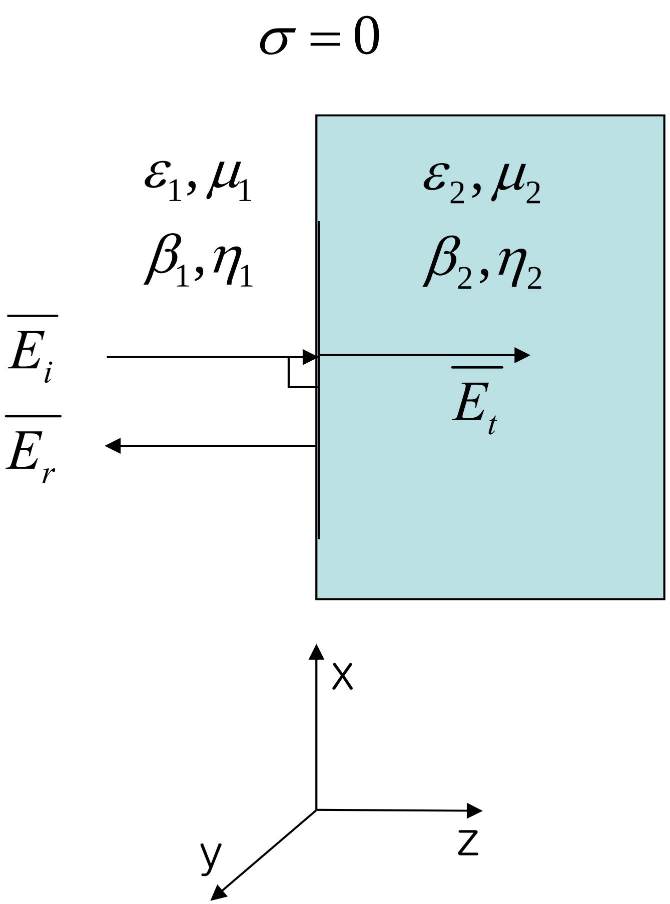
# Lect. 5: Normal Incidence at Dielectric Interface

(Cheng 8-8)




$$\overline{E}_i = \overline{y} E_0 \exp(-j\beta_1 z), \quad \overline{H}_i = -\overline{x}\frac{E_0}{\eta_1}\exp(-j\beta_1 z)$$

$$\overline{E}_r = \overline{y} E_r \exp(j\beta_1 z), \quad \overline{H}_r = \overline{x}\frac{E_r}{\eta_1}\exp(j\beta_1 z)$$

$$\overline{E}_t = \overline{y} E_t \exp(-j\beta_2 z), \quad \overline{H}_t = -\overline{x}\frac{E_t}{\eta_2}\exp(-j\beta_2 z)$$

Determine $E_r$, $E_t$

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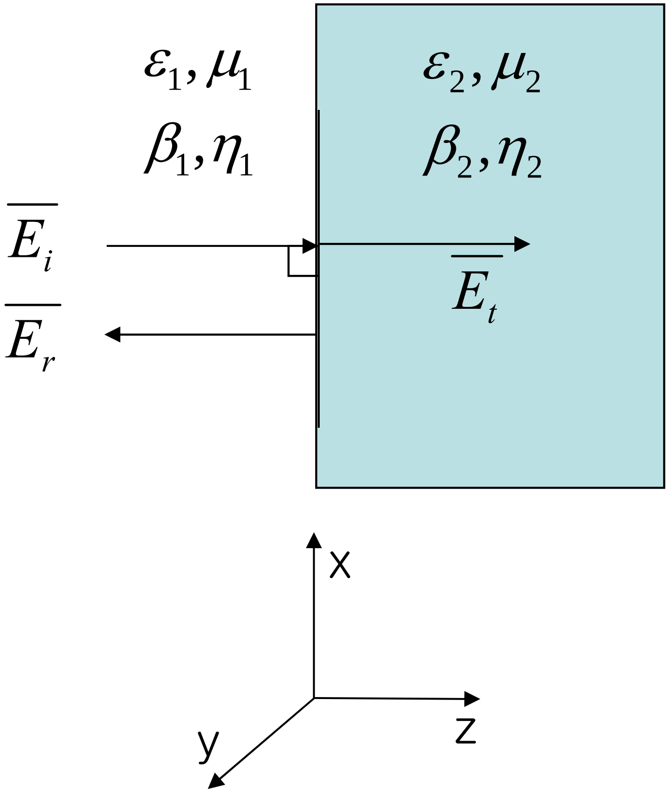


