
Larger σ ➔ Larger α

$1/\alpha$ penetration depth, skin depth

Lect. 4: Light Propagation in Media

What happens σ becomes infinitely large (perfect conductor)?

α becomes infinitely large ➔ No EM wave inside the conductor ➔ Reflection



Determine other fields when

$$\overline{E}_i = \overline{x} E_0 \exp(-jkz)$$

(1) $\overline{H}_i$?

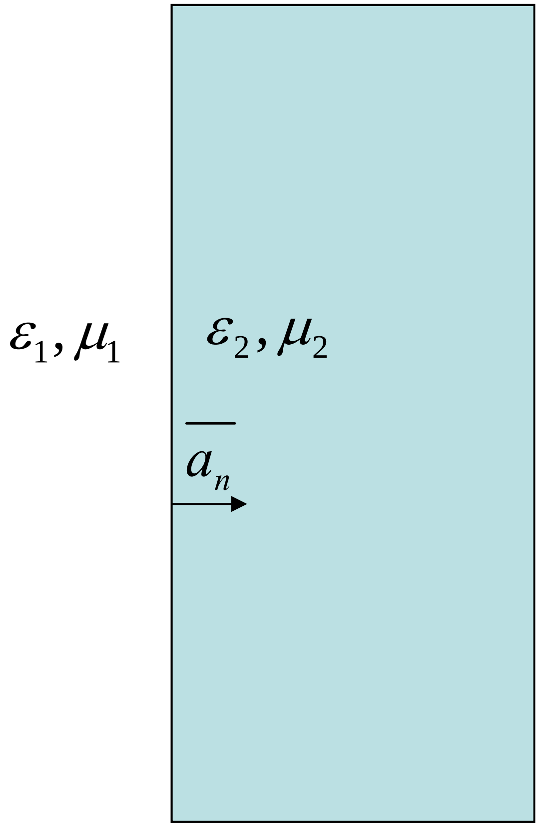
(2) $\overline{E}_r$?

(3) $\overline{H}_r$?