$$\overline{E}_i = \overline{y} E_0 \exp(-j\beta_1 z), \quad \overline{H}_i = -\overline{x}\frac{E_0}{\eta_1}\exp(-j\beta_1 z)$$

$$\overline{E}_r = \overline{y} E_r \exp(j\beta_1 z), \quad \overline{H}_r = \overline{x}\frac{E_r}{\eta_1}\exp(j\beta_1 z)$$

$$\overline{E}_t = \overline{y} E_t \exp(-j\beta_2 z), \quad \overline{H}_t = -\overline{x}\frac{E_t}{\eta_2}\exp(-j\beta_2 z)$$

1) $E_{tan}$ should be continuous at z=0

$$E_0 + E_r = E_t$$

Define $E_r = \Gamma E_0$, and $E_t = \tau E_0$

| |
|---|
| $\Gamma$: reflection coef. |
| $\tau$ : transmission coef. |

$\Rightarrow \quad 1 + \Gamma = \tau$

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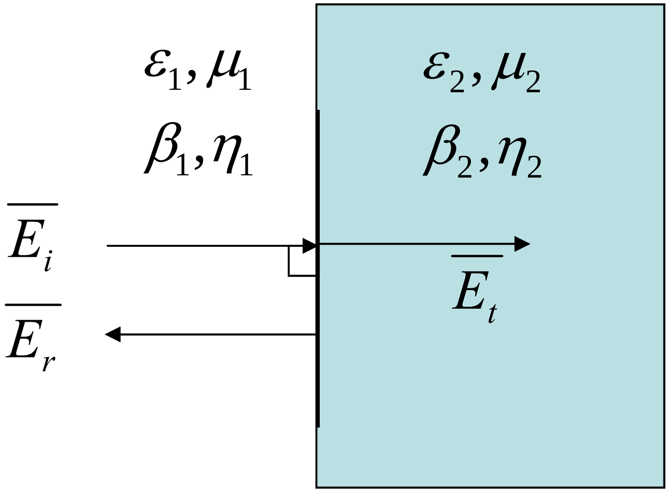


$$\overline{H}_i = -\overline{x}\frac{E_0}{\eta_1}\exp(-j\beta_1 z)$$

$$\overline{H}_r = \overline{x}\frac{E_r}{\eta_1}\exp(j\beta_1 z)$$

$$\overline{H}_t = -\overline{x}\frac{E_t}{\eta_2}\exp(-j\beta_2 z)$$

2) $H_{tan}$ should be continuous at z=0

$$\overline{a}_n \times (\overline{H}_2 - \overline{H}_1) = \overline{J}_s \;\; = 0$$

$$H_i + H_r = H_t$$

$$-\frac{E_0}{\eta_1} + \frac{E_r}{\eta_1} = -\frac{E_t}{\eta_2}$$

$$\therefore -\frac{1}{\eta_1} + \frac{\Gamma}{\eta_1} = -\frac{\tau}{\eta_2}, \qquad 1 + \Gamma = \tau$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}, \quad \tau = \frac{2\eta_2}{\eta_2 + \eta_1}$$

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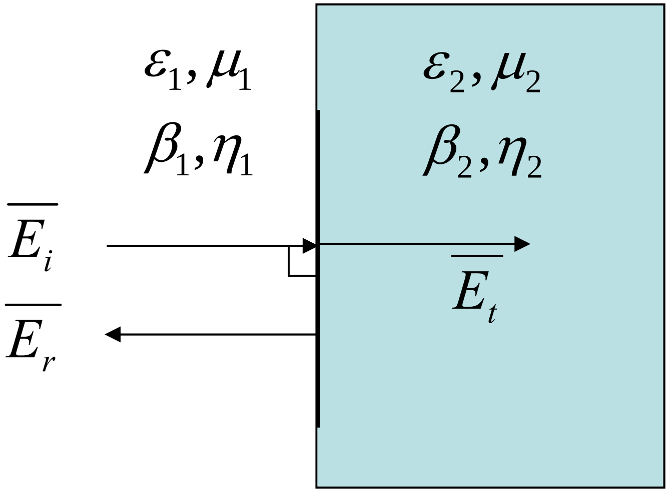


$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}, \quad \tau = \frac{2\eta_2}{\eta_2 + \eta_1}$$

Max. $\Gamma$: 1, $\eta_2 >> \eta_1$

$\varepsilon_2 << \varepsilon_1$ (Assuming $\mu_1 = \mu_2$)

$\tau$: 2

Min. $\Gamma$: -1, $\eta_2 << \eta_1$

$\varepsilon_2 >> \varepsilon_1$ (Assuming $\mu_1 = \mu_2$)

$\tau$: 0

➔ Same as the case for EM wave incident on a perfect conductor

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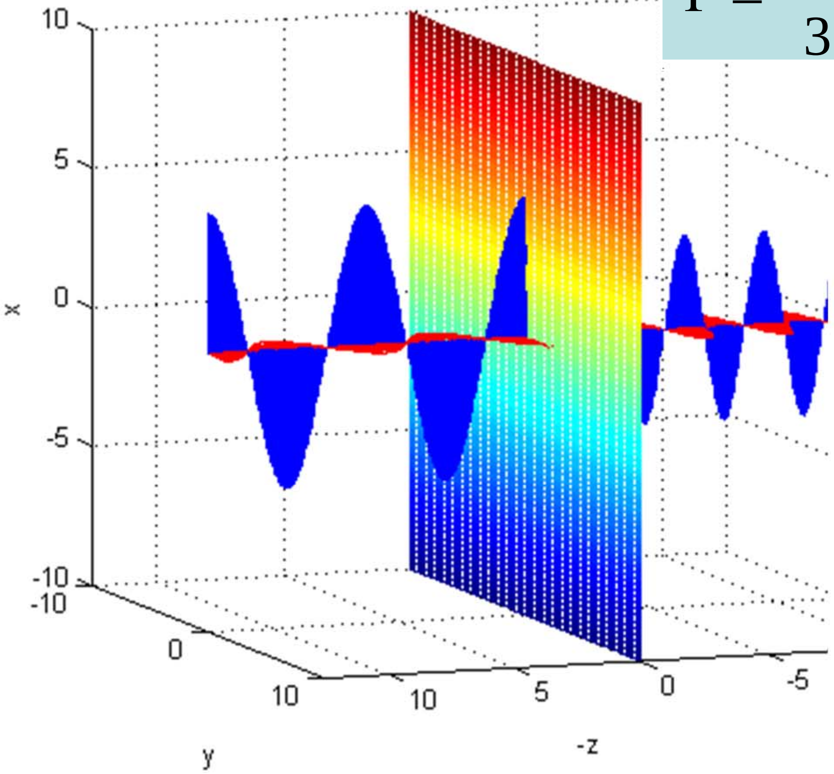
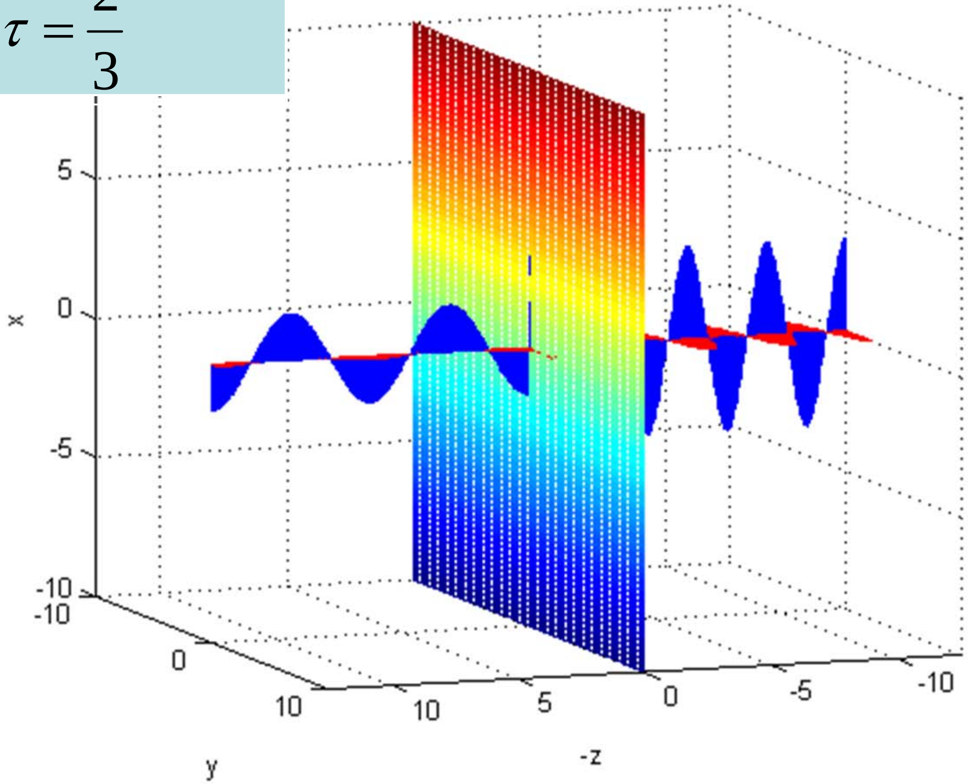
$$4\varepsilon_1 = \varepsilon_2 \quad \text{and} \quad \mu_1 = \mu_2$$