$$\overline{H}_i = \overline{y} \frac{E_0}{\eta} \exp(-jkz)$$

✓ Boundary Conditions: Constraints on E,H fields at a boundary

➔ Each Maxwell's Eq. provides one constraint on E or H across the boundary

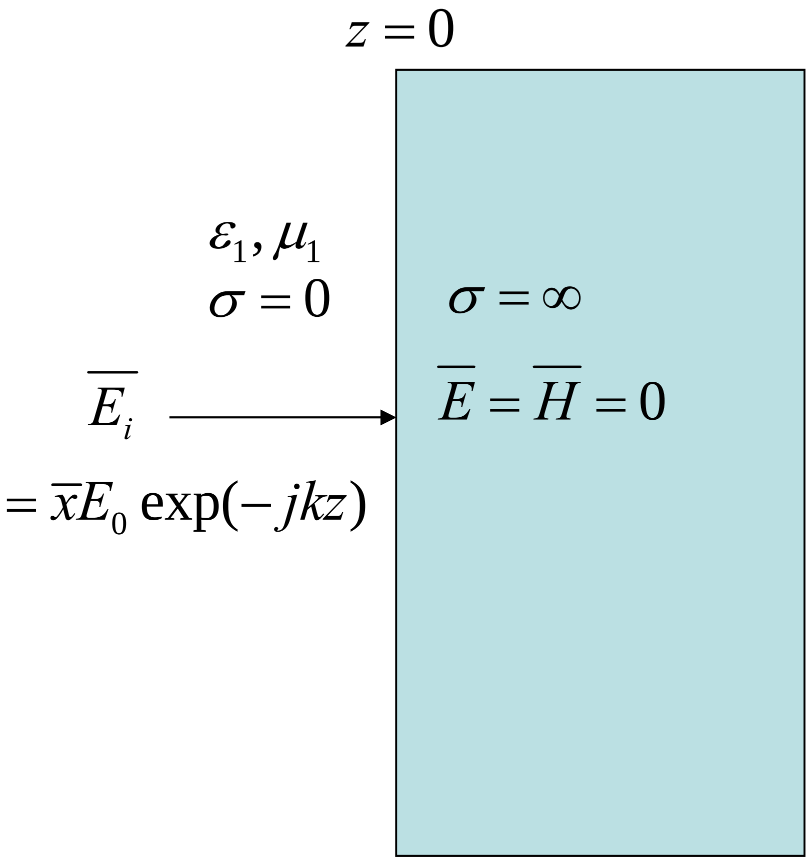


$$\nabla \cdot \overline{D} = \rho \qquad D_{2,n} - D_{1,n} = \rho_s \quad (\varepsilon_2 E_{2,n} - \varepsilon_1 E_{1,n} = \rho_s)$$

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t} \qquad E_{2,t} - E_{1,t} = 0$$

$$\nabla \cdot \overline{B} = 0 \qquad B_{2,n} - B_{1,n} = 0 \quad (\mu_2 H_{2,n} - \mu_1 H_{1,n} = 0)$$

$$\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t} \qquad \overline{a_n} \times (\overline{H}_2 - \overline{H}_1) = \overline{J}_s \quad (H_{2,t} - H_{1,t} = J_s)$$



(2) $\overline{E}_r = ?$

Apply B.C. at $z = 0$,

$$E_{2,t} - E_{1,t} = 0$$

$$\overline{E}_i(z=0) + \overline{E}_r(z=0) = 0$$

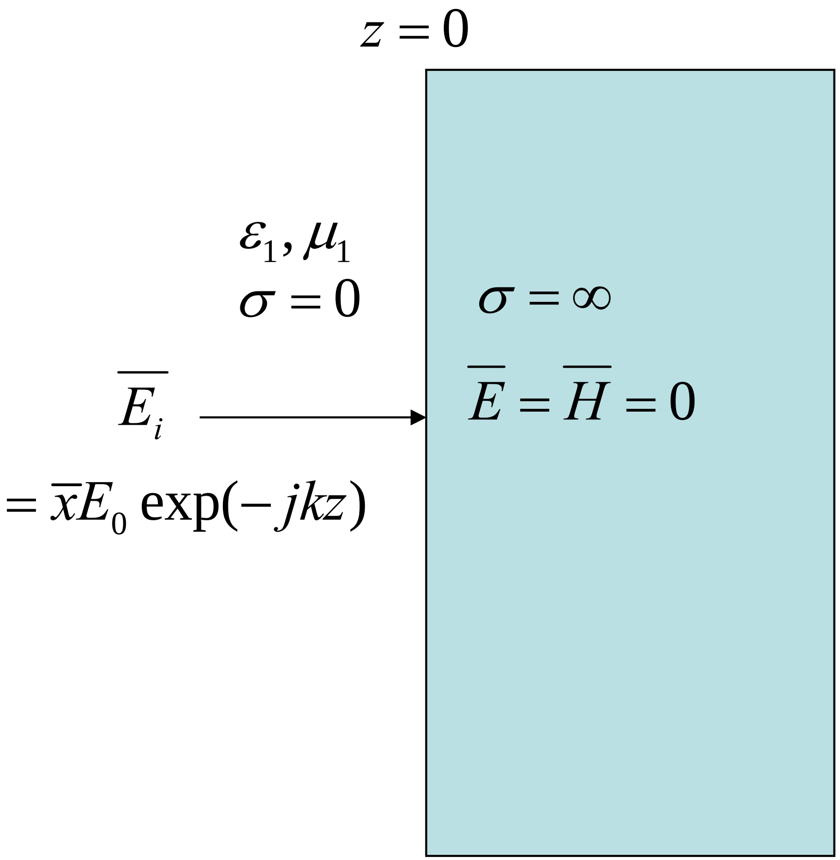
$$\therefore \overline{E}_r = -\overline{x}E_0 \exp(jkz)$$

Other BC?