$$\Gamma = -\frac{1}{3}, \;\; \tau = \frac{2}{3}$$




Incident and Transmitted waves




Reflected and Transmitted waves

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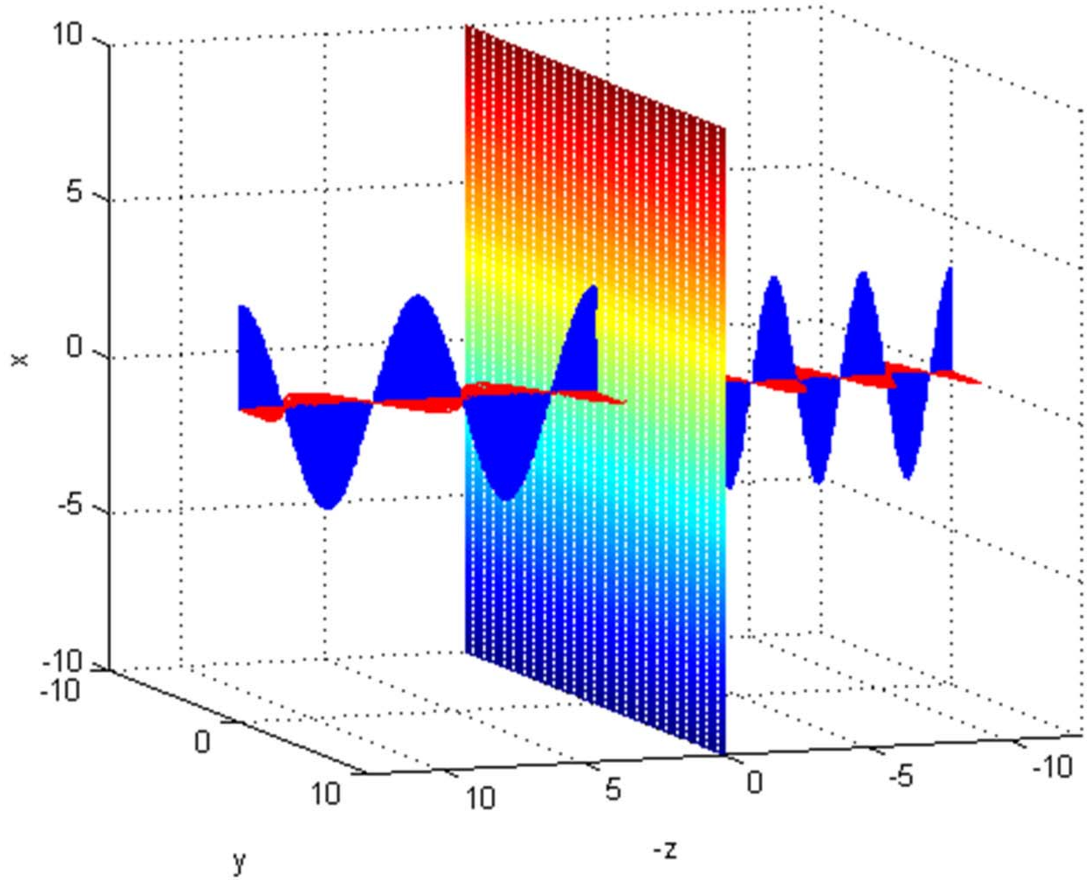


Total waves

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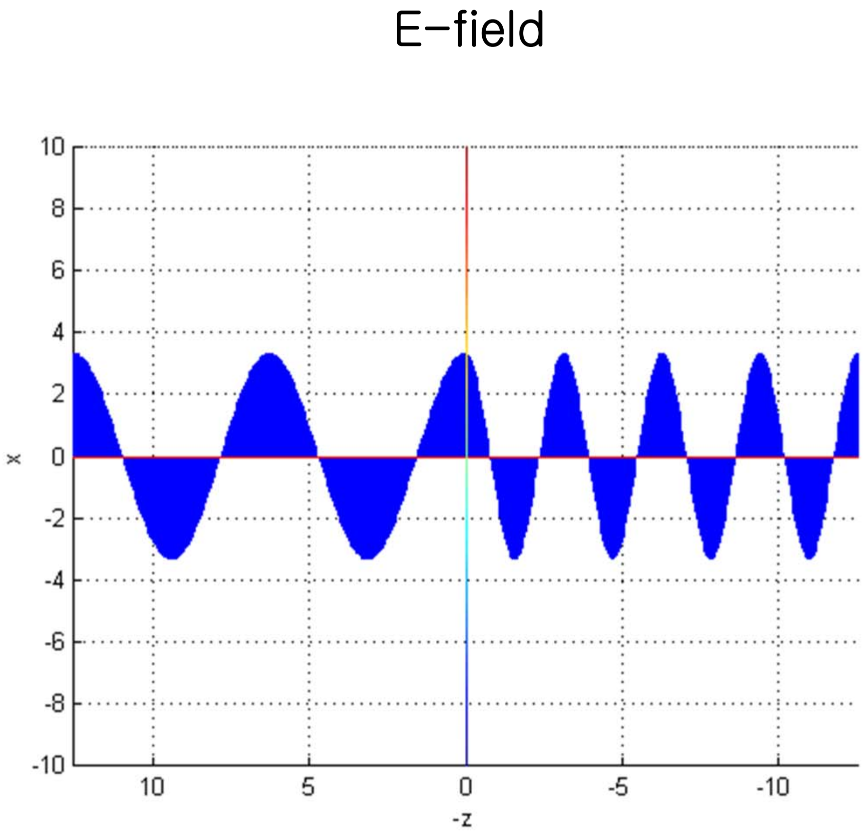