$$D_{2,n} - D_{1,n} = \rho_s \qquad \varepsilon_2 E_{2,n} - \varepsilon_1 E_{1,n} = \rho_s$$

$$\rho_s = -\varepsilon_1 E_0$$

Surface charges induced for terminating normal E-field



(3) $\overline{H}_r = ?$

From $\overline{E}_r = -\overline{x} E_0 \exp(jkz)$

$$\overline{H}_r = \overline{y} \frac{E_0}{\eta_1} \exp(jkz)$$

Lect. 4: Light Propagation in Media

$z = 0$

ε_1, μ_1
$\sigma = 0$

$\sigma = \infty$

$\overline{E} = \overline{H} = 0$

$\overline{E}_i$

$= \overline{x} E_0 \exp(-jkz)$

$$\overline{H}_i = \overline{y} \frac{E_0}{\eta_1} \exp(-jkz)$$

$$\overline{H}_r = \overline{y} \frac{E_0}{\eta_1} \exp(jkz)$$

Other BC?

$$\overline{a}_n \times (\overline{H}_2 - \overline{H}_1) = \overline{J}_s$$

$$\overline{z} \times \left(0 - \overline{y} \frac{2E_0}{\eta_1} \right) = \overline{J}_s \qquad \overline{J}_s = \overline{x} \frac{2E_0}{\eta}$$

Surface currents induced for blocking tangential H-filed

✓ Expression for the total E-field for z < 0

$$\bar{E}_{total}(z) = \bar{E}_i + \bar{E}_r \;\; = \bar{x}E_0 \exp(-jkz) - \bar{x}E_0 \exp(jkz) \;\; = \bar{x}E_0(-2j)\sin(kz)$$

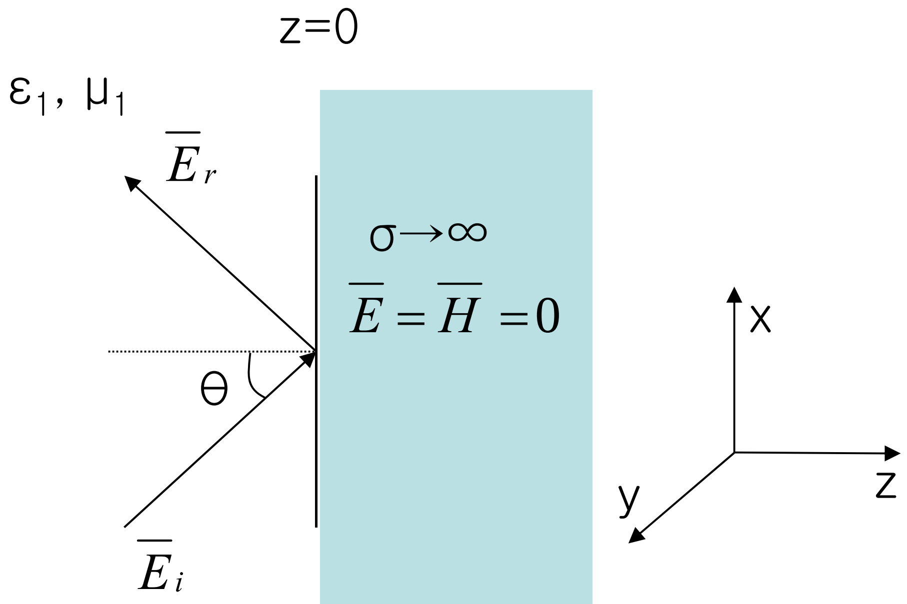
$$\bar{E}_{total}(z,t) = \bar{x}E_0(-2j)\sin(kz)\exp(j\omega t)$$

$$\mathrm{Re}\left[\bar{E}_{total}(z,t)\right] = \bar{x}E_0 2\sin(kz)\sin(\omega t)$$

➔ Standing Wave!

Lect. 4: Light Propagation in Media

Homework: Determine $\overline{E}_r, \overline{H}_i, \overline{H}_r$ when $\overline{E}_i = \overline{y} E_0 e^{-jk_x x} e^{-jk_z z}$



(See Cheng 8-7 or Lect. 11 of 2016-1 전자기2)