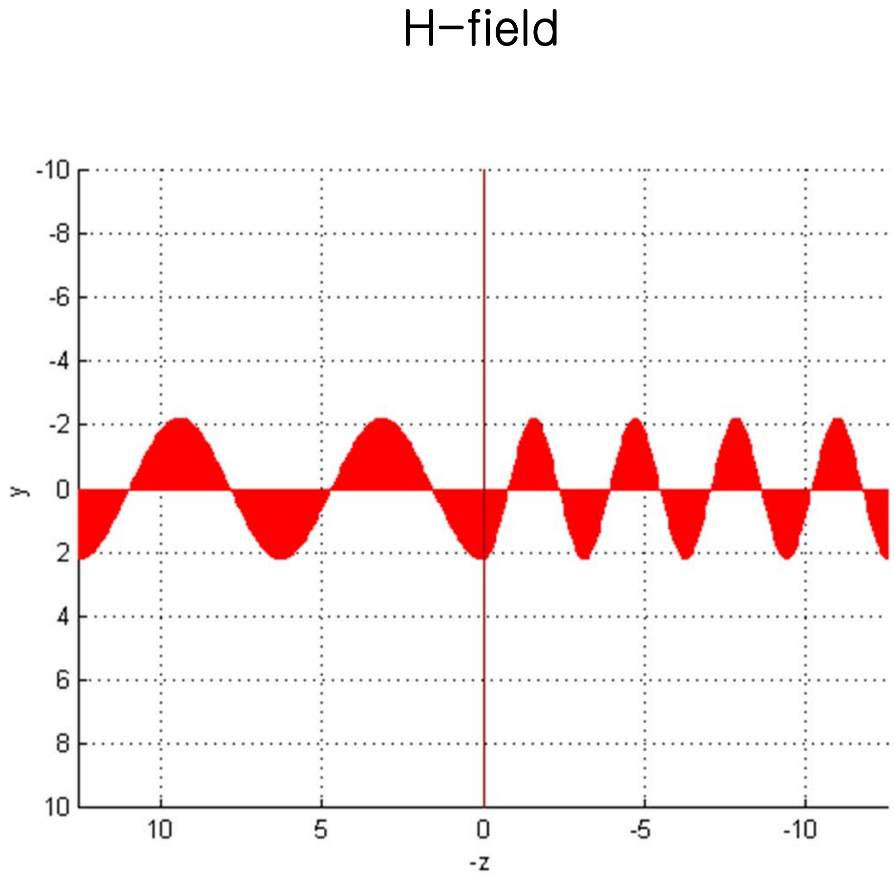

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Total E-field for z<0

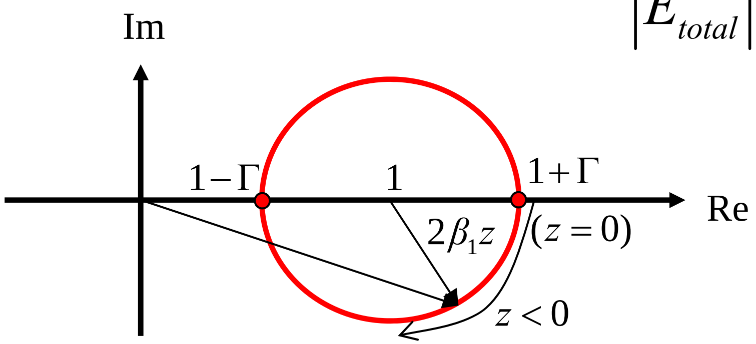
$$\overline{E}_{total}(z<0)=\overline{E}_i+\overline{E}_r=\overline{y}E_0(e^{-j\beta_1 z}+\Gamma e^{j\beta_1 z})$$

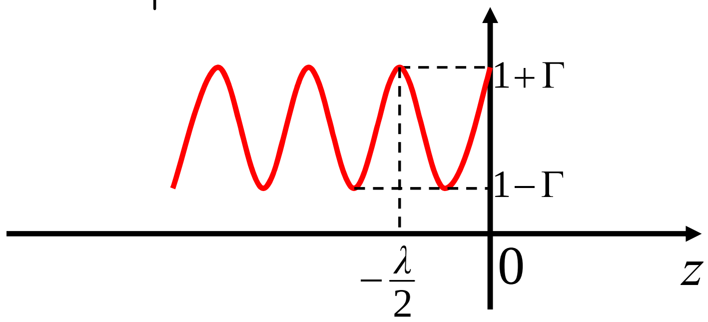
$$=\overline{y}E_0\left\{(1+\Gamma)e^{-j\beta_1 z}+\Gamma(e^{j\beta_1 z}-e^{-j\beta_1 z})\right\}$$

transmitting wave — standing wave

$$\overline{E}_{total}(z<0)=\overline{E}_i+\overline{E}_r=\overline{y}E_0(e^{-j\beta_1 z}+\Gamma e^{j\beta_1 z})=\overline{y}E_0 e^{-j\beta_1 z}(1+\Gamma e^{j2\beta_1 z})$$

$$\left|\overline{E}_{total}\right|=E_0\left|1+\Gamma e^{j2\beta_1 z}\right|$$



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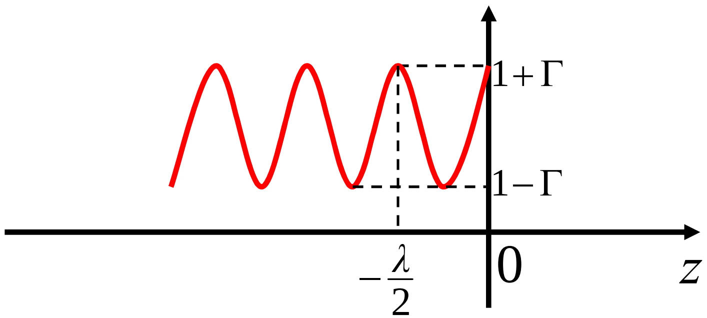


✓ Define standing wave ratio

$$S = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$|\Gamma| = 1 \Rightarrow S \rightarrow \infty$$

$$\Gamma = 0 \Rightarrow S = 1$$

$$1 \leq S < \infty$$

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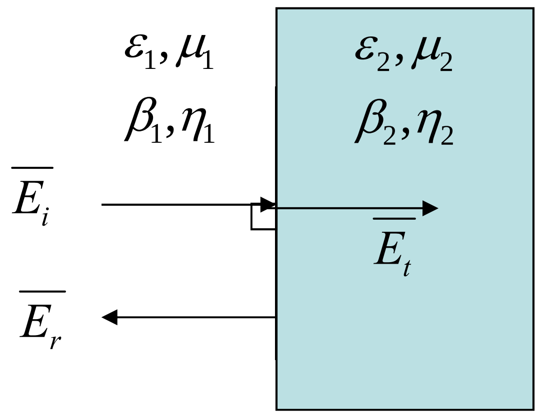


$$\overline{E}_i = \overline{y} E_0 \exp(-j\beta_1 z), \quad \overline{H}_i = -\overline{x} \frac{E_0}{\eta_1} \exp(-j\beta_1 z)$$

$$\overline{E}_r = \overline{y} \Gamma E_0 \exp(j\beta_1 z), \quad \overline{H}_r = \overline{x} \frac{\Gamma E_0}{\eta_1} \exp(j\beta_1 z)$$

$$\overline{E}_t = \overline{y} \tau E_0 \exp(-j\beta_2 z), \quad \overline{H}_t = -\overline{x} \frac{\tau E_0}{\eta_2} \exp(-j\beta_2 z)$$

Average power density propagation $\left(\overline{P}_{av} = \frac{1}{2} \mathrm{Re}\left[\overline{E} \times \overline{H}^*\right]\right)$

In medium 2,

$$\overline{P}_{av} = \overline{z} \frac{{E_0}^2}{2\eta_2} \tau^2$$

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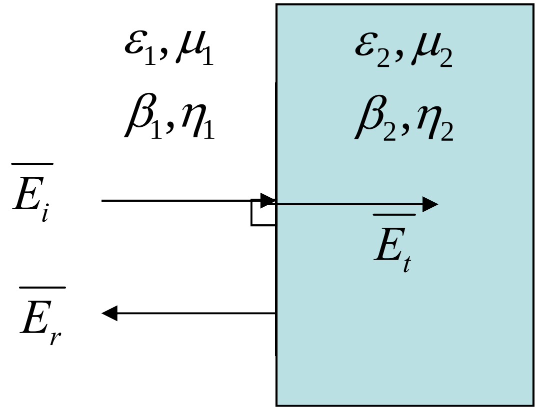


$$\overline{E}_i = \overline{y} E_0 \exp(-j\beta_1 z), \quad \overline{H}_i = -\overline{x}\frac{E_0}{\eta_1}\exp(-j\beta_1 z)$$

$$\overline{E}_r = \overline{y}\Gamma E_0 \exp(j\beta_1 z), \quad \overline{H}_r = \overline{x}\frac{\Gamma E_0}{\eta_1}\exp(j\beta_1 z)$$

$$\overline{E}_t = \overline{y}\tau E_0 \exp(-j\beta_2 z), \quad \overline{H}_t = -\overline{x}\frac{\tau E_0}{\eta_2}\exp(-j\beta_2 z)$$

Average power density propagation $\left(\overline{P}_{av} = \frac{1}{2}\mathrm{Re}\left[\overline{E}\times\overline{H}^*\right]\right)$

In medium 2,

$$\overline{P}_{av} = \overline{z}\frac{E_0^{\ 2}}{2\eta_2}\tau^2$$

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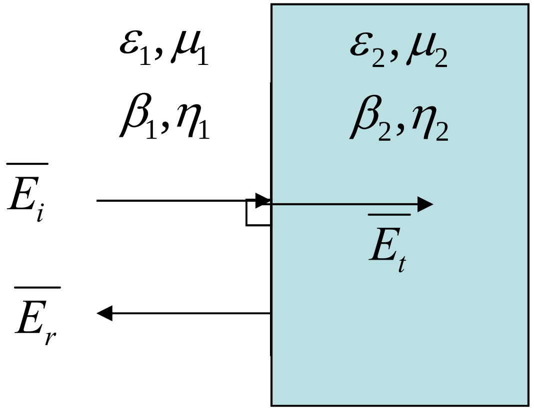


Homework:
Determine average power density propagation in medium 1 and show it is same as that of medium 2.

$$(\overline{P}_{av} = \frac{1}{2}\mathrm{Re}\left[\overline{E} \times \overline{H}^*\right])$$